

VALLIAMMAI ENGINEERING COLLEGE
DEPARTMENT OF MATHEMATICS
SUB CODE/ TITLE:MA7169- ADVANCED NUMERICAL METHOD
QUESTION BANK
M.ECAD/CAM

UNIT- I (ALGEBRAIC EQUATIONS)

PART A

1. Explain Gauss – Elimination method?
2. Solve the system of equations by Gauss elimination method $11x + 3y = 17$, $2x + 7y = 16$.
3. What are the advantages of Gauss Seidel method over Jacobi method?
4. What is the limitation of power method?
5. State the two differences between direct and iterative methods for solving system of equations?
6. Explain the term pivoting?
7. Define partial pivoting?
8. Define Complete Pivoting?
9. Define round off error?
10. State the principle used in Gauss- Jordan method?
11. For solving a linear system, compare Gauss elimination method and Gauss Jordan method?
12. Define tridiagonal Matrix?
13. Gauss – Seidel method is better than Gauss Jacobi method. Why?
14. Write down formula for SOR iterative method?
15. Write the Iterative Formula of Newton – Raphson method?
16. Obtain an iterative formula to find the square root of a natural number N using Newton's method.
17. What is the rate of convergence in NR method?
18. If $g(x)$ is continuous in $[a,b]$ then under what condition the iterative method $x = g(x)$ has a unique solution?
19. What type of eigen value can be obtained using power method?
20. If the eigen values of A are -3,3,1 then the dominant eigen value of A is ----- ?
21. How will you find the smallest eigen value of a square matrix A?
22. Determine the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ correct to two decimal places using power method?
23. Explain power method to find the dominant eigen value of a square matrix?
24. Write down formula for the Faddeev – Levertier method?

PART B

1. Solve using Gauss Elimination

$$2x_1 + x_2 = 1$$

$$x_1 + 2x_2 + x_3 = 2$$

$$x_2 + x_3 = 4$$

2. Solve using gauss – Jordan elimination

$$x - y + 2z = -8$$

$$x + y + z = -2$$

$$2x - 2y + 3z = -20$$

3. Solving the system of equation

$$4x_1 + x_2 + x_3 = 2$$

$$x_1 + 5x_2 + 2x_3 = -6$$

$$x_1 + 2x_2 + 3x_3 = -4$$

using jacobi method

4. Using SOR method solve

$$2x_1 - x_2 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$-x_2 + 2x_3 = 1 \quad \text{with } W = 1.17$$

5. Solve the system of equation

$$4x + 2y + z = 14$$

$$x + 5y - z = 10$$

$$x + y + 8z = 10$$

using Gauss – Seidal iteration method

6. Find a real of equation $x^3 + x^2 - 1 = 0$ by iteration

7. Solve

$$\text{i } 3x_1 + 2x_2 = 12$$

$$2x_1 + 3x_2 + 2x_3 = 17$$

$$2x_2 + 3x_3 + 2x_4 = 14$$

$$2x_3 + 3x_4 = 7$$

$$\text{ii. } 2.04x_1 - 2x_2 = 40.8$$

$$-x_1 + 2.04x_2 - x_3 = 0.8$$

$$-x_2 + 2.04x_3 - x_4 = 0.8$$

$$-x_3 + 2.04x_4 = 200.8$$

Using Thomas algorithm

8. Find the root of $x^4 = x + 10$, correct to three decimal places using NM
9. Find the largest eigen value in modulus and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$

10. Find the smallest eigen value in magnitude of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Using inverse power method

11. Obtain the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 0 & 2 & 3 \\ -10 & -1 & 2 \\ -2 & 4 & 7 \end{bmatrix}$$

Using Faddev – Leverrier method.

12. Use Faddeev’s method to find the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix} \text{ and hence find its inverse.}$$

13. Solve the system of equations $x+y+z=6, 3x+3y+4z=20, 2x+y+3z=13$. Using the Gauss elimination method.

14. Find the smallest eigenvalue in magnitude of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ using four iterations of the inverse power method.

15. Find a zero of the system of non linear equation $x_1^3 + 10x_1 - x_2 - 5 = 0, x_2^3 + x_1 - 10x_2 + 1 = 0$. by fixed point iteration method.

16. Solve the trididagonal system by Thomas method $2x-y=1, -x+2y-z=0, -y+2z-w=0, z+2w=1$.

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SUB CODE/ TITLE:MA7169- ADVANCED NUMERICAL METHOD
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UNIT II(ORDINARY DIFFERENTIAL EQUATIONS)

PART A

1. What is single step methods
2. Write down the Runge-Kutta formula of fourth order .
3. When a single step method is applied to the test equation $u' = \lambda u$, what are the conditions for absolutely stable and relatively stable?
4. When will you say that a system of ODE of the form $u' = f(t, u_1, u_2, u_3, \dots, u_m), u(t_0) = \eta_0$ is stiff?
5. State the special advantage of Runge-kutta method.
6. Write down Adams-Bashforth predictor formula.
7. Write down Adams-Bashforth corrector formula.
8. Define Stiff
9. Give example of Stiff ODE equation.
10. What is IVP
11. What is multistep methods
12. What is BVP
13. What is meant by Stability analysis.
14. Define Shooting Method.
15. What are the steps involved in the shooting method?
16. Write first four derivatives of finite difference method
17. Define collocation
18. What is meant by orthogonal collocation.
19. Define FEM.
20. What is Global Matrix.
21. What is meant by Galerkin method
22. Define orthogonal collocation in Galerkin Method.
23. Write down Uses of FEM
24. State advantage of Orthogonal collocation.

PART B

1. Solve the initial value problem $u' = -2tu^2$ $u(0) = 1$ With $h = 0.2$ on the interval $[0, 0.4]$ using R. K method.
2. Using Runge-kutta method of fourth order, find $y(0.8)$ correct to 4 decimal places if $y' = y - x^2, y(0.6) = 1.7379$

3. Solve the system of equations $u' = 3u + 2v$, $u(0) = 0$, $v' = 3u - 4v$, $v(0) = 0.5$ with $h = 0.2$ on the interval $[0, 0.4]$ using R.K method
4. Given $\frac{dy}{dx} = x^2(1+y)$ at $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$ Find $y(1.4)$ by Adams - Bashforth method.
5. Using the shooting method solve the BVP $y''(x) = y(x)$, $y(0) = 0$ and $y(1) = 1.17$
6. Solve the following BVP using shooting method
 $y'' + y + x = 0$, $y(0) = y(1) = 0$
7. Using Shooting method solve the equation $y'' - 2y = 8x(9-x)$. $y(0) = 0$, $y(9) = 0$.
8. Using Shooting method, solve the BVP $y'' + y + x = 0$, $0 < x < 1$, $y(0) = 0$, $y(1) = e-1$
9. Using the finite difference method find $y(0.25)$, $y(0.5)$ at $y(0.75)$ satisfying the differential equation $\frac{d^2y}{dx^2} + y = x$ subject to the boundary condition $y(0) = 0$, $y(1) = 2$
10. Solve by orthogonal collocation method $y''(x) = y(x)$, $y(0) = y(1) = 0$.
11. Solve by orthogonal collocation method $y'' + (1+x^2)y + 1 = 0$ With $y(-1) = y(1) = 0$.
12. Explain BVP for solving with Finite Difference method.
13. Solve the following BVP $u'' + u - x = 0$, $u(0) = 0$, $u(1) = 0$. Using FEM for two and three elements of equal length
14. Write short notes on oc with example.
15. Explain Galerkin Fem with example.
16. Write short notes on FEM.
17. Solve the differential equation $y'' + y + x = 0$, $0 < x < 1$, $y(0) = 0$, $y(1) = 1$ Using Galerkin FEM.
18. Use the Galerkin method approximate the solution of equation $y'' + y + x = 0$, subject to the boundary condition $y(0) = 0$, $y(1) = 0$
19. Solve the following BVP $y'' + 2y - x = 0$, $y(0) = 1$, $y(1) = 2$. Using FEM for two and four elements of equal length
20. Solve $\nabla''[x] + \nabla[x] + 2x(1-x) = 0$. Use the boundary values $\nabla[0] = 0$ and $\nabla[1] = 0$.
21. Solve the BVP $\frac{d^2y}{dx^2} = y$ with $y(0) = 0$ and $y(2) = 3.627$ by finite difference method. (Take $h=0.5$)
22. Solve the IVP $y' = t+y$, $y(0) = 1$ by classical fourth order Runge-Kutta method with $h = 0.1$ to get $y = 0.1$.
23. Given $\frac{dy}{dx} = 1+y^2$ where $y = 0$ when $x = 0$, $y(0.2) = 0.2027$, $y(0.4) = 0.4228$, $y(0.6) = 0.6841$. Compute $y(0.8)$ by Adams-Bashforth multistep method.
24. Consider the BVP $u'' + (1+x^2)u + 1 = 0$. $U(\pm 1) = 0$. Use the Galerkin method to determine the co-efficients of the approximate solution $w(x) = u_0(1+x^2) (1-4x^2) + \frac{16}{3}(x^2 - x^4)$ where u_0 and u_1 are the unknown solution values at the nodes 0 and $\frac{1}{2}$ respectively.

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QUESTION BANK
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UNIT III(FINITE DIFFERENCE METHOD FOR TIME DEPENDENT PARTIAL
DIFFERENTIAL EQUATION)

PART A

1. Give an example of a parabolic equation
2. State Schmidts's explicit formula for solving heat flow equation
3. Bender-Schmidt recurrence scheme is useful to solve _____ equation.
4. What is the classification of one dimensional heat flow equation.
5. What is the significance of Neumann boundary conditions of the Laplace equation $\nabla^2 u = 0$?
6. Verify by direct substitution that $u(x,t) = \sin(n\pi x) \cdot \cos(2n\pi t)$ is a solution to the wave equation $u_{tt}(x,t) = 4u_{xx}(x,t)$. For each positive integers $n = 1, 2, \dots$.
7. What is weighted residual
8. Define Finite difference methods
9. Write formula for Finite difference methods
10. What is numerical stability
11. State Crank-Nicolson difference scheme to solve parabolic equation. Also write the simplified formula when $k = ah^2$.
12. Write down the Crank-Nicolson formula to solve $u_{xx} = u_t$
13. When explicit method is stable only if ?
14. What is the condition for stability of the solution for hyperbolic equations?
15. Write the formula For Implicit formula.
16. Write difference between Implicit and explicit methods.
17. Write equation for 2 dimensional heat flow equation
18. Write formula for 2 dimensional heat flow equation
19. Write ADI method formula.
20. Define Dirichlet's condition
21. Define Neumann Condition.
22. Write Characteristics of PDE.
23. Write a note on the stability and convergence of the solution of the difference equation corresponding to the hyperbolic equation $u_{tt} = a^2 u_{xx}$.
24. State the explicit scheme formula for the solution of the wave equation.
25. Write down the general and simplest forms of the difference equation corresponding to the hyperbolic equation $u_{tt} = a^2 u_{xx}$.

PART B

1. Explain Explicit method.
2. Solve the parabolic equation $u_t = u_{xx}$ subject $u(0,t) = 0, u(1,t) = 0, u(x,0) = 0$
3. Solve the equation $u_t = 1/2 u_{xx}$ $0 \leq x \leq 12, 0 \leq t \leq 12$ with boundary and initial condition $u(x,0) = \frac{1}{4}x(15-x), 0 \leq x \leq 12, u(0,t) = 0, u(12,t) = 9, 0 \leq t \leq 12$. Using Schmidt relation
4. Solve by implicit method $u_t = \frac{1}{16} u_{xx}$ $0 < x < 1, t > 0, u(x,0) = 0 = u(0,t), u(1,t) = 100t$. Compute u for one step with $h = \frac{1}{4}$
5. Solve $u_t = u_{xx}$ subject $u(0,t) = 0, u(1,t) = 0, u(x,0) = \sin \pi x, 0 < x < 1$ Using Bender schmidt Method.
6. Find the solution of the two dimensional heat conduction equation $u_t = u_{xx} + u_{yy}$ Subject to the initial condition $u(x,y,0) = \sin \pi x \sin \pi y, 0 \leq x, y \leq 1$ and the boundary condition for $t \geq 0$ Using ADI Method. Assume $\lambda = \frac{1}{8}, h = \frac{1}{4}$ and integrate for one time step.
7. Write procedure for ADI Method.
8. Discuss ADI method to solve two dimensional parabolic equations.
9. Solve the first order hyperbolic PDE $\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0$ subject to the initial condition $u(x,0) = u^0(x)$
10. Solve $u_t = u_{xx}$ $0 < x < 2, t > 0, u(0,t) = u(2,t) = 0, t > 0, u(x,0) = \sin \frac{\pi x}{2}, 0 \leq x \leq 2$ using $\Delta x = 0.5, \Delta t = 0.25$ for two time steps by Implicit method
11. Explain Implicit method
12. Solve $u_{xx} = u_{tt}$ up to $t = 0.5$ with a spacing of 0.1 subject to $y(0,t) = 0, y(1,t) = 0, y_t(x,0) = 0$ and $y(x,0) = 10 + x(1-x)$.
13. Solve $u_t = u_{xx}$ subject $u(0,t) = 0, u(1,t) = 0, u(x,0) = \sin \pi x, 0 < x < 1$ Using Bender schmidt Method.
14. Solve $u_{tt} = u_{xx}$ subject $u(0,t) = 0, u(1,t) = 0, u_t(x,0) = 0, u(x,0) = \sin \pi x, 0 < x < 1$ Using finite difference method
15. Solve $4u_{xx} = u_{tt}$ $u(0,t) = 0, y(4,t) = 0, u_t(x,0) = 0$ and $u(x,0) = x(4-x)$.
16. Use the explicit difference scheme (Schmidt scheme) to determine the numerical solution to the IBVP. $u_t = u_{xx}, U(x,0) = \sin \pi x, 0 < x < 1, u(0,t) = u(1,t) = 0, t \geq 0$.
17. Find the solution of $u_t + u_x = 0$ subject to the initial condition $U(x,0) = 0, x < 0$
 $= x, 0 \leq x \leq 1$
 $= 2-x, 1 \leq x \leq 2$
 $= 0, x > 2$ and $r = \frac{1}{2}$. Compute upto two time steps.

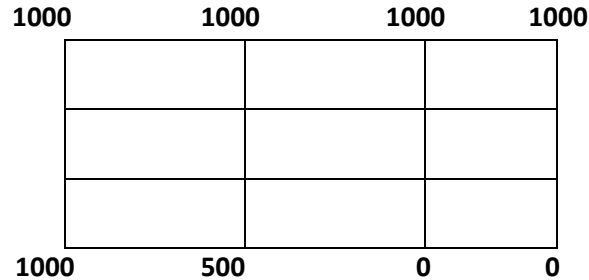
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SUB CODE/ TITLE:MA7169- ADVANCED NUMERICAL METHOD
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UNIT IV(FINITE DIFFERENCE METHODS FOR ELLIPTIC EQUATIONS)

PART A

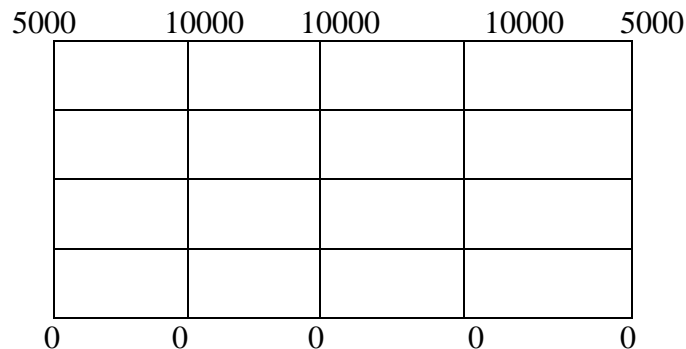
1. The number of conditions required to solve the Laplace equation is
2. Define a difference quotient .
3. State Liebmann's iteration process formulae.
4. State the general form of Poisson's equation in partial derivatives.
5. If u satisfies Laplace equation and $u = 100$ on the boundary of square what will be the value of u at an interior grid
6. For what value of r , the explicit forward, difference equation $u_{i,j+1} = u_{ij}(1 - 2r) + r(u_{i-1,j} + u_{i+1,j})$ is stable?
7. To solve $\nabla^2 u = 0$ in the rectangle R which is divided into 10 by 10 squares,
8. Write the diagonal five-point formula to solve the Laplace equation.
9. Write down the finite difference form of the equation $\nabla^2 u = f(x, y)$.
10. What is the purpose of Liebmann's iterative formula?
11. To solve $\nabla^2 u = 0$. let us use
12. The solution of the elliptic equation $\nabla^2 u = 0$. is given by
13. State the difference equation that approximate elliptic equation.
14. State Dirichlet's conditions to solve Laplace equations.
15. Write down the five point finite difference scheme to solve Laplace equations.
16. Write the difference scheme for solving the Laplace's equation.
17. What is difference between Laplace's and poisson equations.
18. Write down the short note on boundary curved using square mesh.
19. What is forward finite difference method.
20. Write the formula for backward finite difference method.
21. Write the formula for central finite difference method.
22. Define Dirichlet's condition
23. Define Neumann Condition.
24. Write down the formula for polar form of Laplace equation.
25. Write the formula for Finite difference method Poisson equation using polar coordinates

PART B

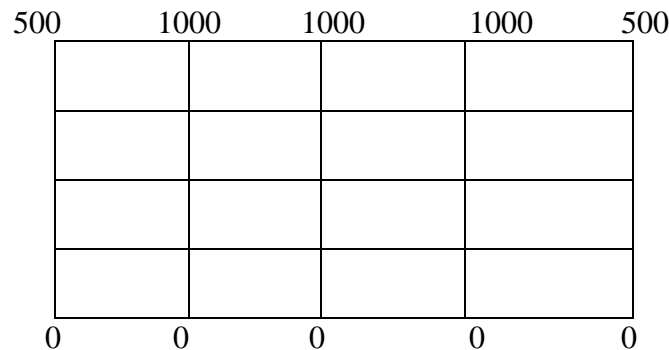
- By iteration method, Solve the Laplace equation $u_{xx} + u_{yy} = 0$ over the square region satisfying the boundary condition i. $u(0,y) = 0$ $0 \leq y \leq 3$. li. $u(3,y) = 9 + y$ $0 \leq y \leq 3$.
iii. $u(x,0) = 3x$ $0 \leq x \leq 3$. Iv $u(x,3) = 4x$ $0 \leq x \leq 3$
- Solve the Poisson equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ subject to the condition $u = 0$ at $x = 0$ and $x = 3$ $u = 3$ $u = 0$ at $y = 0$ and $u = 1$ at $y = 3$ for $0 < x < 3$. Find the solution taking $h = 1$ with a square.
- Obtain a finite difference scheme to solve the $\nabla^2 u = 0$ at the pivotal points in the square shown fitted with square mesh. Use Liebmann's.



- Explain solution of Laplace's Equation.
- Solve the Poisson equation $\nabla^2 u = -36(x^3 + y^3 + 5)$ subject to the condition $u = 0$ at $x = 0$ and $x = 3$ $u = 3$ $u = 0$ at $y = 0$ and $u = 1$ at $y = 3$ for $0 < x < 3$. Find the solution taking $h = 1$ with a square.
- Explain solution of poisson Equation.
- Solve the Laplace equation at the interior points of the square region given below.



8. Solve the Laplace equation at the interior points of the square region given below.



9. Solve the Poisson's equation $u_{xx} + u_{yy} = -81xy$ $0 < x < 1$ $0 < y < 1$ given that $u(0,y) = 0$ $u(x,0) = 0$ $u(1,y) = 100$ $u(x,1) = 100$ and $h = 1/3$.
10. Solve the Poisson's equation $u_{xx} + u_{yy} = 8x^2y^2$ for the square mesh of the given figure with $u(x,y) = 0$ on the boundary and mesh length = 1
11. Solve $u_{xx} + u_{yy} = 0$ $0 \leq x, y \leq 1$, with $u(0,y) = 10 = u(1,y)$ and $u(x,0) = 20 = u(x,1)$. Take $h = 0.25$ and apply Liebmann method to 3 decimal accuracy.
12. Solve the heat equation $u_{xx} + u_{yy} = u_t$ considering the region Ω lies between the unit square, the circle with centre at $(0.5, 0.5)$ and radius 0.33 which form the boundary $\partial\Omega$, $u(x, y, t) = 0$ on $\{x + y \geq 1\} \cap \partial\Omega = 0$ on $\{x + y < 1\} \cap \partial\Omega$ and the initial condition $u(x,y,0) = 0$.
13. Explain Finite difference method for Poisson equation using polar coordinates.
14. Find the solution of the 2D heat conduction equation $u_t = u_{xx} + u_{yy}$ subject to the initial condition $u(x,y,0) = \sin(\pi x), = \sin(\pi y), 0 \leq x, y \leq 1$ and the boundary conditions $u = 0$ on the boundary for $t \geq 0$ using the Peaceman- Rachford ADI method. Assume $h = \frac{1}{4}$ $\lambda = \frac{1}{8}$ and integrate for one time steps.
15. Solve $u_{xx} + u_{yy} = 0$ for (x, y) in the set $R = [(x, y) 0 < x < 0.5, 0 < y < 0.5]$ with the boundary conditions $u(0,y) = 0$ $u(x,0) = 0$ $u(x,0.5) = 200x$ $u(0.5,y) = 200y$. using Liebmann's iterative method.

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UNIT V(FINITE ELEMENT METHOD)

PART A

1. What is meant by finite elements?
2. Define FEM
3. Write formula for Galerkin Finite element method.
4. What is meant by orthogonal collocation.
5. What is weighted residual method.
6. What do you mean by collocation method
7. Define conforming elements.
8. Define IVP.
9. Define BVP.
10. What is least square method.
11. What is stiffness matrix.
12. What is trial solution.
13. What is difference between ODE and PDE FEM.
14. Write down the advantage of Orthogonal Collocation FEM.
15. What is difference between ODE and PDE in OC.
16. What is Galerkin Finite element method.
17. What is one dimensional finite element.
18. What is triangular finite element
19. Write formula for triangular finite element
20. Write formula for tetrahedral finite element
21. Write uses of FEM
22. Write merit of OC.
- 23.** What is difference between OC and FEM

PART B

1. Solve $(T_{xx} + T_{yy}) = x$, $0 \leq x, y \leq 1$, with $u = 0$ on the boundary c of the regions Using Orthogonal collocation FEM.
2. Solve $-(u_{xx} + u_{yy}) = 2$, $0 \leq x, y \leq 1$, with the condition on the boundary of square $0 \leq x \leq 1$, $0 \leq y \leq 1$ using FEM.
3. Solve by orthogonal collocation method $(u_{xx} + u_{yy}) = -2$, $0 \leq x, y \leq 1$,
4. Solve $(u_{xx} + u_{yy}) = k$, $0 \leq x, y \leq 1$, with $u = 0$ on the boundary c of the regions Using Orthogonal collocation FEM.
5. Solve $(u_{xx} + u_{yy}) = 2x$, $0 \leq x, y \leq 1$, with $u = 0$ on the boundary c of the regions Using Galerkin FEM.
6. Solve $-(u_{xx} + u_{yy}) = 3$, $0 \leq x, y \leq 1$, with the condition on the boundary of square $0 \leq x \leq 1$, $0 \leq y \leq 1$ using FEM.
7. Solve by orthogonal collocation method $(u_{xx} + u_{yy}) = -1$, $0 \leq x, y \leq 1$,
8. Solve $(u_{xx} + u_{yy}) = x$, $0 \leq x, y \leq 1$, with $u = 0$ on the boundary c of the regions Using Galerkin FEM.
9. Solve the poisson equation $u_{xx} + u_{yy} = -2$, $0 \leq x, y \leq 1$, with th condition $u = 0$ on the boundary of the square $0 \leq x \leq 1$, $0 \leq y \leq 1$, using FEM.
10. Solve the BVP $u_{xx} + u_{yy} = -1$, $|x| \leq 1$, $|y| \leq 1$ and $u = 0$ on $|x| = 1$, $|y| = 1$. Using the Galerkin FEM to determine the solution values at the node $(0,0)$, $(0.5,0)$ and $(0.5,0.5)$
11. Write short notes on oc with example.
12. Explain Galerkin Fem with example.
13. Write short notes on FEM.
14. Solve the mixed BVP equation $\nabla^2 u = 0$, $0 \leq x, y \leq 1$

$$u_x - u = 1 + y, x = 0$$

$$u_x - u = 1 + y, x = 0, 0 \leq y \leq 1$$

$$u_y - u = -1 - x, y = 0, u_y + u = -2 + x, y = 1, 0 \leq x \leq 1$$
15. Solve by the FEM the BVP $u'' + (1+x^2)u + 1 = 0$, $u(\pm 1) = 0$.
16. Use the Finite element Galerkin method to derive the difference schemes for the BVP $u'' - ku' = 1$, $u(1) = 0$ where $k > 0$ is assumed constant. Also obtain the characteristic equation of difference schemes.