Department of Electronics and Instrumentation Engineering
M. E- CONTROL AND INSTRUMENTATION ENGINEERING
CL7101 CONTROL SYSTEM DESIGN
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Unit I- BASICS AND ROOT-LOCUS DESIGN

PART-A (2 marks)

1. What are the time domain & frequency domain specifications needed to design a control system?
2. What is compensation and compensators?
3. When lag/lead/lag-lead compensation is employed?
4. Discuss the effect of adding a zero to open loop transfer function of a system?
5. What are the advantages of design in root locus?
6. What are the advantages and disadvantages in frequency domain design?
7. What is lag compensation?
8. Write the transfer function of lag compensator and draw its pole-zero plots?
9. What is the relation between ϕm and β in lag compensator?
10. What is lead compensator? Give an example.
11. Draw the bode plot of lead compensator.
12. What is the relation between ϕm and α in lead compensator?
13. When maximum phase lead occurs in the lead compensator? Give the expressions for maximum lead angle and the corresponding frequency.
14. Write the two equations that relates α and ϕm of lead compensator.
15. Write the transfer function of lag-lead compensator and its pole-zero plots.
16. Write the transfer function of PI-controller.
17. What is PD-controller and what are its effect on system performance?
18. Write the transfer function of PD-controller.
19. What is the PID controller and what are its effect on system performance?
20. Write the transfer function of PID controller.

PART-B (16 marks)

1. Discuss in detail the design of PID controllers using root locus techniques.
2. Consider a unity feedback system with open loop transfer function **G(s) = 10/s(s+4).**The dominant poles are **$-2 \pm j\sqrt{6}$.** Design a suitable PD controller.
3. Consider a unity feedback system with open loop transfer function **G(s) = 10/ (s+1)(s+2).** Design a PI controller so that the closed loop has a damping ratio of 0.707 and natural frequency of oscillation as **1.2 rad/sec.**
4. Consider a unity feedback system with **G(s) = 50/(s+2)(s+10).** Design a PID controller to satisfy the following specifications.
   (i) **$K_v \geq 2$** (ii) damping ratio =**0.6** (iii) **ω_n = 2 rad/sec.**
5. The controlled plant of a unity feedback system **G(s) = K/s(s+5).**It is specified that velocity error constant of the system be equal to **15**,while the damping ratio is **0.6**and velocity error is less than **0.25 rad** per unit ramp input. Design a suitable lag compensator.

6. The controlled plant of a unity feedback system **G(s) = K/s(s+10)²**. It is specified that velocity error constant of the system be equal to **20**,while the damping ratio of the dominant roots be **0.707**. Design a suitable cascade compensation scheme to meet the specifications.
7. A unity feedback system with open loop transfer function **G(s) =K/s² (s+1.5)** is to be lead compensated to satisfy the following specifications.
   (i) **Kv =30** (ii) damping ratio =**0.45** (iii) **ωn = 2.2 rad/sec.**
8. Design a lead compensator for a unity feedback system with open loop transfer function **G(s) = K/s (s+4)(s+7)** to meet the following specifications.(i) % peak overshoot =**12.63%**
   (ii) Natural frequency of oscillation =**8 rad/sec.** (iii) **Kv ≥ 2.5**
9. Consider a unity feedback control system whose forward transfer function is **G(s) = K/s (s+2)(s+8**).Design a lag-lead compensator so that **Kv = 80 s^{-1}** and dominant closed loop poles are located at **-2 ± j 2√3**
10. Design a lag- lead compensator for a unity feedback system with open loop transfer function **G(s) = K/s (s+1)(s+4)** to meet the following specifications (i) **Kv ≥ 5** (ii) damping ratio =**0.4**

Unit II- FREQUENCY RESPONSE BASED DESIGN

Part A

1. What is sampling?
2. Explain the effects of sampling.
3. What is hold circuit? Give the types of hold circuit.
4. Give the expression for ZOH?
5. Define Nyquist stability criterion .
6. Explain about aliasing effect.
7. With a neat block diagram, explain the conversion of continuous time signal to discrete time signal.
8. What are the methods of discretisation?
9. Give the advantages of digital controllers over analog controllers.
10. How to eliminate aliasing effect in sampled signal?
11. Give the steps involved in converting continuous time signal into discrete time signal using impulse invariance transformation.
12. Give the relationship between S-domain and Z-Domain in Bilinear Transformation.
13. Give the magnitude and angle criterion of root locus plot in Z-Domain.
14. Give the stability criteria of a system in S-Plane and Z-Plane.
15. What are the techniques used in sampling?
16. What is the effect of adding a lag compensator to a discrete system?
17. What is the effect of adding a lead compensator to a discrete system?
18. How to reconstruct a sampled signal?
19. Transfer function of a system is

    Y(z)/F(z)=(Z-0.3)/(Z^2-1.6Z+0.7)

    How will you choose the poles, so that the system meets the following requirements: Damping ratio > 0.6, Natural frequency >0.4rad/sample?
20. Give the mathematical expression of sampled signal.

Part B

1. Design a phase lag network for a system having $\mathbf{G(s) = K/s(1+0.2s)^2}$ to have a phase margin of $\mathbf{30^0}$.
2. The open loop transfer function of a certain unity feedback control system is given by $\mathbf{G(s) = K/s\ (s+1)}$. It is desired to have the velocity error constant $\mathbf{K_v = 10}$,and the phase margin to be atleast $\mathbf{60^0}$ .Design a phase lag series compensator.
3. Consider a unity feedback control system with an open loop transfer function is $\mathbf{G(s) = K/s\ (s+2)^2}$ .Design a suitable lead compensator so that phase margin is atleast $\mathbf{50^0}$ and velocity error constant is $\mathbf{20s^{-1}}$.
4. The open loop transfer function of a certain unity feedback control system is given by $\mathbf{G(s) = K/s\ (0.1s+1)(0.2s+1)}$.It is desired to have the phase margin atleast $\mathbf{30^0}$. Design a suitable phase lead series compensator.
5. Consider a unity feedback control system with an open loop transfer function is $\mathbf{G(s) = K/s\ (2s+1)(0.5s+1)}$.Design a suitable lag-lead compensator to meet the following specifications. (i) $\mathbf{Kv = 30}$ (ii) phase margin $\geq$ **50**
6. Consider a unity feedback control system with an open loop transfer function is $\mathbf{G(s) = K/s\ (s+2)^2}$ .Design a suitable lead compensator so that phase margin is atleast $\mathbf{50^0}$ and velocity error constant is $\mathbf{20s^{-1}}$.
7. The open loop transfer function of a certain unity feedback control system is given by $\mathbf{G(s) = K/s\ (0.1s+1)(0.2s+1)}$.It is desired to have the phase margin atleast $\mathbf{30^0}$. Design a suitable phase lead series compensator.
8. Consider a unity feedback control system with an open loop transfer function is $\mathbf{G(s) = K/s\ (2s+1)(0.5s+1)}$.Design a suitable lag-lead compensator to meet the following specifications. (i) $\mathbf{Kv = 30}$ (ii) phase margin $\geq$ **50**

9.Use Routh stability criterion to determine the no .of roots in the left half plane, the right half plane and on imaginary axis for the given characteristic equation:

(i) $\mathbf{S^5+4S^4+8S^3+8S^2+7S+4 = 0}$

(ii) $\mathbf{S^7+5S^6+9S^5+9S^4+4S^3+20S^2+36^S+36 = 0}$

10.The open loop transfer function of certain unity feedback system are given below. In each case ,discuss the stability of the closed loop system as a function of K > 0. Determine the values of K which will causesustained oscillations in the closed loop system .what are the corresponding oscillating frequencies?

(i) $$\frac{K}{(S+2)(S+4)(S^2+6S+25)}$$

(ii) $$\frac{K\,(S+1)}{S(S-1)(S2+4S+16)}$$

6. By use of Nyquist stability criterion determine whether the closed loop system having the following open loop transfer functions are stable or not. If not,How many closed loop poles lie in the right half of the S-plane?

(i)

$$G(s)H(s) = \frac{180}{(S+1)(S+2)(S+5)}$$

(ii)

$$G(s)H(s) = \frac{1+4S}{(S^2(1+S)(1+2S)}$$

11. Sketch the Nyquist plot for a feed back system with open loop transfer function

$$G(s)H(s) = \frac{K(S+10)^2}{S^3}$$

Find the range of values of K for which the system is stable.

12. Consider a unity feedback system with open loop transfer function **G(s) = 1/S(S+1).** Design a P D controller so that the phase margin of the system is **30^0** at a frequency of **2 rad/sec.**

13. A unity feedback system has an open loop transfer function as **G(s) = 50/(S+3)(S+1).** Design a PI controller so that the phase margin of the system is **35^0** at a frequency of **1.2 rad/sec.**

14. Consider a unity feedback system with open loop transfer function **G(s) = 20/(S+2)(S+4)(S+0.5).** Design a PID controller so that the phase margin of the system is **30^0** at a frequency of **2 rad/sec** and the steady state error for unit ramp input is **0.1**

Unit III- DESIGN IN DISCRETE DOMAIN

Part A

1. Define optimal control with example
2. Why do we use state variable technique for optimal control analysis?
3. Define tracking problem
4. State minimum time problem
5. Define minimum control effort problem
6. Define state regulator problem
7. State optimal control problem
8. Give the performance index of output regulator problem
9. What are the main theoretical approaches for optimal control?
10. What is the fundamental theorem of calculus of variation?
11. Write down the Euler Lagrange's equation.
12. What is meant by transversality condition?
13. Give the boundary value relationship used for minimization of functional when

a. When t1 is free and x(t1) is fixed

b. When t1 and x(t1) both are free.

14. What is the necessary and sufficient condition for optimal control "u" to minimize the Hamiltonian function?

15. State Pontryagin's minimum principle

16. Give the control and state variable inequality constraints

17. Distinguish the terms Hamiltonian function and Hamiltonian matrix

18. State the Hamiltonian-Jacobi equation

19. Write down the expression for optimal control using Riccati equation.

20. Give the performance index of regulator problem and solution of Matrix Riccati equation.

Part B

1. The characteristic Equation of a sampled data control system is given below. Check the stability of the system using Jury's stability test.

(i) $Z^4 - 1.2 Z^3 + 0.22 Z^2 + 0.066 Z - 0.008 = 0$

(ii) $Z^3 - 1.4 Z^2 + 0.53 Z - 0.04 = 0$

2.The digital process of a unity feedback system is described by the transfer unction

$$Gh_0 G(z) = \frac{K (z + 0.717)}{(z-1)(z-0.368)} ; T = 1s$$

Sketch the root locus plot for $0 \leq K \leq \infty$ and from there obtain the value of K that results in marginal stability. Also find the frequency of oscillations.

3. Given

$$Gh_0 G(z) = \frac{K (z - 0.9048)}{(z-1)^2} ; T = 1s$$

Sketch the root locus plot for $0 \leq K \leq \infty$. Using the information in the root locus plot, determine the range of values of K for which the closed loop system is stable.

4 (i)For a system with the following Z-transfer function, find the normalized steady state error between input and output , for a step input

$$D(z) = \frac{20 Z^2}{12Z^2 + 7 Z + 1}$$

(ii)Write a short note on direct discrete design

5 (i) Let **G(s) = 2/(s+2).** Discuss the effect of discretizing G(s) for control purpose using ZOH and sampling the output at every **0.02 sec** and **0.05 sec.**

Briefly explain the properties of Root Loci in Z-plane

Unit IV- DISCRETE STATE VARIABLE DESIGN

Part-A

1. What is pole placement by state feedback?
2. How control system design is carried in state space?
3. What is the necessary condition to be satisfied for designing state feedback?
4. Draw the block diagram of the system with feedback.
5. What is the control law used in state variable feedback?
6. Write down the Ackermann's formula used to find the state feedback gain matrix k.
7. What do you mean by pole placement by output feedback?
8. Give the control law used in output feedback.
9. What is the need for output feedback.
10. Write short notes on dynamic programming.
11. Give the problem statement of dynamic programming.
12. Give the schematics of state transition for discrete time system used in dynamic programming.
13. State the advantages of using dynamic programming approach for solving optimal control problem.
14. What are the principles on which dynamic programming is based on?
15. Give the minimum cost of Jth stage process using dynamic programming.
16. Give the necessary and sufficient condition to determine optimal control by dynamic programming.
17. State the disadvantages of using dynamic programming approach for solving optimal control problem.
18. Give a short note on principle of casuality.
19. Give a short note on principle of invariant embedded.
20. Give a short note on principle of optimality.

Part-B

1. Given the state equations of a linear system is **dx(t)/dt = A x(t) + B u(t)**
   With u(t) = u(kT)= constant for $kT \le t < (k+1)T$ .The system is discretized ,resulting in the following discrete-data state equations : **x(k+1)T = Φ(T) x(kT) + θ(T) u(kT)**
   Find the matrices **Φ(T)** and **θ(T).** **A =** $\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$ **B =** $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

2. A discrete data control system is described by the state equation : **x(k+1) = A x(k) + B u(k)**
   A= $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.04 & -0.53 & 1.4 \end{bmatrix}$ **B =** $\begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix}$ Determine the controllability of the system **.**

3 .A linear controlled process of a discrete data control system is described by the following state equations: **dx(t)/dt = A x(t) + B u(t)** where **A =** $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ **B =** $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ u(t) = u(kT)

for $kT \le t < (k+1)T$. Determine the values of the sampling period T that makes the system uncontrollable.

4. Given the discrete data control system **x(k+1) = A x(k) + B u(k)** **c(k) = D x(k)**

Where **A** = A = $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ **B** = $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ **D = [1 -1]** the control is realized through the state feedback u(k) = -G x(k) = -[g_1 g_2] x(k) where g_1 and g_2 are real constants. Determine the value of g_1 and g_2 that must be avoided for the system to be completely observable.

5. The dynamic equations of a digital process are given as **x(k+1) = A x(k) + B u(k)** **c(k) = D x(k)**

Where **A** = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ **B** = $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ **D = [1 1]** the state feedback control is u(k) = -G x(k) where G = [g_1 g_2] . Design a full order observer so that x(k) is observed from c(k)

6. Consider a digital control system **x(k+1)T = A x(kT) + B u(kT)** where **A** = $\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$ **B** = $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$**.** The state feedback control is described by u(kT) = -K x(kT) where K = [k_1 k_2].Find the values of k_1 and k_2 so that the roots of the characteristic equation of the closed loop system are at 0.5 and 0.7.

7. Find the controller gain for the discrete system described by **A** =$\begin{bmatrix} 0 & 0.1 \\ 0 & 1 \end{bmatrix}$ and **B** = $\begin{bmatrix} 0.003 \\ 0.1 \end{bmatrix}$ **t**o place the closed loop system poles at Z= 0.8 ± j 0.25 using direct pole placement.

8.(i) Let **A** = $\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$ **B** = $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ Find the state feed back gain matrix such that the eigen values of **A- BG** are at **0** and **0.3**

(ii) Write notes on full order Observer.

9 Given the state equations of a linear system is **dx(t)/dt = A x(t) + B u(t)**
With u(t) = u(kT)= constant for kT ≤ t< (k+1)T .The system is discretized ,resulting in the following discrete-data state equations : **x(k+1)T = Φ(T) x(kT) + θ(T) u(kT)**
Find the matrices **Φ(T)** and **θ(T).** **A** = $\begin{bmatrix} -1 & 2 \\ -1 & 0 \end{bmatrix}$ **B** = $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

10. Investigate the controllability and observability of the following system:

(i) x(k+1) = $\begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$ x(k) + $\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ u(k)

y(k) = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ x(k)

(ii) x(k+1) = $\begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ x(k) + $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ u(k)

y(k) = $\begin{bmatrix} 1 & 1 \end{bmatrix}$ x(k)

Unit-V- LQR AND LQG DESIGN

Part-A

1. What is the need for observer?
2. Write a note on full order observer.
3. Write a note on reduced order observer.
4. Give the modified state and output equation of a system with Luenberger observer.

5. What are the criteria involved in selecting Luenberger gain matrix L?
6. Give the limitation of Luenberger observer.
7. State separation principle
8. Explain Bucy filter.
9. State any two rules used for assessing an acceptable estimate.
10. Write down the PDF function for jointly Gaussian Variable.
11. Write a note on Weiner filter
12. Give the design procedure for LQR controller
13. Write down the Ackermann’s formula for finding the observer gain matrix.
14. What do you mean by whit noise?
15. Draw the structure of optimal estimator.
16. Draw the system representation including input distribution and measurement noise.
17. Give the expression of Kalman gain.
18. Draw the block diagram of Discrete Kalman Filter.

Part-B

11. Derive the necessary and sufficient condition to be satisfied along the optimal trajectory using Hamiltonian formulation starting from the results of calculus variation approach for a state tracking problem of a linear time invariant system.

12. Derive the matrix Riccati equation and state the necessary and sufficient condtion for optimal solution.

13. (a) Derive the optimal control policy for the following optimal control problem

$$\dot{x} = -2x + u$$

$$\min J = \tfrac{1}{2}\int_0^{\infty}\left((x - \sin t)2 + u2\right)dt$$

(b) Write notes on kalman filter.

14. Find the optimal control law for the system $\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} u$ with the performance index $J = \int (x_1{}^2 + u_1{}^2 + u_2{}^2)\, dt$

15. Determine the optimal control law for the system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$y = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} x$ such that the following performance index is minimized

$$J = \int (y_1{}^2 + y_2{}^2 + u^2)\, dt$$