VALLIAMMAI ENGINEERING COLLEGE

DEPARTMENT OF MATHEMATICS

BRANCH / SEM : M.E. COMMUNICATION SYSYEMS – FIRST SEMESTER

MA7158–Applied Mathematics for Communication Engineers

QUESTION BANK

UNIT –I(LINEAR ALGEBRA)

PART – A

1.Write a note on least square solution

2.Define singular value matrix

3.If A is a non singular matrix ,then what is A^+ ?

4.What is meant by singular value of a matrix?

5.Compute the norms of $x = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 2 \end{pmatrix}$ and $y = \begin{pmatrix} 3 \\ -1 \\ 0 \\ -1 \end{pmatrix}$.Also verify that x and y are orthogonal . Find $< x ,y >$

6.State Singular value decomposition theorem

7.Give an example of a Toeplitz matrix of order 3.

8.Explain least square solution

9.Define Toeplitz matrix with an example

10.Find the least square solution to the system $x_1 + x_2 = 3$, $-2x_1 + 3x_2 = 1$ and $2x_1 - x_2 = 2$

11.Explain singular value decomposition in matrix theory

12. What is the advantage in matrix factorization methods?

13. Check whether the given matrix is positive definite or not $\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{pmatrix}$

14.Define pseudo inverse of a matrix A

15.Give some properties of generalized inverse

16.Define Hermitian matrix

17.Define singular value of a matrix

18.If A is a non singular matrix , then what is A^+

19.Define an inner product space

20. Write the general form of the toeplitz matrix of order n. Also write any two applications of it.

PART –B

1.Find the QR decomposition of $\begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{pmatrix}$

2.Find the eigen values of $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 18 & -1 & -7 \end{pmatrix}$ **by using QR transformation**

3. Construct QR decomposition for the matrix i) $\begin{pmatrix} -4 & 2 & 2 \\ 3 & -3 & 3 \\ 6 & 6 & 0 \end{pmatrix}$ **ii)**$A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$

4. Construct the singular value decomposition for the matrix $\begin{pmatrix} -3 & 1 \\ -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix}$

5.Find the QR factorization of(i) $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix}$ **ii)** $A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$

6.Obtain the singular value decomposition of (i) $A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 3 \end{pmatrix}$ **(ii)** $A = \begin{pmatrix} 2 & -1 \\ -1 & 1 \\ 4 & -3 \end{pmatrix}$ **iii)** $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 2 & 2 \end{pmatrix}$

iv) $A = \begin{pmatrix} 1 & 2 \\ 2 & 2 \\ 2 & 1 \end{pmatrix}$

7.Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{pmatrix}$ **.Compute the singular values of and singular value decomposition of A .Also find A^{-1}.**

8.Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} = A_1$ **.Compute A_2 and A_3 using QR algorithm.**

9. Solve the following system of equations in the least square sense
$2x_1 + 2x_2 - 2x_3 = 1 , 2x_1 + 2x_2 - 2x_3 = 3, - 2x_1 - 2x_2 + 6x_3 = 2$

10.Find the least square squares solution of $x + 2y + z = 1$, $3x - y = 2$, $2x + y - z = 2$,

$x + 2y + 2z = 1$

11.Find the generalized inverse of $\begin{pmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 6 \end{pmatrix}$ by least square method

12.If x *and* y are any two vectors in an inner product space , then prove that $|< x, y >| \leq \|x\|\|y\|$

UNIT – II (LINEAR PROGRAMMING)

PART –A

1.What is degeneracy in a transportation model

2.Differentiate between balanced and unbalanced cases in Assignment model

3.List any two basic differences between a transportation and assignment problem

4.What do you mean by degeneracy?

5.Explian optimal solution in L.P.P

6.Solve the following L.P.P by using graphical method

$Maximize\ Z = 5x_1 + 3x_2$, Subject to $3x_1 + 5x_2 \leq 15$, $5x_1 + 2x_2 \leq 10$, $x_1, x_2 \geq 0$

7.Obtain an initial basic feasible solution to the following transportation problem by using Matrix minima method

	D_1	D_2	D_3	D_4	capacity
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
	4	6	8	6	

Demand

8.What is the difference between feasible solution and basic feasible solution?

9.What is an assignment problem? Give two applications

10.Write down the mathematical formulation of L.P.P.

11.When will you say a transportation problem is said to be unbalanced?

12.What is a travelling sales man problem?

13.Enumerate the methods to find the initial basic feasible solution for transportation problem

14.Define transshipment problem

Part - B

1.Solve the L.P.P by Simplex method

$Maximize\ Z = 3x + 2y$ **Subject to** $2x + y \leq 6\ , x + 2y\ \leq 6\ , x\ , y\ \geq 0$

2.Solve by Simplex method.

Maximize = $5x_1 + 4x_2$ **, Subject to**

$4x_1 + 10x_2 \leq 10, 3x_1 + 2x_2 \leq 9\ , 8x_1 + 3x_2 \leq 12, x_1, x_2 \geq 0$

3.Maximize $Z = 4x_1 + 10x_2$ **Subject to**

$2x_1 + x_2 \leq 10\ , 2x_1 + 5x_2 \leq 20\ , 2x_1 + 3x_2 \leq 18\ , x_1, x_2 \geq 0$

4.Use two phase simplex method to minimize $Z = x_1 + x_2$ **, Subject to**

$2x_1 + x_2 \geq 4\ , x_1 + 7x_2 \geq 7\ \ x_1, x_2 \geq 0$

5.Solve the following LPP by graphical method Maximize $Z =\ x_1 + x_2$ **,Subject to**

$x_1 + x_2 \leq 1\ , -3x_1 + x_2 \geq 3\ , x_1, x_2\ \geq 0$

6.Using simplex method solve the L.P.P , Maximize $Z = 2x_1 + 3x_2$ **, Subject to**

$x_1 - x_2 \leq 2\ , x_1 + x_2 \leq 4, \quad x_1, x_2\ \geq 0\ ,$

7.Solve the following L.P.P. by using Simplex method : Maximize $Z =\ 20x_1 + 6x_2 + 8x_3$

Subject to the constraints $8x_1 + 2x_2 + 3x_3 \leq 250, 4x_1 + 3x_2 \leq 150, 2x_1 + x_2 \leq 50, x_1, x_{2,} x_3\ \geq 0$

8.Use two phase Simplex method to solve Maximize $Z = 5x_1 + 8x_2$ **,Subject to**

$3x_1 + 2x_2 \geq 3\ , x_1 + 4x_2\ \geq 4\ , x_1 + x_2\ \leq 5\ , x_1, x_2\ \geq 0$

9.Solve the following LPP by using Simplex method ,Maximize $Z = 4x_1 + x_2 + 3x_3 + 5x_4$

Subject to ; $4x_1 - 6x_2 - 5x_3 + 4x_4 \geq -20, 3x_1 - 2x_2 + 4x_3 + x_4 \leq 10\ , 8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20\ , x_1, x_2\ , x_3\ , x_4\ \geq 0$

10.Solve the L.P.P by graphical method , Maximize $Z = 100x_1 + 40x_2$

Subject to ; $5x_1 + 2x_2 \leq 1000\ , 3x_1 + 2x_2 \leq 900, x_1 + 2x_2 \leq 500\ , x_1, x_2\ \geq 0$

11.Solve by Simplex method Maximize $Z = 15x_1 + 6x_2 + 9x_3 + 2x_4$**,Subject to**

$2x_1 + x_2 + 5x_3 + 6x_4 \leq 20, 3x_1 + x_2 + 3x_3 + 25x_4 \leq 24, 7x_1 + x_4 \leq 70, x_1, x_2, x_3, x_4 \geq 0$

12.Solve the L.P.P by using Simplex method , Maximize $Z = 5x_1 + 4x_2$,Subject to

$4x_1 + 5x_2 \leq 10, 3x_1 + 2x_2 \leq 9, 8x_1 + 3x_2 \leq 12, x_1, x_2 \geq 0$

13.Find the initial solution to the following TP using Vogel's approximation method

Destination

	D_1	D_2	D_3	D_4	Supply
F_1	3	3	4	1	100
F_2	4	2	4	2	125
F_3	1	5	3	2	75
Demand	120	80	75	25	300

14. Determine the basic feasible solution to the transportation problem

Distribution Centers

Sources	D_1	D_2	D_3	D_4	Supply
S_1	2	3	1	7	6
S_2	1	0	6	1	1
S_3	5	8	1	9	10
Requirement	7	5	3	2	

15.A military equipment is to be transportated from origins x,y,z to the destinations A ,B ,C and D The supply at the origins, the demand at te destinations and time of shipment is sown in the table. Work out the transportation plan so that the time required for shipment is the minimum

Destinations

		A	B	C	D	Supply
Origin	X	10	22	0	22	8
	Y	15	20	12	8	13
	Z	21	12	10	15	11
Demand		5	11	8	8	32

16.Find an optimal solution to the following transportation problem

16.Find an optimal solution to the following transportation problem

	A	B	C	D	
X	2	3	11	7	6
Y	1	0	6	1	1
Z	5	8	15	9	10
	7	5	3	2	

17.Solve the transportation problem using Vogel's approximation method and check its optimality using MODI method

To

	A	B	C	Availability
From I	50	20	220	1
II	90	45	170	3
III	250	200	50	4
Requirement	4	2	2	

18.A travelling sales man has to visit 5 cities .He wishes to start from a particular city, visit each city once and then returns to his starting point. Cost of going from one city to another is shown below. Find the least cost route

To City

	A	B	C	D	E
A	∞	4	10	14	2
B	12	∞	6	10	4
C	16	14	∞	8	14
D	24	8	12	∞	10
E	2	6	4	16	∞

19.Solve the assignment problem represented by the matrix

	1	2	3	4	5	6
A	9	22	58	11	19	27
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	49	39	32
E	36	11	57	22	25	18
F	3	56	53	31	17	28

20.A company has a team of four sales man and four districts where the company wants to start its business. After taking into account the capabilities of sales men and the nature of districts, the company estimates that the profit per day in hundreds of rupees for each salesman in each district is as below. Find the assignment of sales man to various district which will yield maximum profit

	1	2	3	4
A	16	10	14	11
B	14	11	15	15
C	15	15	13	12
D	13	12	14	15

21. Solve the assignment problem

	P	Q	R	S
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

UNIT- III - (ORDINARY DIFFERENTIAL EQUATIONS)

PART –A

1. What is a predictor- corrector method?

2. What is the difference between initial and boundary value problems?

3. Write down Adam Bashforth 's predictor and corrector method

4. What are the steps involved in shooting method?

5. What is weighting function of collocation method?

6. Explain stiff ordinary differential equations

7. When a single step method is applied to the test equation $u' = \lambda u$, what are the conditions for absolutely stable and relatively stable?

8. When will you say that a system of ordinary differential equations of the form

$u' = f(t, u_1, u_2, \dots, u_n), u(t_0) = \eta_0$ is stiff?

9. Write down Runge- Kutta method of order four for solving initial value problems in solving ordinary differential equations

10. State the stability region for R-K method of fourth order

11. Differentiate single step and multi step method

12. Use R-K method of second order to find y (0.4) given y' = xy , y(0) = 1

13.Mention the methods available to solve boundary value problems

14. Discuss the stability of Euler's method

15. Discuss the orthogonal collaction method.

16. Discuss the finite element method

PART-B

1. **Solve :** $\frac{dy}{dx} - y = 2$ **where** $y(0) = 0$ *and* $y(1) = 1$ **by orthogonal collocation method**
2. **Solve :**$\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1.0954, y(0.2) = 1.1832, y(0.3) = 1.2649$ **, compute** $y(0.4)$ **by Adam's predictor and corrector method**
3. **Use Runge- Kutta method of fourth order to find** $y(0.1)$ **given** $\frac{dy}{dx} = x + y$

With $y(0) = 1$

4.Solve the boundary value problem $\frac{d^2y}{dx^2} - 64\,y + 10 = 0\,, y(0) = y(1) = 1$ **using shooting method**

5.Solve the boundary value problem $y^{//} + yx = x^2\,, y(0) = 1$ *and* $y(1) = 1$ **by Runge-Kutta method**

6.Find $y(4.4)$ **by Adam –Bashforth multi step method given**

$5xy^{/} + y^2 = 2\,, y(4) = 1.2, y(4.1) = 1.2003\,, y(4.2) = 1.012\,, y(4.3) = 1.023$

7.Consider the boundary value problem $\nabla^2 u = -1, |x| \le 1, |y| \le 1$ **and** $u = 0, |x| = 1, |y| = 1$ **use the Galerkin method to determine the solution values at the nodes** $(0,0)\,, \left(\frac{1}{2}, 0\right)\left(\frac{1}{2}, \frac{1}{2}\right)$

8.Use the shooting method to solve $\frac{d^2T}{dx^2} + \mathrm{h}'(\mathrm{T_a} - \mathrm{T`}) = 0$ **for a 10-m rod with** $\mathrm{h}' = 0.01\mathrm{m}^{-2}, T_a = 20$ **and the boundary conditions** $T(0) = 40\,, T(10) = 20$

9. Using Runge- Kutta method of fourth order, find $y(0.8)$ **correct to 4 decimal places if**

$y' = y - x^2, y(0.6) = 1.7379$

10.Solve the differential equation $y^{''} + y = -x, 0 < x < 1\,, y(0) = y(1) = 1$ **using Galerkin finite element method**

11.Using shooting method , solve the BVP : $y'' = y + 1 , 0 < x < 1, y(0) = 0, y(1) = e - 1$

12.Solve the initial value problem $y' = t + y, y(0) = 1$, by classical Runge- Kutta method of fourth order with $h = 0.1$, $to\ get\ y(0.1)$

13.Solve the BVP $\frac{d^2y}{dx^2} = y\ with\ y(0) = 0\ and\ y(2) = 3.627$ by finite difference method [Take $h = 0.5$]

14.Given $\frac{dy}{dx} = 1 + y^2$, where $x = 0, y(0.2) = 0.2027, y(0.4) = 0.4228, y(0.6) = 0.6841.$ Compute $y(0.8)$by Adams- Bashforth multistep method

15.Consider the boundary value problem $u'' + (1 + x^2)u + 1 = 0, u(\pm 1) = 0$. Use Galerkin method to determine the coefficients of the approximate solution, $u(x) = u_0(1 - x^2)(1 - 4x^2) + \frac{16}{3}, u_1(x^2 - x^4)$ where $u_0\ and\ u_1$ are the unknown solution values at the nodes 0 and ½

respectively

UNIT –IV (TWO DIMENSIONAL RANDOM VARIABLES)

PART-A

1.Let X and Y be continuous RVs with joint density function f(x,y) =$\frac{x(x-y)}{8}$, $0 < x < 2 , -x < y < x\ and\ f(x, y) = 0\ elsewhere$.Find f(y /x)

2.Find P (X +Y < 1) for the RVs whose joint pdf is f(x,y) =$\begin{cases} 2xy + \left(\frac{3}{2}\right) y^3, 0 < x < 1 , 0 < y < 1 \\ 0, \quad otherwise \end{cases}$

3.If f(x,y) =e^{x+y} , x> 0 ,y > 0 is the joint pdf of the RVs X and Y then find P [X <1]

4.If f(x,y) =$e^{-(x+y)}$, x> 0 ,y > 0 is the joint pdf of the RVs X and Y,check whether X and Y are independent

5.The regression equations are 3x+2y =26 and 6x+ y =31.Find the correlation coefficient between them

6.Find the acute angle between the two lines of regression

7..Find the value of k , if f(x,y) = k (1-x) (1-y) in 0< x,y <1 and f(x,y) = 0 otherwise ,is to be the joint density function

8.If $\bar{X} = 970$,$\bar{Y} = 18$, $\sigma_x = 38$, σ_y= 2 and r =0.6 ,find the line of regression of X on Y

9.If f (x ,y) = 8xy , 0<x<1 , 0<y<x is the joint pdf of X and Y , find f (y/x)

10.If the joint pdf of (X ,Y) is given by 2-x-y , in 0<x<y<1 , find E(X)

11.If Y = 2x +3 , find the COV (X,Y)

12.If X has mean 4 and variance 9 while Y has mean -2 and variance 5 and the two are independent find Var (2X + Y -5)

13. The joint probability density function of the random variable (X,Y) is given by f(x,y) =
$Kxye^{-(x^2 + y^2)}$, $x > 0, \quad y > 0$ **Find the value of K**

14.Let X and Y be two independent RVs with joint pmf P (X =x ,Y =y) = $\begin{cases} \frac{x+2y}{18}, & x = 1, 2, y = 1, 2 \\ 0, & otherwise \end{cases}$

.Find the marginal probability mass function of X and E (X)

15.Define joint pdf of two RVs X and Y and state its properties

16.If two RV Xand Y have the pdf f (x,y) = k (2x+y) for 0 <x<2 and 0< y<3 ,evaluate k

17.Find the marginal density functions of X and Y if f(x,y) = $\begin{cases} \frac{2(2x+5y)}{5} & 0 \le x, y \le 1 \\ 0, & otherwise \end{cases}$

18.If two random variables X and Y have probability density function f(x,y) = k(2x + y) for 0≤ x ≤ 2 and 0 ≤ y ≤ 2, evaluate k

19.Let X and Y be random variables with joint density function

$f_{XY}(x,y) = \begin{cases} 4xy, & 0 < x < 1, 0 < y < 1 \\ 0, & otherwise \end{cases}$ **find E(XY)**

20.If X and Y have joint pdf f(x,y) = $\begin{cases} x + y, 0 < x < 1, 0 < y < 1 \\ 0, otherwise \end{cases}$ **check whether X and Y are independent.**

21.Can the joint distributions of two random variables X and Y be got if their marginal distributions are known?

22..If two RVs X and Y have probability density function f (x,y) = k $e^{-(2x+y)}$ **for x ,y > 0 ,evalutate k**

23.Distinguish between correlation and regression

24.Let X and Y be integer valued random variables with P(X=m, Y=n) = $q^2 p^{m+n-2}$**, n, m = 1,2,… and p+q = 1. Are X and Y independent ?**

25.If X and Y are random variables having the joint density function f(x,y) = 1/8 (6 – x – y),

0 < x < 2, 2 < y < 4, find P(X+Y < 3)

PART - B

1.The joint pdf of Rv X and Y is given by f(x,y) = $\begin{cases} \frac{8xy}{9}, & 1 \le x \le y \le 2 \\ 0, & otherwise \end{cases}$

Find the conditional density functions of X and Y

2. Two random variables X and Y have the joint density f(x,y) = $\begin{cases} 2-x-y, & 0<x<1, 0<y<1 \\ 0, & otherwise \end{cases}$

Show that Correlation coefficient between X and Y is -1 /11

3.X and Y are independent with a common pdf f (x) =e^{-x} ,x> 0 ,0 otherwise , f(y) =e^{-y} y> 0 , 0, otherwise .Find the PDF for X –Y

4.The regression equation of X on Y is 3Y -5X +108 = 0 .If the mean of Y is 44 and the variance of X is 9/16 th of the variance of Y.Find the mean value of X and the correlation coefficient

5.If X and Y are independent RVs with density function f(x) = 1 ,1 <x<2 , 0 otherwise and

f (y) =y / 6 , 2 < y<4 , 0 otherwise .Find the density function of Z =XY

6.Two RVs X and Y have the joint pdf given by f(x,y) = k(1 – x^2y), 0<x< 1 , 0 <Y <1 , 0 otherwise. 1.Find the value of K 2.Obtain the marginal pdf of X and Y 3.Also find the correlation coefficient between them.

7.The joint pdf of a two dimensional RV is given by f(x,y) = 1 /3 (x+ y) , 0<x,1 ,)<y<2.Find the 1.Correlation coefficient 2.The equation of the two lines of regression lines.

9.The joint density function of (X ,Y) is f(x,y)=$\begin{cases} \frac{x^3y^3}{16} & , 0 \le x \le 2, 0 \le y \le 2, \\ 0, & otherwise \end{cases}$

find 1.The marginal density functions of X and Y 2.Conditional density of X given Y and Y given X

10. If the joint pdf of (X,Y) is given by f(x,y) = x+y, 0 ≤ x , y ≤ 1, find the pdf of U=XY

11.The equation of two regression lines obtained by in a coreelation analysis is as follows :

3x + 12 y = 19 , 3y + 9x =46.Obtain the correlation coefficient 2.Mean value of X and Y

15..If (X ,Y) is a two dimensional RV uniformly distributed over the triangular region R bounded by y =0 , x =3 and y = $\frac{4x}{3}$ **Find the marginal density function of X and Y**

16.For the bivariate probability distribution of (X ,Y) given below

X \ Y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

Find the marginal distributions , conditional distributions of X given Y =1 and conditional distribution of Y given X = 0.

17.Find the covariance of X and Y , if the RV (X,Y) has the joint pdf f(x,y) = x+ y , 0<x<1 , 0 < y<1 and f(x,y) =0 ,otherwise

18.The joint pdf of two dimensional RV (X,Y) is given by $f(x,y) = \frac{8}{9}xy, 0 \le x \le y \le 2$ and

f(x,y) =0 ,otherwise .Find the densities of X and Y and the conditional densities f (x/y) and f (y/x)

19.Find the correlation coefficient r_{xy} for the bivariate RV (X,Y) having the pdf f (x,y) = 2xy , 0<x< 1 , 0< y<1 , 0 otherwise

20. Let X and Y be non-negative continuous random variables having the joint probability density function f(x,y) = $\begin{cases} 4xy\ e^{-(x^2+y^2)}, & x>0,\ y>0 \\ 0, & otherwise \end{cases}$ **find the pdf of U =** $\sqrt{X^2+Y^2}$

UNIT –V (QUEUING THEORY)

PART –A

1.What is the probability that a customer has to wait more than 15 minutes in an $(M/M/1):(\infty/FIFO)$ queueing model with $\lambda = 6$ *per hour and* $\mu = 10$ per hour?

2.What is the effective arrival rate in an (M/M/1):(K/FIFO) model?

3.What do you mean by steady state and transient state in queuing theory?

4.For an M/M/1 queuing system if $\lambda = 6$ *per hour and* $\mu = 8$ per hour, what is the probability that at least 10 customers in the system?

5.Write Little's formula for infinite capacity queuing system

6.There are 3 typists in an office .Each can type an average 6 letters per hour. If the letters arrive for being typed at the rate of 15 letters per hour What fraction of time all the typists will be busy?

7.For an (M/M/S) (N/FIFO) queuing system write down the formula for 1)Average number of customers in queue 2) Average waiting time of customers in queue.

8.If people arrive to purchase cinema tickets at the average rate of 6 per minute ,it takes an average of 7.5 seconds to purchase a ticket . If a person arrives 2 minutes before the picture starts and it takes exactly 1.5 minutes to reach the correct seat after purchasing the ticket. Can he expect to be seated for the start of the picture?

9.Define M/M/2 queuing model. Why the notation M is used?

10.What is the probability that an arrival to an infinite capacity 3 server Poisson queue with $\frac{\lambda}{c\mu} = \frac{2}{3}$ *and* $P_0 = \frac{1}{9}$ enters the server without waiting?

11.Define traffic intensity of an M/M/1 queuing model .What is the condition for steady state in terms of the traffic intensity?

12.Consider an M/M/C queuing system .Find the probability that an arriving customer is forced to join the queue.

13.In a given M/M/1 /∞ /FCFS queue , $\rho = 0.6$,what is the probability that the queue contains 5 or more customers?

14.Suppose that customers arrive at a Poisson rate of one per every twelve minutes, and that the service time is exponential at a rate of one service per 8 minutes a) What is the average no. of customers in the system? b) What is the average time of a customers spends in the system?

15.Obtain the steady state probabilities of an (M/M/1):(∞ /FIFO) queuing model

16.In the usual notation of an M/M/1 queuing system, if $\lambda = 3$ per hour and $\mu = 4$/hour,find $P(X \geq 5)$ where X is the number of customers in the sytem.

17. What is the probability that a customer has to wait more than 15 minutes to get his service (M/M/l):(°° /FIFO) queue system if $\lambda = 6$ per hour and $\mu =$ *10* per hour?

18.Define the effective arrival rate for M | M | 1/N FCFS queueing system.

19.In a 3 server infinite capacity Poisson queue model if $\lambda/s\,\mu = 2/3$ find P_0

20.In an M/M/1 / K /FIFO queuing system if $\lambda = 4$ / hour and $\mu = 12$ /hour, find the probability that there is no customer in the system. If $\lambda = \mu$ what is the value of this probability?

PART- B

1.Arrivals at a telephone booth are considered to be Poisson with an average time of 12 min.between one arrival and the next .The length of a phone call is assumed to be distributed exponentially with mean 4 min.

a) Find the average number of persons waiting in the system

b) What is the probability that a person arriving at the booth will have to wait in the queue ?

c) What is the probability that it will take him more than 10 min altogether to wait for the phone and complete his call?

d) Estimate the fraction of the day when the phone will be in use

e) The telephone department will install a second booth ,when convinced that an arrival has to wait on the average for atleast 3 min. for phone .By how much the flow of arrivals should increase in order to justify a second booth?

f) What is the average length of the queue that forms from time to time?

2. Arrivals at a telephone booth are considered to be Poisson with an average time of 10 min. between one arrival and the next .The length of a phone call is assumed to be distributed exponentially with mean 3 min.a) Find the average number of persons waiting in the system

b) What is the probability that a person arriving at the booth will have to wait in the queue ?

c) What is the probability that it will take him more than 10 min altogether to wait for the phone and complete his call?

d) Estimate the fraction of the day when the phone will be in use

e) The telephone department will install a second booth ,when convinced that an arrival has to wait on the average for atleast 3 min. for phone . By how much the flow of arrivals should increase in order to justify a second booth?

f) What is the average length of the queue that forms from time to time?

3.Customers arrive at a one man barber shop according to a Poisson with a mean inter arrival time of 20 min .Customers spend an average of 15 min in the barber's chair

1) What is the expected number of customers in the barber shop ?in the Queue?

2) What is the probability that a customer will not have to wait for a hair cut?
3) How much can a customer expect to spend in the barbershop?
4) What is the average time customers spend in the queue?
5) What is the probability that the waiting time in the system is greater than 30 min?
6) What is the probability that there are more than 3 customers in the system?

4. In a given M / M / 1 queueing system ,the average arrivals is 4 customers per minute , ρ= 0.7 . What are 1) mean number of customers L_s in the system
2) mean number of customers L_q in the queue
3) probability that the server is idle
4) mean waiting time W_s in the system .

5.Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every 10 minutes , and the service time is exponential random variable with 8 minutes
a) Find the average number of customers L_s in the shop
b) Find the average number of customers L_q in the queue'
c) Find the average time a customer spends in the system in the shop W_s
d) What is the probability that the server is idle ?

6.A repairman is to be hired to repair machines which breakdown at the average rate of 3 per hour The breakdown follow Poisson distribution .Non –productive time of machine is considered to cost Rs 16/hour. Two repair man have been interviewed . one is slow but cheap while the other is fast and expensive.The slow repairman charges Rs.8 per hour and he services at the rate of 4 per hour The fast repairman demands Rs .10 per hour and services at the average rate 6 per hour . Which repairman should be hired?

7. On average 96 patients per 24 hour day require the service of an emergency clinic .Also an average , a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time .Suppose that it costs the clinic Rs.100 per patient treated to obtain an average service time of 10 minutes, and that each minute of decrease in this average time would cost Rs . 10 per patient treated .How much would have to be budgeted by the clinic to decrease the average size of the queue from 1 1/3 patients to ½ patient?

8. There are three typists in an office .Each typist can type an average of 6 Letters per hour .If letters arrive for being typed at the rate of 15 letters per hour,
a)What fraction of the time all the typists will be busy ?
b)What is the average number of letters waiting to be typed?
c)What is the average time a letter has to spend for waiting and for being typed ?
d)What is the probability that a letter will take longer than 20 min waiting to be typed ?

9. A telephone company is planning to install telephone booths in a new airport It has established the policy that a person should not have to wait more than 10 % of the times hew tries to use a phone. The demand for use is estimated to be Poisson with an average of 30 per hour. The average phone call has an exponential distribution with a mean time of 5 min. how many phone should be installed?

10.A bank has two tellers working on savings accounts. The first teller handles withdrawals only. The second teller handles deposits only .It has been found that the service time distributions for both deposits and withdrawals are exponential with mean time of 3 min per customer. Depositors are found to arrive in Poisson fashion throughout the day with mean arrival rate of 16 per hour Withdrawers also arrive in a Poisson fashion with mean arrival rate 14 per hour. What would be the effect on the average waiting time for the customers if each teller handles both withdrawals and deposits. What Would be the effect, if this could only be accomplished by increasing the service time to 3.5 min?

11. A super market has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4 min and if people arrive in Poisson fashion at the rate of 10 per hour
a) What is the probability that a customer has to wait for service?
b) What is the expected percentage of idle time for each girl?
c) If the customer has to wait in the queue, what is the expected length of the waiting time?

12. A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean of 6 min and cars arrive for service in a Poisson process at the rate of 30 cars per hour .
(a) What is the probability that an arrival would have to wait in line ?
(b) Find the average waiting time , average time spent in the system and the average number of cars in the system . (c) For what percentage of time would a pump be idle on an average?
13.Automatic car wash facility operates with only one bay . Cars arrive according to a Poisson process with mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy . If the service time for all cars is constant and equal to 10 min, Find L_S, Lq , W_S, Wq .

14. In a single server queueing system with Poisson input and exponential service times,if the mean arrival rate is 3 calling units per hour ,the expected service time is 0.25 h and the maximum possible number of calling units in the system is 2, find P_n ($n \geq 0$) ,average number of calling units in the system and in the queue and the average waiting time in the system and in the queue

15.At railway station, only one train is handled at a time.The railway yard is sufficient only for 2 trains to wait, while the other is given signalto leave station .Trains arrive at the station at an average of 6 per hour.Assuming poisson arrivals and exponential service distribution, find the average waiting time of a new train coming into the yard .If the handling rate is doubled, how will the above results get modified?

16.Patients arrive at a clinic according to Poisson distribtution at a rate of 30 patients per hour .The waiting room does not accomodate more than 14 patients.Examination time per patient is exponential with mean rate of 20 per hour
a)Find the effective arrival rate at the clinic
b) What is the probability that an arriving patient will not wait?
c)What is the expected waiting time until a patient is discharged from the clinic?

17. A 2 – person barber shop has 5 chair to accommodate waiting customers.Potential customers ,who arrive when all 5 chairs are full,leave without entering barber shop .Customers arrive at the average rate of 4 per hour and spend an average of 12 min in the barber's chair .Compute P_0, P_1 P_7,E(N_q) and E (w) .
18. A car servicing station has 2 bays where service can be offered simultaneously .Because of space limtation,only 4 vcars are accepted for servicing . The arrival pattern is Poisson with 12 cars per day.The service time in both bays is exponentially distributed with μ = 8 cars per day per bay. Find the average number of cars in the service station , the average number of cars waiting for service time a car spends in the system.
19. At a port there are 6 unloading berths and 4 unloading crews.When all the berths are full, arriving ships are diverted to an overflow facility 20 kms down the river .Tankers arrive acccording to a Poisson process with a mean rate of 1 every 2 hours.It takes for an unloading crew ,on the average ,10 h to unload a tanker ,the unloading time following an exponential distribution.Find
a) how many tankers are at the port on the average ?
b) how long does a tanker spend at the port on the average?
c)What is the arrival rate at the overflow facility?