Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Previous)
Paper Code:MT-02
Real Analysis and Topology
Section – A
(Very Short Answers Questions)

1. Define a denumerable set.
A. A set S is said to be a denumerable set if it is equipotent to the set N of natural numbers.
2. Define an algebra of sets.
A. A non-void collections & of subsets of a set X is said to be an algebra of sets if & is closed under finite intersections and complementation.
3. The outer measure of the set of natural number is :
A. Zero
4. Define measurable sets.
A. A set E is said to beLebesque measurable or simply measurable if for each set $A \subset R$,
$$m^*(A) = m^*(A \cap E) + m^*(A \cap E^c)$$
5. If E_1 *and* E_2 are two measurable sets, then,
$m^*(E_1 \cup E_2) + m^*(E_1 \cap E_2) =$
A. $m^*(E_1) + m^*(E_2) =$
6. What is measurable function?
A. An extended real valued function f on a measurable set E is said to be Lebesgue measurable on E if the set $\{x \in E : f(x) > x\}$ is a measurable set for each $x \in R$.
7. What is step function?
A. A real valued function f defined on an interval (a, b) is said to be a step function if there exists a partition $a = x_0 < x_1 < x_2 < \cdots \ldots\ldots < x_n = b$ such that the function f assumes one and only one value in each sub interval.
8. Define uniform convergence.
A. Let $< f_n >$ be a sequence of measurable functions defined on measurable set. Then $< f_n >$ is said to converge pointwise to a measurable function f on E if for any $\in > 0$ *and* $x \in E, \exists\, n_0 \in N$ s.t.
$$n > n_0 \implies |f_n(x) - f(x)| < \in$$
9. What is Bernstein polynomial?
A. Let f be a finite function defined on the closed interval [0, 1]. The polynomial $B_n(x) = \sum_{k=0}^{n} f\left(\frac{k}{n}\right) C_k^n\, x^k (1-x)^{n-k}$
is called the Bernstein polynomial of degree for the function f.

10. What is Lebesgue integral function.

A. Let f be a bounded function defined on a measurable set E, then f is said to be Lebsgue integrable on $E = [a,b] if\ L \underline{\int_a^b} f(x)dx = L \overline{\int_a^b} f(x)dx$

11. Let $< f_n >$ be a sequence of measurable functions defined on a measurable set E and $\lim_{n\to\infty} f_n(x) = f(x) a.e. on\ E$ then

A. f is measurable on E.

12. Let $f(x) = \frac{1}{\sqrt{x}}\ fo\ x \in [0,1] and\ f(0) = 0$. Then define $[f(x)]_n$

A. $[f(x)]_n = \begin{cases} \frac{1}{\sqrt{x}} , when\ \frac{1}{n^2} \le x \le 1 \\ n , when\ 0 \le x \le \frac{1}{n^2} \\ 0, \quad when \quad x = 0 \end{cases}$

13. Define square summable function.

A. A measurable function f defined on a measurable set $E \subset R$ is said to be square summable function of :

$$\int_E f^2(x)dx\ exist\ finitely$$

14. Define Cauchy sequence in L_2.

A. A sequence $< f_n >\ in\ L_2$ is a Cauchy sequence is L_2 if for each $\in <$ $0, \exists\ n_0(\in) \in N\ s.t. m,n \ge n_0 \implies \|f_m - f_n\| < \in$.

15. Define scalar product in L_2.

A. The scalar product of two functions $f \in L_2, f \in L_2\ denoted\ as < f,g >$ is defined as $< f,g > = \int_a^b f(x)g(x)dx$

16. What is closed orthonormal system?

A. An orthonormal system $\{\phi_i\}$ is said to be closed if it satisfies Parseval's identity $\sum_{i=1}^{\infty} a_i^2 = \|f\|^2$ for the function f, where ai are fourier coefficients for f with respect to ϕ_i.

17. The inequality $\sum_{i=1}^{\infty} a_i^2 \le \|f\|^2$ is known as:

A. Bessel's inequality.

18. Define convergence in norm in L^p space.

A. Let $< f_n >$ be sequence of functions in $L^p, 1 \le p, \infty. Then < f_n >$ is said to converge in norm to $f \in L^p$ if for each $\in > 0$, there exist $n_0 \in N$ such that $\|f_n - f\|\ p < e, \quad \forall\ n \ge n_0$

19. State Minkowski's inequality.

A. $\|f + g\|_p \le \|f\|_p + \|g\|_p \quad ; \ f,g \in L^p, p > 1$

20. Define co-countable topology.

A. Let X be a non-void set. Let J be the family consisting of ϕ and all those non-void subsets of X, where complements are countable. Then J is a topology, known as co-countable topology.

21. What is comparable topologies?

A. Two topologies $J_1\ and\ J_2$ on a non-void set X are comparable of either $J_1 \subset J_2\ or\ J_2 \subset J_1$.

22. What is closed set?

A. A subset F of a topological space {X, J} is closed if its complement F^c ins open.

23. Define neighbourhood.

A. Let (X, J) be a topological space and $x \in X.$ A subset N of X is said to be a neighbourhood of x if ∃ an open set G such that $X \in G \subset N.$

24. Define Hereditary property.

A. A property of a topological space is said to be a hereditary property, if it is satisfied by every sub space of the given space.

25. Define base for a topology.

A. Let (X, J) be a topology. A sub family B of J is said to form a base for J, if for each open set G and each $x \in G$ ∃ a member B of B such that $x \in B \subset G.$

26. What is first countable?

A. A topological space (X, J) is said to be first countable, if each point of X possesses a countable local base.

27. What is an open mapping?

A. If (X, J) and (γ, ξ) be topological spaces and $f : X \rightarrow Y$ then f is said to be an open iff the image of each open set is open.

28. If $f : (X, J) \rightarrow (\gamma, \xi)$ be one one onto such that: $\overline{f(A)} = f\,\overline{(A)}, for\ any\ A \subset X$ then.

A. F is a homeomorphism.

29. Define topological property.

A. A property which when satisfied by a topological space is also satisfied by every homeomorphic image of this space, is called a topological property.

30. Define second countable space.

A. A topological space (X, J) is said to be second countable space if there exist a countable base for J.

31. Define T_1- space.

A. A topological space {X, J} is said to be a T_1- space iff for any two distinct points $x, y \in X, \exists\ open\ set\ G\ and\ H\ s.t. x\ \ G, y \notin G\ and\ y \in\ H, x \notin H.$

32. Define T_2- space.

A. A topological space {X, J} is said to be a T_2- space iff for any two distinct points $x, y \in X, \exists\ G, H \in J\ s.t. x \in G, y \in H, G \cap H = \phi.$

33. Every singletion set in T_2- space is:

A. Closed

34. What is a regular space?

A. A toplogical space (X, J) is said to be a regular space iff for every closed subset F of X and each $x\ (\notin F) \in X\ , ther$ ∃ open sets G and H s.t. $F \subset G,$ $x \in H\ ang\ G \cap H = \emptyset.$

35. What is a T_2- space?

A. A normal T_1- space is called T_4- space.

36. What is an open cover of a set?

A. Let A be a subset of a topological space. A collection $C = \{G_k\}$ of open subsets X is aid to be an open cover of A if $A \subset U_\alpha\ G_\alpha$

37. Define a compact space.

A. A topological space (X, J) is called compact iff every open cover of X is reducible to a finite sub-cover.

38. Define locally compact space.

A. A topological space (X, J) is said to be locally compact iff every point in X has at least one neighbourhood whose closure is compact.

39. What is Bolazano weirstrass property?

A. Every infinite subset of the space has a limit point.

40. What is finite intersection property?

A. A family J of sets is said to have the finite intersection property if every finite sub family of J has a non-void intersection.

41. The one point compactification of the interval (0, 1) is :

A. [0, 1]

42. The one point compactification of the set complex numbers is called:

A. Extended complex plane.

43. Let $Y = X\ \cup \{\infty\}$ be the one point compactification of X then $\overline{X}\ =$

A. Y

44. If (X_∞, T_∞) be the one point compactification of a topological space (X, J) and X is Hausdarff and locally compact the X_∞ :

A. Is a Hausdraff space

45. What is separated sets?

A. Let (X, J) be a topological space. Two non-void subsets A and B of X are said to be separated iff $A\ \cap\ \overline{B}\ =\ \emptyset\ and\ \overline{A}\ \cap B = \emptyset$.

46. What is a disconnected set ?

A. A subset E of a topological space (X, J) is said to be disconnected off ∃ two non-void seoarates sets A and B s.t. $E = A \cup B$.

47. If $X = \{1, 2, 3\}\ and\ J = \{\emptyset, \{1\}, \{2, 3\}, X\} then\ X\ is$:

A. Disconnecter as {1} is a proper subset of X which is both open and closed.

48. Define locally connected space at a point.

A. A topological space {X, J} is said to be locally connecter at a point $x\ \in X$ iff every open set containing x contains a connecter open set containing x.

49. What is a disconnection?

A. A subset γ of a topological space (X, J) is called disconnecter iff ∃ two non-void sets G and H open in X s.t.

$$\gamma\ \cap G =\ \emptyset\ , \qquad \gamma\ \cap H = \emptyset\ ,\ \ \gamma \cap (G \cap H) = \emptyset\ and\ \gamma \cap (G \cup H) = \gamma$$

In such case we say that $G \cup H$ forms a disconnection.

50. Define a component?

A. A maximal connection subspace C of a topological space (X, J) is called a component of X.

51. Define product topology.

A. Let (X, J) and (γ , ξ) be any two topological spaces. Then the topology P for $X \times Y$ whose base is the collection $B = \{G \times H: G \in J, H \in \xi\}$ is called a product topology.

52. What is the projection mappings?

A. The mappings $\pi_x: X \times \gamma \rightarrow X\ s.t.\ \pi_x\ (x,y) = x, \forall\ (x,y), \in X \times \gamma$ and $\pi_y: X \times \gamma \rightarrow \gamma\ s.t.\ \pi_y(x,y) = y,\ \forall\ (x,y) \in X \times \gamma$ are called the projection mappings or projections of $X \times \gamma\ om\ X\ and\ \gamma$ respectively.

53. Define finitely short family.

A. Let (X, J) be a topological space then a collection C of substes of X is said to be short iff C does not cover X. Also C is said to be finitely short iff no finite sub family of C covers X.

54. What is a Quotient space?

A. Let (X, J) be a topological space and Let R be an equivalence relation on X. Let $\pi: X \rightarrow \frac{X}{R} s.t. \pi(x) = [x], \forall\ x \in X$ then, the family X/R with quotient topology is called the quotient space.

55. What is Embedding?

A. A mapping $f : X \rightarrow Y$ which defines a homeomorphism of X onto f(x) is said to be an embedding of a space X into another space Y.

56. Define a residual set.

A. Let $(A, \geq)$ be a directed set and Let $E \subset A$. Then E is said to be a residual subset of A iff $\exists\ n \in A\ s.t.\ m \geq n \Rightarrow m \in E$.

57. Define eventually net.

A. Let $(f, X, A, \geq)$ be a net in X and Let $Y \subset X$. Then the net f is said to be eventually in Y iff $\exists$ a residual subset E of A s.t. $f(E) \subset Y, ie. iff \exists\ n \in A\ s.t. \forall\ m \in A, m \geq n \Rightarrow f_a \in Y$.

58. What is Ultranet?

A. Let $(f, X, A, \geq)$ be in a set X is said to be an ultranet or universal net iff for each subset y of X, the net is eventually in Y or eventually in Y^c.

59. What is filter?

A. Let X be a non-void set, then a non-void family J of subsets of X is called a filter on X iff the following axions are satisfied :

(i) $\emptyset \notin J$

(ii) If $F \in J\ amd\ H \supset F, then\ H \in J$

(iii) If $F_1 \in J, F_2 \in J, then\ F_1 \cap F_2 \in J$

60. Define subbase of a filter.

A. Let J be a filter on X, then a sub family B^* of J is called a subbase of J iff the family of all finite ntersections of membered of B^* is a base for J.

61. Describe adherent point of a filter.

A. Let J be a filter on a topological space (X, J). A point $x \in X$ is called an adherent point if J iff J is frequently in every $nbd\ of\ x$.

62. Define a σ-ring.

A. A ring R is said to be a σ-ring if R is closed under countable unions and intersections of the collections of subsets in R.

63. A ring R an algebra of sunsets of X iff:

A. $X \in R$

64. Every super set of an uncountable set is :

A. Uncountable

65. The outer measure of an interval is:

A. Its length

66. If E_1 and E_2 are measurable sets then $m^*(E_1) - m^*(E_2) =$

A. $m^*(E_1 - E_2)$

67. If $< f_n >$ is a convergent sequence of measurable functions defined on a measurable set E, then the limit function of $< f_n >$ is:

A. Measurable

68. If $f : R \rightarrow R$ is measurable and $g : R \rightarrow R$ is continuous then gof is :

A. Measurabe.

69. Define partition of a measurable set.

A. Let E be a measurable set. Then a finite collection $P = \{E_1, E_2, \ldots \ldots, E_n\}$ of measurable subsets of E, where $E = \cup_{i=1}^{n} E_i$ and $E_i \cap E_j = \emptyset, \forall i, j, i \neq j$ is said to be a measurable partition of E.

70. If f is a measurable function on the measurable set E. Then U (f, P) and L (f, P) can be defined only when :

A. F is bounded on E.

71. If f is Lebsegue integrable then $-\int f(-x)dx =$

A. $\int f(x)dx$

72. L_2 space is not a Banach space. It is true or false.

A. False.