AE 1007 –FINITE ELEMENT METHOD

PART A QUESTIONS WITH ANSWERS

UNIT 1

1. What is meant by finite element?

   A small units having definite shape of geometry and nodes is called finite element.

2. What is meant by node or joint?

   Each kind of finite element has a specific structural shape and is inter- connected with the adjacent element by nodal point or nodes. At the nodes, degrees of freedom are located. The forces will act only at nodes at any others place in the element.

3. What is the basic of finite element method?

   Discretization is the basis of finite element method. The art of subdividing a structure in to convenient number of smaller components is known as discretization.

4. What are the types of boundary conditions?

   Primary boundary conditions

   Secondary boundary conditions

5. State the methods of engineering analysis?

   Experimental methods

   Analytical methods

   Numerical methods or approximate methods

6. What are the types of element?

7. 1D element

   2D element

   3D element

8. State the three phases of finite element method.

   Preprocessing

   Analysis

   Post Processing

9. What is structural problem?

   Displacement at each nodal point is obtained. By these displacements solution stress and strain in each element can be calculated.

10. What is non structural problem?

    Temperature or fluid pressure at each nodal point is obtained. By using these values properties such as heat flow fluid flow for each element can be calculated.

10. What are the methods are generally associated with the finite element analysis?

    Force method

    Displacement or stiffness method.

11. Explain stiffness method.

    Displacement or stiffness method, displacement of the nodes is considered as the unknown of the problem. Among them two approaches, displacement method is desirable.

12. What is meant by post processing?

    Analysis and evaluation of the solution result is referred to as post processing. Postprocessor computer program help the user to interpret the result by displaying them in graphical form.

13. Name the variation methods.

    Ritz method.

    Ray-Leigh Ritz method.

14. What is meant by degrees of freedom?

    When the force or reaction act at nodal point node is subjected to deformation. The deformation includes displacement rotation, and or strains. These are collectively known as degrees of freedom

15. What is meant by discretization and assemblage?

    The art of subdividing a structure in to convenient number of smaller components is known as discretization. These smaller components are then put together. The process of uniting the various elements together is called assemblage.

16. What is Rayleigh-Ritz method?

    It is integral approach method which is useful for solving complex structural problem, encountered in finite element analysis. This method is possible only if a suitable function is available.

17. What is Aspect ratio?

    It is defined as the ratio of the largest dimension of the element to the smallest dimension. In many cases, as the aspect ratio increases the in accuracy of the solution increases. The conclusion of many researches is that the aspect ratio should be close to unity as possible.

18. What is truss element?

The truss elements are the part of a truss structure linked together by point joint which transmits only axial force to the element.

19. What are the h and p versions of finite element method?

It is used to improve the accuracy of the finite element method. In h version, the order of polynomial approximation for all elements is kept constant and the numbers of elements are increased. In p version, the numbers of elements are maintained constant and the order of polynomial approximation of element is increased.

20. Name the weighted residual method

Point collocation method

Sub domain collocation method

Least squares method

Galerkins method.

UNIT 2

21. List the two advantages of post processing.

Required result can be obtained in graphical form. Contour diagrams can be used to understand the solution easily and quickly.

22. During discretization, mention the places where it is necessary to place a node?

Concentrated load acting point

Cross-section changing point

Different material interjections

Sudden change in point load

23. What is the difference between static and dynamic analysis?

Static analysis: The solution of the problem does not vary with time is known as static analysis

Example: stress analysis on a beam

Dynamic analysis: The solution of the problem varies with time is known as dynamic analysis

Example: vibration analysis problem.

24. Name any four FEA softwares.

ANSYS

NASTRAN

COSMOS

25. Differentiate between global and local axes.

Local axes are established in an element. Since it is in the element level, they change with the change in orientation of the element. The direction differs from element to element.

Global axes are defined for the entire system. They are same in direction for all the elements even though the elements are differently oriented.

26. Distinguish between potential energy function and potential energy functional

If a system has finite number of degree of freedom (q_1,q_2,and q_3), then the potential energy expressed as,

$\pi = f$ (q_1,q_2,and q_3)

It is known as function. If a system has infinite degrees of freedom then the potential energy is expressed as

$$\int f\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2} \ldots\right)dx$$

27. What are the types of loading acting on the structure?

Body force (f)

Traction force (T)

Point load (P)

28. Define the body force

A body force is distributed force acting on every elemental volume of the body Unit: Force per unit volume.

Example: Self weight due to gravity

29. Define traction force

Traction force is defined as distributed force acting on the surface of the body. Unit: Force per unit area.

Example: Frictional resistance, viscous drag, surface shear

30. What is point load?

Point load is force acting at a particular point which causes displacement.

31. What are the basic steps involved in the finite element modeling.

Discretization of structure.

Numbering of nodes.

32. Write down the general finite element equation.

$$\{F\}=[K]\{u\}$$

33. What is discretization?

The art of subdividing a structure in to a convenient number of smaller components is known as discretization.

34. What are the classifications of coordinates?

Global coordinates

Local coordinates

Natural coordinates

35. What is Global coordinates?

The points in the entire structure are defined using coordinates system is known as global coordinate system.

36. What is natural coordinates?

A natural coordinate system is used to define any point inside the element by a set of dimensionless number whose magnitude never exceeds unity. This system is very useful in assembling of stiffness matrices.

37. Define shape function.

Approximate relation $\varphi (x,y) = N_1 (x,y) \varphi_1 + N_2 (x,y) \varphi_2 + N_3 (x,y) \varphi_3$

Where $\varphi 1$, φ_2, and φ_3 are the values of the field variable at the nodes N_1, N_2, and N_3 are the interpolation functions.

N_1, N_2, and N_3 are also called shape functions because they are used to express the geometry or shape of the element.

38. What are the characteristic of shape function?

It has unit value at one nodal point and zero value at other nodal points. The sum of shape function is equal to one.

39. Why polynomials are generally used as shape function?

Differentiation and integration of polynomial are quit easy.

The accuracy of the result can be improved by increasing the order of the polynomial. It is easy to formulate and computerize the finite element equations

40. How do you calculate the size of the global stiffness matrix?

Global stiffness matrix size = Number of nodes X Degrees of freedom per node

UNIT 3

41. Write down the expression of stiffness matrix for one dimensional bar element.

$$[K]=\frac{AE}{l}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

42. State the properties of stiffness matrix

It is a symmetric matrix

The sum of elements in any column must be equal to zero
It is an unstable element. So the determinant is equal to zero.

43. Write down the expression of stiffness matrix for a truss element.

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

44. Write down the expression of shape function N and displacement u for one dimensional bar element.

U= $N_1u_1+N_2u_2$

N_1= 1-X /

l N_2 = X / l

45. Define total potential energy.

Total potential energy, π = Strain energy (U) + potential energy of the external forces (W)

46. State the principle of minimum potential energy.

Among all the displacement equations that satisfied internal compatibility and the boundary condition those that also satisfy the equation of equilibrium make the potential energy a minimum is a stable system.

47. Write down the finite element equation for one dimensional two noded bar element.

$$[K]=\frac{AE}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

48. What is truss?

A truss is defined as a structure made up of several bars, riveted or welded together.

49. States the assumption are made while finding the forces in a truss.

All the members are pin jointed.

The truss is loaded only at the joint

The self weight of the members is neglected unless stated.

50. State the principles of virtual energy?

A body is in equilibrium if the internal virtual work equals the external virtual work for the every kinematically admissible displacement field

51. What is essential boundary condition?

Primary boundary condition or EBC Boundary condition which in terms of field variable is known as Primary boundary condition.

52. Natural boundary conditions?

Secondary boundary natural boundary conditions which are in the differential form of field variable is known as secondary boundary condition

53. How do you define two dimensional elements?

Two dimensional elements are define by three or more nodes in a two dimensional plane. The basic element useful for two dimensional analysis is the triangular element.

54. What is CST element?

Three noded triangular elements are known as CST. It has six unknown displacement degrees of freedom (u1, v1, u2, v2, u3, v3). The element is called CST because it has a constant strain throughout it.

55. What is LST element?

Six nodded triangular elements are known as LST. It has twelve unknown displacement degrees of freedom. The displacement function for the elements are quadratic instead of linear as in the CST.

56. What is QST element?

Ten nodded triangular elements are known as Quadratic strain triangle. It is also called as cubic displacement triangle.

58. What meant by plane stress analysis?

Plane stress is defined to be a state of stress in which the normal stress and shear stress directed perpendicular to the plane are assumed to be zero.

60. Define plane strain analysis.

Plane strain is defined to be state of strain normal to the xy plane and the shear strains are assumed to be zero.

UNIT 4

61. Write down the stiffness matrix equation for two dimensional CST elements.

Stiffness matrix $[K]=[B]^T[D][B]At$

$[B]^T$ -Strain displacement $[D]$-Stress strain matrix $[B]$-Strain displacement matrix

62. Write down the stress strain relationship matrix for plane stress conditions.

$$\frac{E}{(1+\nu)(1-2\nu)}\begin{pmatrix} 1-\nu & \nu & 0 \\ 0 & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{pmatrix}$$

63. What is axisymmetric element?

Many three dimensional problem in engineering exhibit symmetry about an axis of rotation such type of problem are solved by special two dimensional element called the axisymmetric element

64. What are the conditions for a problem to be axisymmetric?

The problem domain must be symmetric about the axis of revolution All boundary condition must be symmetric about the axis of revolution All loading condition must be symmetric about the axis of revolution

65. Give the stiffness matrix equation for an axisymmetric triangular element. Stiffness matrix $[K]=[B]^T[D][B]2\pi rA$

66. What is the purpose of Isoparametric element?

It is difficult to represent the curved boundaries by straight edges finite elements. A large number of finite elements may be used to obtain reasonable resemblance between original body and the assemblage.

67. Write down the shape functions for 4 noded rectangular elements using natural coordinate system.

$$N_1 = \frac{1}{4}(1-\varepsilon)(1-\eta) \qquad N_2 = \frac{1}{4}(1+\varepsilon)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\varepsilon)(1+\eta) \qquad N_4 = \frac{1}{4}(1-\varepsilon)(1+\eta)$$

68. Write down Jacobian matrix for 4 noded quadrilateral elements.

$$[J] = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}$$

69. Write down stiffnes matrix equation for 4 noded isoparametric quadrilateral elements.

Stiffness matrix $[K] = t\int_{-1}^{1}\int_{-1}^{1} [B]^T[D][B]|J|\partial\varepsilon\partial\eta$

70. Define super parametric element.

If the number of nodes used for defining the geometry is more than of nodes used for defining the displacement is known as super parametric element

71. Define sub parametric element.

If the number of nodes used for defining the geometry is less than number of nodes used for defining the displacement is known as sub parametric element.

72. What is meant by Isoparametric element?

If the number of nodes used for defining the geometry is same as number of nodes used for defining the displacement is known as Isoparametric element.

73. Is beam element an Isoparametric element?

Beam element is not an Isoparametric element since the geometry and displacement are defined by different order interpretation functions.

74. What is the difference between natural coordinate and simple natural coordinate?

L1 = 1-x/l

L2 = x/l

75. What is Area coordinates?

L1 = A1/A L2 = A2/A L3 = A3/A

76. What is simple natural coordinate?

A simple natural coordinate is one whose value between -1 and 1.

77. Give example for essential boundary conditions.

The geometry boundary condition are displacement, slope.

78. Give example for non essential boundary conditions.

The natural boundary conditions are bending moment, shear force

79. What is meant by degrees of freedom?

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UNIT 5

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82. Define plane strain analysis.

Plane strain is defined to be state of strain normal to the x,y plane and the shear strains are assumed to be zero.

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Concentrated load acting point

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Example: stress analysis on a beam

Dynamic analysis: The solution of the problem varies with time is known as dynamic analysis

Example: vibration analysis problem.

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94. State the principles of virtual energy?

A body is in equilibrium if the internal virtual work equals the external virtual work for the every kinematically admissible displacement field.

95. Define Eigen value problem.

The problem of determining the constant is called eigen value problem.

$$-\frac{d}{dx}\left(a\frac{du}{dx}\right) = \lambda u$$

96. What is non-homogeneous form?

When the specified values of dependent variables are non-zero, the boundary conditi said to be non-homogeneous.

97. What is homogeneous form?

When the specified values of dependent variables is zero, the boundary condition are said to be homogeneous.

98. Define initial value problem.

An initial value problem is one in which the dependent variable and possibly is derivatives are specified initially.

99. Define boundary value problem.

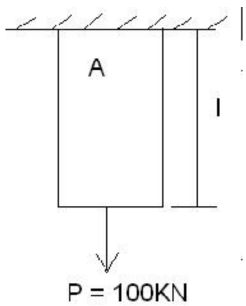
A differential equation is said to describe a boundary value problem if the dependent variable and its derivatives are required to take specified values on the boundary.

100. Define governing equation.

$$\frac{d}{dx}\left(EA\frac{du}{dx}\right) + \rho A = 0$$

16 MARKS QUESTIONS & ANSWER

1. Using Rayleigh Ritz methods calculate the deflection at the middle and end for the following cantilever beam.



A=80 sq.cm, l=300 cm, E= $2x10^7$ N/ sq.cm, $\rho = 0.075$ N/ cm^3

$$EA\begin{bmatrix} l & l^2 \\ l^2 & \frac{4}{3}l^3 \end{bmatrix}\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \rho A \begin{Bmatrix} \frac{l^2}{2} \\ \frac{l^3}{3} \end{Bmatrix} + P\begin{Bmatrix} l \\ l^2 \end{Bmatrix}$$

$$2\,X\,10^7\,X\,80 = \begin{bmatrix} 300 & 300^2 \\ 300^2 & \frac{4}{3}300^3 \end{bmatrix}\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = 0.075X\,80\begin{Bmatrix} \frac{300^2}{2} \\ \frac{300^3}{3} \end{Bmatrix} + 10^5\begin{Bmatrix} 300 \\ 300^2 \end{Bmatrix}$$

a_1= 6.36X10^{-5} a_2= -1.88X10^{-9}

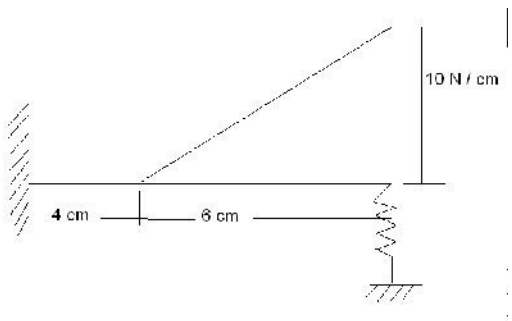
2. Solve the given problem for temperature distribution consider the end convection coefficient .Take t_1=170^0C , h_1= 0.0025 w/ cm^2 ^{0}c, h_2= 0.0625 w/ cm^2 ^{0}c Tα=25^0c, l=10 cm, k= 0.17 w/ cm ^{0}c

$$\frac{KA}{he}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hphe}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} t_1 \\ t_2 \end{Bmatrix} = \frac{hpT\alpha he}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\frac{0.17\,X\,\pi X\,1^2}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{0.0025\,X\,\pi X\,2\,X\,2}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} t_1 \\ t_2 \end{Bmatrix} = \frac{0.0025\,X\,\pi X\,25\,X\,2\,X\,2}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

T_1=170^0 C, T_2=73^0C

3. A beam with clamped support at one end and spring support at the other end. A linearly varying transverse load of max.magnitude 100 N/cm is applied in the span 4 cm <x<10 cm. solve the problem by FEM method EI= 2X10^7 N cm^2

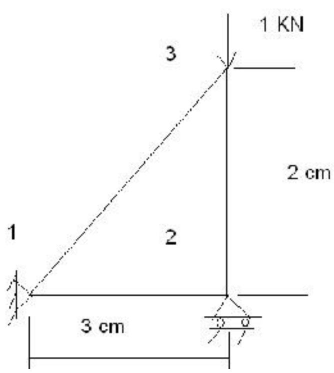


$$[K]=\frac{EI}{h^3}\begin{bmatrix}12 & -6h_e & -12 & -6h_e\\ -6h_e & 4h_e^2 & 6h_e & 2h_e^2\\ -12 & 6h_e & 12 & 6h_e\\ -6h_e & 2h_e^2 & 6h_e & 4h_e^2\end{bmatrix}\begin{Bmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{Bmatrix} = f(x)\begin{Bmatrix}\frac{h_e}{2}\\ -\frac{h_e^2}{2}\\ \frac{h_e}{2}\\ \frac{h_e^2}{2}\end{Bmatrix}+\begin{Bmatrix}Q_1\\ Q_2\\ Q_3\\ Q_4\end{Bmatrix}$$

$$\frac{2X10^7}{4^3}\begin{bmatrix}12 & -24 & -12 & -24\\ -24 & 64 & 24 & 32\\ -12 & 24 & 12 & 24\\ -24 & 32 & 24 & 64\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\\ u_3\\ u_4\end{Bmatrix}=\begin{Bmatrix}-90\\ 120\\ -210\\ 180\end{Bmatrix}+\begin{Bmatrix}Q_1\\ Q_2\\ Q_3\\ Q_4\end{Bmatrix}$$

U_1= 0.0001 mm, U_2= 0.0001 mm, U_3= 0.0001 mm, U_4= 0.0001 mm

4. Evaluate the displacement at node 1, 2. Take t=0.5 cm, E=$2X10^7$ N/cm^2 μ = 0.27 using plane stress condition.



$$[K]=tAB^TDB$$

$$D=\frac{E}{1-\mu^2}\begin{bmatrix}1 & \mu & 0\\ \mu & 1 & 0\\ 0 & 0 & \frac{1-\mu}{2}\end{bmatrix}$$

$$B=\frac{1}{2A}\begin{bmatrix}y_{23} & 0 & y_{31} & 0 & y_{12} & 0\\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21}\\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12}\end{bmatrix}$$

$$A=\frac{1}{2}bh$$

$$A=\frac{1}{2}X\,3\,X\,2=3cm\,,\quad B=\frac{1}{6}\begin{bmatrix}2 & 0 & 0 & 0 & -2 & 0\\ 0 & -3 & 0 & 3 & 0 & 0\\ -3 & 2 & 3 & 0 & 0 & -2\end{bmatrix},\ B^T=\frac{1}{6}\begin{bmatrix}2 & 0 & -3\\ 0 & -3 & 2\\ 0 & 0 & 3\\ 0 & 3 & 0\\ -2 & 0 & 0\\ 0 & 0 & -2\end{bmatrix},$$

$$D=\frac{2\,X10^7}{1-0.27^2}\begin{bmatrix}1 & 0.27 & 0\\ 0.27 & 1 & 0\\ 0 & 0 & \frac{1-0.27}{2}\end{bmatrix}$$

5. Tow point Gauss integration method

$$\int_{-1}^{1} f(r)dr=w_1 f(r_1)+w_2 f(r_2)$$

$$\int_{-1}^{1} f(r)dr=w_1 f(r_1)+w_2 f(r_2)$$

$$\int_{-1}^{1} f(r)dr-w_1 f(r_1)+w_2 f(r_2)=0$$

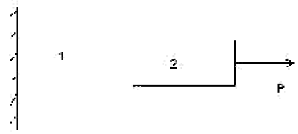
$$\int_{-1}^{1}(1+r+r^2+r^3)dr-\left[w_1 f(r_1)+w_2 f(r_2)=0\right]$$

$W_1+W_2=2$

$W_1=W_2=1$

$r_1 = r_2 = +\text{or} - 0.57735$

6. Consider the bar element as shown in fig. An axial load of 200kN is applied at point P. Take A_1= 2400 mm^2 , $E_1 = 70 * 10^9$ N/m^2, A_2= 600 mm^2, E_2=200 * 10^9 N/m^2.



Calculate the following: The nodal displacement at point P, Stress in each material, Reaction force.

Finite element equation for one dimensional two nodded bar element is

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

Element 1 $\frac{A_1 E_1}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$ $1 \times 10^5 \begin{bmatrix} 5.6 & -5.6 \\ -5.6 & 5.6 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$

Assemble the finite elements,

$$1 \times 10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

Boundary Conditions: $u_1 = u_3 = 0$, $F_1 = F_3 = 0$

$$1 \times 10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2 \times 10^5 \\ 0 \end{Bmatrix}$$

$1 \times 10^5 \, [8.6]\{u_2\} = \{2 \times 10^5\}$ $u_2 = 0.2325$ mm

Stress in each element, $\sigma = E \frac{du}{dx}$

$\sigma_1 = E_1 \frac{u_2 - u_1}{l_1}$ $\sigma_1 = 54.25$ N/mm^2

$\sigma_2 = E_2 \frac{u_3 - u_2}{l_2}$ $\sigma_2 = -116.25$ N/mm^2

Reaction Force:

$\{R\} = [K]\{u\} - \{F\}$ $R_1 = -1.302 \times 10^5$ N, $R_2 = 0$, $R_3 = -0.6975 \times 10^5$ N

7. Consider the bar element Area of element 1, A_1=300 mm^2 Area of element 2, A_2= 300 mm^2 ,Area of element 3, A_3=600 mm^2 ,Length of element 1, l_1=200mm,Length of element 2, l_2=200mm, Length of element 3, l_3=400mm, E = 2*10^5 N/mm^2,Point Load p = 400kN =400 * 10^3 N Calculate The nodal displacement at point P,

Finite element equation for one dimensional two nodded bar element is

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{AE}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

Element 1: $$1 \times 10^5 \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

Element 2: $$\frac{A_2E_2}{l_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} \quad 1 \times 10^5 \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}$$

Element 3: $$\frac{A_3E_3}{l_3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} \quad 1 \times 10^5 \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}\begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix}$$

Assemble the finite elements, $$1 \times 10^5 \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 6 & -3 & 0 \\ 0 & -3 & 6 & -3 \\ 0 & 0 & -3 & 3 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

Boundary Conditions: $u_1 = u_4 = 0$, $F_1 = F_3 = F_4 = 0$

$$1 \times 10^5 \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 6 & -3 & 0 \\ 0 & -3 & 6 & -3 \\ 0 & 0 & -3 & 3 \end{bmatrix}\begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 400 \times 10^3 \\ 0 \\ 0 \end{Bmatrix}$$

u_3 = 0.4444 mm, u_2 = 0.8888 mm

8. For a tapered plate of uniform thickness t=10mm, find the displacement at the nodes by forming into two element model. The bar has mass density ρ=7800 kg/m3, Youngs modulus, E=2*10^5MN/m2.In addition to self-weight, the plate is subjected to the point load p=10kN at its centre.

$$\{F\} = \frac{\rho A l}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$$

Element 1: $\begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \frac{\rho_1 A^*{}_1 l_1}{2}\begin{Bmatrix}1\\1\end{Bmatrix} \quad \begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \begin{Bmatrix}4.017\\4.017\end{Bmatrix}$

Element 2: $\begin{Bmatrix}F_2\\F_3\end{Bmatrix} = \begin{Bmatrix}2.869\\2.869\end{Bmatrix}$

Global Force Vector,

$$\begin{Bmatrix}F_1\\F_2\\F_3\end{Bmatrix} = \begin{Bmatrix}4.017\\10006.886\\2.869\end{Bmatrix}$$

Finite element equation for one dimensional two nodded bar element is

$$\begin{pmatrix}F_1\\F_2\end{pmatrix} = \begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix} \qquad \frac{A_1E_1}{l_1}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix} = \begin{pmatrix}F_1\\F_2\end{pmatrix}$$

$$2\times 10^5\begin{bmatrix}4.666 & -4.666\\-4.666 & 4.666\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix} = \begin{pmatrix}F_1\\F_2\end{pmatrix}$$

$$\frac{A_2E_2}{l_2}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\begin{pmatrix}u_2\\u_3\end{pmatrix} = \begin{pmatrix}F_2\\F_3\end{pmatrix} \qquad 2\times 10^5\begin{bmatrix}3.333 & -3.333\\-3.333 & 3.333\end{bmatrix}\begin{pmatrix}u_2\\u_3\end{pmatrix} = \begin{pmatrix}F_2\\F_3\end{pmatrix}$$

Assemble the finite elements,

$$2\times 10^5\begin{bmatrix}4.666 & -4.666 & 0\\-4.666 & 7.999 & -3.333\\0 & -3.333 & 3.333\end{bmatrix}\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix} = \begin{Bmatrix}F_1\\F_2\\F_3\end{Bmatrix}$$

Boundary Conditions: $u_1 = 0$

$$2\times 10^5\begin{bmatrix}4.666 & -4.666 & 0\\-4.666 & 7.999 & -3.333\\0 & -3.333 & 3.333\end{bmatrix}\begin{Bmatrix}0\\u_2\\u_3\end{Bmatrix} = \begin{Bmatrix}4.017\\10006.886\\2.869\end{Bmatrix}$$

$u_3 = u_2 = 0.01073$ mm

9. A rod subjected to an axial load p = 600kN is applied at end Divide the domain into two elements. Determine the following: The nodal displacement at point P, Stress in each material. Take A=250mm^2,E=2*10^5N/mm, The gap between the wall and

node 3 is 1.2mm.

The gap between the wall and node 3 is 1.2mm.(i.e.) u_3 = 1.2mm

Finite element equation for one dimensional two nodded bar element is

Element

$$\frac{A_1E_1}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix}=\begin{pmatrix}F_1\\ F_2\end{pmatrix} \quad 1\times 10^5\begin{bmatrix}3.333 & -3.333\\ -3.333 & 3.333\end{bmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix}=\begin{pmatrix}F_1\\ F_2\end{pmatrix}$$

$$\frac{A_2E_2}{l_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{pmatrix}u_2\\ u_3\end{pmatrix}=\begin{pmatrix}F_2\\ F_3\end{pmatrix} \quad 1\times 10^5\begin{bmatrix}3.333 & -3.333\\ -3.333 & 3.333\end{bmatrix}\begin{pmatrix}u_2\\ u_3\end{pmatrix}=\begin{pmatrix}F_2\\ F_3\end{pmatrix}$$

Assemble the finite elements,

$$1\times 10^5\begin{bmatrix}3.333 & -3.333 & 0\\ -3.333 & 6.666 & -3.333\\ 0 & -3.333 & 3.333\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\\ u_3\end{Bmatrix}=\begin{Bmatrix}F_1\\ F_2\\ F_3\end{Bmatrix}$$

Boundary Conditions:$u_1 = 0$, $u_3 = 1.2$ mm, $F_1 = F_3 = 0$

$$1\times 10^5\begin{bmatrix}3.333 & -3.333 & 0\\ -3.333 & 6.666 & -3.333\\ 0 & -3.333 & 3.333\end{bmatrix}\begin{Bmatrix}0\\ u_2\\ 0\end{Bmatrix}=\begin{Bmatrix}0\\ 6\times 10^5\\ 0\end{Bmatrix}$$

u_2 = 1.50 mm

Stress in each element, $\sigma = E\frac{du}{dx}$ $\sigma_1 = E_1\frac{u_2-u_1}{l_1}$ σ_1 = 2000 N/mm^2

$$\sigma_2 = E_2\frac{u_3-u_2}{l_2}$$ σ_2 = - 400 N/mm^2

10. A thin plate of uniform thickness 25mm is subjected to a point load of 25N at mid depth, the plate is also subjected to self-weight. If E=2*10^5 N/mm2, ρ=.8*10^{-4} N/mm^3. Calculate, Nodal displacement

Area at node, A=Width*Thickness

A1=2500 mm2, A2=2000 mm2

Body Force Vector , $\{F\} = \frac{\rho A l}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$

Element 1: $\begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \frac{\rho_1 A_1 l_1}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$ $\begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \begin{Bmatrix}20\\20\end{Bmatrix}$

Element 2: $\begin{Bmatrix}F_2\\F_3\end{Bmatrix} = \begin{Bmatrix}16\\16\end{Bmatrix}$

Global Force Vector, $\begin{Bmatrix}F_1\\F_2\\F_3\end{Bmatrix} = \begin{Bmatrix}20\\20+16+420\\16\end{Bmatrix}$

Finite element equation for one dimensional two nodded bar element is

$$\begin{pmatrix}F_1\\F_2\end{pmatrix} - \begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}$$

Element 1:

$$\frac{A_1 E_1}{l_1}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix} = \begin{pmatrix}F_1\\F_2\end{pmatrix} \quad 2\times 10^5\begin{bmatrix}12.5 & -12.5\\-12.5 & 12.5\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix} = \begin{pmatrix}F_1\\F_2\end{pmatrix}$$

Element 2:

$$\frac{A_2 E_2}{l_2}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\begin{pmatrix}u_2\\u_3\end{pmatrix} = \begin{pmatrix}F_2\\F_3\end{pmatrix} \quad 2\times 10^5\begin{bmatrix}10 & -10\\-10 & 10\end{bmatrix}\begin{pmatrix}u_2\\u_3\end{pmatrix} = \begin{pmatrix}F_2\\F_3\end{pmatrix}$$

Assemble the finite elements,

$$2\times 10^5\begin{bmatrix}12.5 & -12.5 & 0\\-12.5 & 22.5 & -10\\0 & -10 & 10\end{bmatrix}\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix} = \begin{Bmatrix}20\\456\\16\end{Bmatrix}$$

Boundary Conditions: $u_1 = 0$

$$2\times 10^5\begin{bmatrix}12.5 & -12.5 & 0\\-12.5 & 22.5 & -10\\0 & -10 & 10\end{bmatrix}\begin{Bmatrix}0\\u_2\\u_3\end{Bmatrix} = \begin{Bmatrix}20\\456\\16\end{Bmatrix}$$

$u_3 = 1.968 * 10^{-4}$ mm, $u_2 = 1.888 * 10^{-4}$ mm

Stress in each element,

$$\sigma_1 = E_1 \frac{u_2 - u_1}{l_1} \qquad \sigma_1 = .188 \text{ N/mm}^2$$

$$\sigma_2 = F_2 \frac{u_3 - u_2}{l_2} \qquad \sigma_2 = 0.008 \text{ N/mm}^2$$

11. A spring assemblage with arbitrarily numbered nodes. The nodes 1 & 2 are fixed & a force of 500 kN is applied at node 4 in the x direction. Calculate the following; Global stiffness matrix, Nodal displacements

Solution: Finite element equation for spring element is

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = k_1 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

Element 1: $k_1 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{pmatrix} u_1 \\ u_3 \end{pmatrix} = \begin{pmatrix} F_1 \\ F_3 \end{pmatrix} \quad \begin{bmatrix} 100 & -100 \\ -100 & 100 \end{bmatrix} \begin{pmatrix} u_1 \\ u_3 \end{pmatrix} = \begin{pmatrix} F_1 \\ F_3 \end{pmatrix}$

Element 2: $k_2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{pmatrix} u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} F_3 \\ F_4 \end{pmatrix} \quad \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \begin{pmatrix} u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} F_3 \\ F_4 \end{pmatrix}$

Element 3: $k_3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{pmatrix} u_4 \\ u_2 \end{pmatrix} - \begin{pmatrix} F_4 \\ F_2 \end{pmatrix} \quad \begin{bmatrix} 300 & -300 \\ -300 & 300 \end{bmatrix} \begin{pmatrix} u_4 \\ u_2 \end{pmatrix} - \begin{pmatrix} F_4 \\ F_2 \end{pmatrix}$

Assemble the finite elements, $\begin{bmatrix} 100 & 0 & -100 & 0 \\ 0 & 300 & 0 & 300 \\ -100 & 0 & 300 & -200 \\ 0 & -300 & -200 & 500 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$

Boundary Conditions: $u_1 = u_2 = 0$, $F_1 = F_3 = F_2 = 0$

$$\begin{bmatrix} 100 & 0 & -100 & 0 \\ 0 & 300 & 0 & -300 \\ -100 & 0 & 300 & -200 \\ 0 & -300 & -200 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 500 \end{Bmatrix}$$

$u_3 = 0.9091$ mm, $u_4 = 1.364$ mm

12. Determine the nodal displacements in each element & the reactions. E = 70 GPa , A

= 2 * 10 ^ -4 m2, For the bar assemblage, k = 2000 kN/m.

Element 1:

$$\frac{A_1 E_1}{l_1}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \quad 10^3 \begin{bmatrix} 7 & -7 \\ -7 & 7 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$$

Element 2:

$$\frac{A_2 E_2}{l_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{pmatrix} u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} F_2 \\ F_3 \end{pmatrix} \quad 10^3 \begin{bmatrix} 7 & -7 \\ -7 & 7 \end{bmatrix}\begin{pmatrix} u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} F_2 \\ F_3 \end{pmatrix}$$

Element 3:

$$\frac{A_3 F_3}{l_3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{pmatrix} u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} F_3 \\ F_4 \end{pmatrix} \quad 10^3 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{pmatrix} u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} F_3 \\ F_4 \end{pmatrix}$$

Assemble the finite elements,

$$10^3 \begin{bmatrix} 7 & -7 & 0 & 0 \\ -7 & 14 & -7 & 0 \\ 0 & -7 & 9 & -2 \\ 0 & 0 & -2 & 2 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

Boundary Conditions: $u_1 = u_4 = 0$, $F_1 = F_3 = F_4 = 0$

$$10^3 \begin{bmatrix} 7 & -7 & 0 & 0 \\ -7 & 14 & -7 & 0 \\ 0 & 7 & 9 & 2 \\ 0 & 0 & -2 & 2 \end{bmatrix}\begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 8 \\ 0 \\ 0 \end{Bmatrix}$$

$u_3 = 0.727 * 10 ^\wedge -3$ mm, $u_2 = 0.935 * 10 ^\wedge -3$ mm

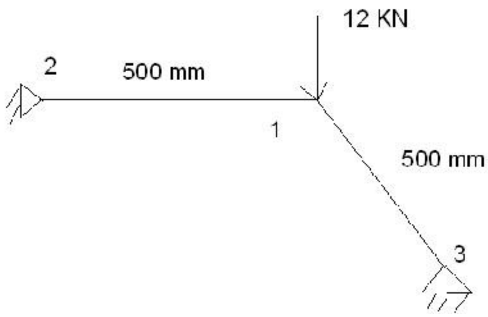
Reaction Force: $\{R\} = [K]\{u\} - \{F\}$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{Bmatrix} = 10^3 \begin{bmatrix} 7 & -7 & 0 & 0 \\ -7 & 14 & -7 & 0 \\ 0 & -7 & 9 & -2 \\ 0 & 0 & -2 & 2 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} - \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{Bmatrix} = 10^3 \begin{bmatrix} 7 & -7 & 0 & 0 \\ -7 & 14 & -7 & 0 \\ 0 & -7 & 9 & -2 \\ 0 & 0 & -2 & 2 \end{bmatrix} \begin{Bmatrix} 0 \\ .935 \times 10^{-3} \\ .727 \times 10^{-3} \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 8 \\ 0 \\ 0 \end{Bmatrix}$$

R_1 = -6.546 k N, R_2 = 0, R_3 = 0, R_4 = -1.45 k N

13. For the two bar truss shown in fig determine the displacement of node 1 & stress in element 1-3.



Element 1:

$l_{e1} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ l_{e1} = 500 mm

$l_1 = \dfrac{x_2 - x_1}{l_{e1}}$, $l_1 = -1$, $m_1 = \dfrac{y_2 - y_1}{l_{e1}}$, $m_1 = 0$

Element 2: $l_{e2} = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$, l_{e1} = 500 mm

$l_2 - \dfrac{x_3 - x_1}{l_{e2}}$, $l_2 = 0.8$, $m_2 - \dfrac{y_3 - y_1}{l_{e2}}$, $m_1 = -0.6$

Stiffness matrix, $[K] = \dfrac{AE}{l_e} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$

$$[K_1] = 28 \times 10^3 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[K_2] = 28 \times 10^3 \begin{bmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.36 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{bmatrix}$$

Assembled Stiffness matrix is,

$$[K] = 28*10\text{^}3 \begin{bmatrix} 1.64 & -0.48 & -1 & 0 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0 & 0 & 0.48 & -0.36 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.64 & 0.48 & 0 & 0 & 0.64 & -0.48 \\ -0.48 & -0.36 & 0 & 0 & -0.48 & 0.36 \end{bmatrix}$$

$\{F\} = [K]\{u\}$

Boundary conditions,

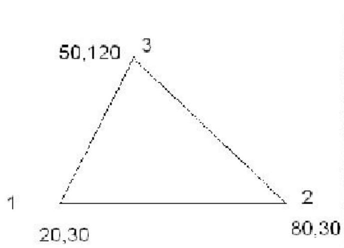
u at 3,4,5,6 is 0, F at 1,3,4,5,and 6 is 0

By elimination,

$28 \times 10^3 \begin{bmatrix} 1.64 & -0.48 \\ -0.48 & 0.36 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} \\ -12\ 10^3 \end{Bmatrix}$ u_1 = -0.571 mm, u_2 = -1.952 mm

$\sigma = \frac{E}{l_e}[-l \quad -m \quad l \quad m] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$ Stress in element 2 is, σ_2 = -100 N /mm^2

14. Determine the stiffness matrix for the CST shown in fig. Assume plane stress conditions. Co ordinates are in mm. Take E = 210 GPa , υ = 0.25, t= 10 mm



$[K] = [B]^T[D][B]A\,t$

Area of element, $A = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$ A= 2700 mm^2

Strain- displacement matrix $[B] = \frac{1}{2A}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$

$$[B] = 5.555 \times 10^{-3} \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

Stress strain relationship matrix,

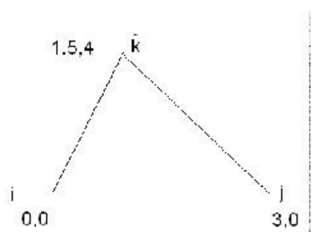
$$[D] = \frac{E}{1-\upsilon^2}\begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon}{2} \end{bmatrix}$$

$$[D] = 56 \times 10^3 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

Stiffness matrix,

$$[K] = 46.656 \times 10^3 \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix} \text{N/mm}$$

15. Evaluate the element stiffness matrix for the triangular element shown in fig. Assume plane stress condition. Take E= 2 * 10^ 5GPa, υ=0.3, t= 10mm.



Stiffness matrix,

$$[K] = [B]^T[D][B]A\,t$$

Area of element, $A = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$ A= 6 mm^2

Strain- displacement matrix

$$[B] = \frac{1}{2A}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$$[B] = \frac{1}{12}\begin{bmatrix} -4 & 0 & 4 & 0 & 0 & 0 \\ 0 & -1.5 & 0 & -1.5 & 0 & 3 \\ -1.5 & -4 & -1.5 & 4 & 3 & 0 \end{bmatrix}$$

Stress strain relationship matrix,

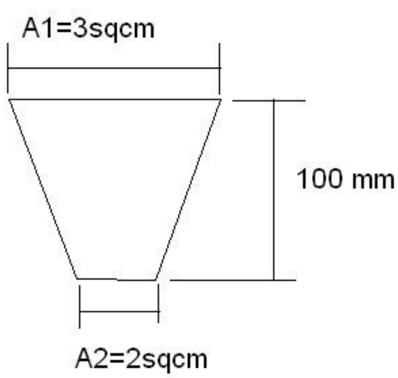
$$[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon}{2} \end{bmatrix}$$

$$[D] - 219.78 \times 10^3 \begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 0.35 \end{bmatrix}$$

Stiffness matrix,

$$[K] = 91.6 \times 10^3 \begin{bmatrix} 16.787 & 3.9 & -15.213 & -0.3 & -1.575 & -3.6 \\ 3.9 & 7.85 & 0.3 & -3.35 & -4.2 & -4.5 \\ -15.213 & 0.3 & 16.787 & -3.9 & -1.575 & 3.6 \\ -0.3 & -3.35 & -3.9 & 7.85 & 4.2 & -4.5 \\ -1.575 & -4.2 & -1.575 & 4.2 & 3.15 & 0 \\ -3.6 & -4.5 & 3.6 & -4.5 & 0 & 0 \end{bmatrix}$$

16. Using two finite elements, find the displacement in a uniformly tapering bar of circular cross-sectional area $3cm^2$ and $2cm^2$ at their ends, length 100mm, subjected to an axial tensile load of 50 N at smaller end and fixed at larger end. Take the value of young's modulus $2x10^5$.



$A_1 = (A_1 + A_2)/2 = 2.75cm^2$

$A_2 = (A_2 + A_3)/2 = 2.25cm^2$

$K^1 = E_1\, A_1/L_1 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$K^1 = 10^6 \begin{bmatrix} 1.1 & -1.1 \\ -1.1 & 1.1 \end{bmatrix}$

$K^2 = 10^6 \begin{bmatrix} 0.9 & -0.9 \\ -0.9 & 0.9 \end{bmatrix}$

$K = K^1 + K^2$

$K = 10^6 \begin{bmatrix} 1.1 & -1.1 & 0 \\ -1.1 & 2 & -0.9 \\ 0 & -0.9 & 0.9 \end{bmatrix}$

$\{F\} = [K]\{u\}$

Applying boundary condition,

Since 1 and 3 neglect 1and 3 row and column,

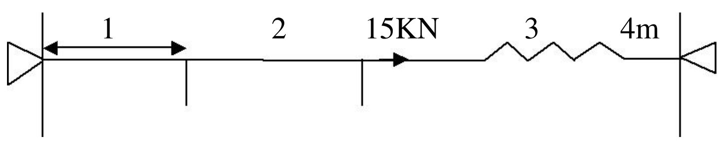
We get,

Result:

$U_2 = 4.545 \times 10^{-5}$ mm

$U_3 = 1.01 \times 10^{-4}$ mm

17. For the bar assemblage as shown in fig. Determine (i)Global stiffness matrix (ii)Nodal displacement



E=70Gpa, $A = 2 \times 10^{-4} m^2$,K = 2000KN/m

$K^1 = E_1 A_1 / L_1 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$K^1 = 10^6 \begin{bmatrix} 7 & -7 \\ -7 & 7 \end{bmatrix}$

$K^2 = 10^6 \begin{bmatrix} 7 & -7 \\ -7 & 7 \end{bmatrix}$

$K^3 = 10^6 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$K = 10^6 \begin{bmatrix} 7 & -7 & 0 & 0 \\ -7 & 14 & -7 & 0 \\ 0 & -7 & 9 & -2 \\ 0 & 0 & -2 & 2 \end{bmatrix}$

$\{F\} = [K]\{u\}$

$$10^6\begin{bmatrix} 7 & -7 & 0 & 0 \\ -7 & 14 & -7 & 0 \\ 0 & -7 & 9 & -2 \\ 0 & 0 & -2 & 2 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 15x10^3 \\ 0 \end{Bmatrix}$$

Applying boundary conditions,

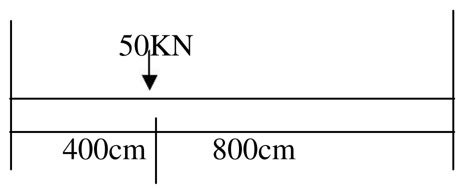
We get,

Result:

U_2=7.27E-4m

U_3=1.454E-3m

18. Calculate the nodal displacement and slope of the beam loading as shown in fig.



$$K_{ij}=\frac{EI}{h^3}\begin{bmatrix} 12 & -6h & -12 & -6h \\ -6h & 4h^2 & 6h & 2h^2 \\ -12 & 6h & 12 & 6h \\ -6h & 2h^2 & 6h & 4h^2 \end{bmatrix}\begin{Bmatrix} u_1 \\ \varphi_1 \\ u_2 \\ \varphi_2 \end{Bmatrix}$$

$$K = \begin{bmatrix} 3E4 & -6E6 & -3E4 & -6E6 & 0 & 0 \\ -6E6 & 1.6E9 & 6E6 & 800E6 & 0 & 0 \\ -3E4 & 6E4 & 3.3E4 & 4.5E6 & -3.7E3 & -1.5E6 \\ -6E6 & 800E6 & 4.5E6 & 2.4E9 & 1.5E6 & 400E6 \\ 0 & 0 & -3.7E3 & 1.5E6 & 3.7E3 & 1.5E6 \\ 0 & 0 & -1.5E6 & 400E6 & 1.5E6 & 800E6 \end{bmatrix}$$

$$\begin{bmatrix} 3E4 & -6E6 & -3E4 & -6E6 & 0 & 0 \\ -6E6 & 1.6E9 & 6E6 & 800E6 & 0 & 0 \\ -3E4 & 6E4 & 3.3E4 & 4.5E6 & -3.7E3 & -1.5E6 \\ -6E6 & 800E6 & 4.5E6 & 2.4E9 & 1.5E6 & 400E6 \\ 0 & 0 & -3.7E3 & 1.5E6 & 3.7E3 & 1.5E6 \\ 0 & 0 & -1.5E6 & 400E6 & 1.5E6 & 800E6 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ u_2 \\ \varphi_2 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -5E4 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Apply the boundary conditions,

Eliminate 1^{st} and 3^{rd} row and column, we get

Result:

U_2= -1.975

$\varphi_2 = 3.703E-3$

19. For the plane strain element the nodal displacements are U1=0.005mm, v1= 0.002mm

U2 =0mm, v2=0.0mm

U3=0.005mm, v3=0mm

Determine the element stresses $\sigma_x, \sigma_y, \tau_{xy}$. Given E=70Gpa, and $v = 0.3$ and use unit thickness for plane strain.

Solution:

A= 100mm^2

$$[B]=\frac{1}{2A}\begin{bmatrix} q1 & 0 & q2 & 0 & q3 & 0 \\ 0 & r1 & 0 & r2 & 0 & r3 \\ r1 & q1 & r2 & q2 & r3 & q3 \end{bmatrix}$$

$$[B]=\frac{1}{200}\begin{bmatrix} -10 & 0 & 0 & 0 & 10 & 0 \\ 0 & 10 & 0 & -20 & 0 & 10 \\ 10 & -10 & -20 & 0 & 10 & 10 \end{bmatrix}$$

$$[D]=26.923E3\begin{bmatrix} 3.5 & 1.5 & 0 \\ 1.5 & 3.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We know that,

$$\{\sigma\} = [D][B]\{U\}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 4.038 \\ 9.423 \\ 10.769 \end{Bmatrix}$$

20. A steel bar of length 800mm is subjected to an axial load of 3 kN 1.Find the elongation of the bar, neglecting self weight.Take E = 2*10^{5}N/mm^2, A = 300mm^2.

Solution:

For one dimensional two noded bar element, the finite element eqn

is $\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \frac{AE}{\_\_} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

For element 1:

Finite element equation is $\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = 150*10^3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

For element 2

Finite element equation is $\begin{pmatrix} F_2 \\ F_3 \end{pmatrix} = 150*10^3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$

$K = K^1 + K^2$

$$[K] = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Applying Boundary conditions,

We get the values

$U_2 = 0.02$mm

$U_3 = 0.04$mm.

$\delta L = 0.04$ mm.

21. A thin steel plate of uniform thickness 25mm is subjected to a point load of 420 N at mid depth. The plate is also subjected to self weight. If Youngs modulus, $E = 2*10^5 N/mm^2$ and unit weight density = $0.8 * 10^{-4} N/mm^3$,

Calculate the following: Displacement at each nodal point, Stresses in each element Solution:

Body force vector, $(F) = \frac{\rho A l}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

For element 1: $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 20 \end{Bmatrix}$

For element 2: $\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 16 \\ 16 \end{Bmatrix}$

Therefore global force vector $\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 456 \\ 16 \end{Bmatrix}$

For one dimensional two noded bar element, the finite element eqn is

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

For element 1: Finite element equation is:

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = 2*10^5 \begin{bmatrix} 12.5 & 12.5 \\ -12.5 & 12.5 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

For element 2;Finite element equation is:

$$\begin{pmatrix} F_2 \\ F_3 \end{pmatrix} = 2*10^5 \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$K = = \begin{bmatrix} 12.5 & -12.5 & 0 \\ -12.5 & 22.5 & -10 \\ 0 & -10 & 10 \end{bmatrix}$$

Applying boundary conditions we get

$U_2 = 1.888 * 10^{-4}$ mm, $U_3 = 1.9698 * 10^{-4}$ mm, $\sigma_1 = 0.188 N/mm^2$, $\sigma_2 = 0.008 N/mm^2$

22. The three bar assemblage. A force of 2500N is applied in the x direction at node 2. The length of each element is 750 mm. Take E = $4*10^5$ N/mm^2. A = 600mm^2 for elements 1 and 2.Take E = $2*10^5$ N/mm^2 and A = 1200mm^2 for element 3. Nodes 1 and 4 are fixed. Calculate the following: Global stiffness matrix, Displacements of nodes 2 and 3. Reactions at nodes 1 and 4.

Solution:

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

For element 1: $\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = 1*10^5 \begin{bmatrix} 3.2 & -3.2 \\ -3.2 & 3.2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

For element 2: $\begin{pmatrix} F_2 \\ F_3 \end{pmatrix} = 1*10^5 \begin{bmatrix} 3.2 & -3.2 \\ -3.2 & 3.2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$

For element 3: $\begin{pmatrix} F_3 \\ F_4 \end{pmatrix} = 1*10^5 \begin{bmatrix} 3.2 & -3.2 \\ -3.2 & 3.2 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix}$

$$K = \begin{bmatrix} 3.2 & -3.2 & 0 & 0 \\ -3.2 & 6.4 & -3.2 & 0 \\ 0 & -3.2 & 6.4 & -3.2 \\ 0 & 0 & -3.2 & 3.2 \end{bmatrix} 10^5$$

Applying boundary conditions:

$U_2 = 5.127*10^{-3}$mm

$U_3 = 2.604*10^{-3}$mm

$R_1 = -1640$ N

$R_4 = -833$ N

23. A steel bar of length 800mm is subjected to an axial load of 3 KN. Find the elongation of the bar, neglect self weight. A= 300 sq mm

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

For element 1: Finite element equation is: $\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = 300*10^3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

For element 2: Finite element equation is: $\begin{pmatrix} F_2 \\ F_3 \end{pmatrix} = 300*10^3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$

For element 3: $\begin{pmatrix} F_3 \\ F_4 \end{pmatrix} = 300*10^3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix}$

For element 4: $\begin{pmatrix} F_4 \\ F_5 \end{pmatrix} = 300*10^3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_4 \\ u_5 \end{Bmatrix}$

Applying boundary conditions:

We get the values

$U_1 = 0$

$U_2 = 0.01mm$

$U_3 = 0.02mm$

$U_4 = 0.03mm$

$U_5 = 0.04mm$

24. Explain Finite element method procedure.

(i) Finite element discretization: the domain circumference is represented as a collection of finite number of n sub domains. This is called discretization of the domain. Each sub domain is called as element. Collection of element is called as finite element mesh. The elements are connected to each other at point is called nodes. The line segment can be same length or of different length. When all the segment are of same length the resulting mesh is called as uniform mesh on the other hand is called as non-uniform mesh

(ii) Element equation: An element is isolated and its required properties ie length is computed by some approximate means

(iii) Assembly of element equation and solution : The approximate values of circumference f the circle is obtained by putting together the element properties in a meaningful way. This process is called assembly of element equation.

(iv) Converge and error estimate: The exact solution is known to us P=2πr

25. A bridge is supported by several concrete piles and the geometry and loads of typical piles. The load 20 KN/sq.m represent weight of the bridge and an assumed distribution of the traffic on the bridge. The concrete weight approximately 25 KN/cu.m and its modulus f is $28X10^6$ KN/sq.m. Analyse the pile for stress and displacement by FEM having more than one element.

$$K_{ij} = \int EA \frac{dN_i}{dx} \frac{dN_i}{dx} dx$$

$$N_1 = 1 - \frac{x}{h_e}; N_2 = \frac{x}{h_e}$$

Element 1:

$$K_{11} = E\int \frac{1}{4}(1+x)\left(\frac{-1}{h_e}\right)^2 dx$$

$$K_{11} = \frac{E}{4}\int_0^{h_e} \frac{1}{h_e} + \frac{x}{h_e^2}\, dx$$

$$K_{11} = 1.5\,\frac{E}{4}$$

$$K_{12} = -1.5\,\frac{E}{4}$$

$$r_i = \int \rho A(x)\, N_i\, dx + P\phi_i(L)$$

$$r_1 = \int_0^1 \frac{25}{4}(1+x)(1-x)\,dx + Q_1$$

$r_1 = 4.167 + Q_1$

$r_2 = 5.167 + Q_1$

$$\frac{E}{A}\begin{bmatrix} 1.5 & 1.5 & 0 \\ -1.5 & 4 & -2.5 \\ 0 & -2.5 & 2.5 \end{bmatrix}\begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{Bmatrix} = \begin{Bmatrix} 4.167 \\ 3.33 \\ 24 \end{Bmatrix} + \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix}$$

$\phi_1 = 1.2173X\,10^{-6}\ mm$

$\phi_2 = 3.4525X\,10^{-7}\ mm$