



## CREST Mathematics Olympiad (CMO)

# Sample Paper

## Pattern and Marking Scheme

Grade	Topic/Section	No. of Questions	Marks per Question	Total Marks
Grade 10	Practical Mathematics	40	1	40
	Achiever's Section	10	2	20
<b>Grand Total</b>		<b>50</b>		<b>60</b>

The total duration of the exam is 60 minutes.

## Syllabus

**Section 1:** Real Numbers, Polynomials, Pair of Linear Equations in Two Variables, Quadratic Equations, Arithmetic Progressions, Triangles, Coordinate Geometry, Introduction to Trigonometry, Some Applications of Trigonometry, Circles, Constructions, Areas Related to Circles, Surface Areas and Volumes, Statistics, Probability.

**Achievers Section:** Higher Order Thinking Questions - Syllabus as per Section 1

For more details, visit <https://www.crestolympiads.com/math-olympiad-cmo>

## Practical Mathematics (Each Question is 1 Mark)

- If  $3y + 4x = 1$ ,  $y = x + 5$  and  $5y + bx = 3$  are concurrent, find the value of 'b'.
  - 1
  - 3
  - 6
  - 0
- Two vertices of a triangle are  $(5, -1)$  and  $(-2, 3)$ . If the orthocentre of the triangle is the origin, find the third vertex.
  - $(4, 7)$
  - $(4, -7)$
  - $(-4, 7)$
  - $(-4, -7)$
- For an acute angle  $\theta$ ,  $\sin \theta + \cos \theta$  takes the greatest value when  $\theta$  is:
  - $30^\circ$
  - $45^\circ$
  - $60^\circ$
  - $90^\circ$
- If the coefficients of  $r^{\text{th}}$  term and  $(r + 1)^{\text{th}}$  term in the expansion of  $(1 + x)^{20}$  are in the ratio 1:2, what is the value of  $r$ ?
  - 6
  - 7
  - 8
  - 9
- Which term is numerically the greatest term in the expansion of  $(3 + 2x)^{49}$ , when  $x = 1/5$ ?
  - 4<sup>th</sup> term
  - 5<sup>th</sup> term
  - 6<sup>th</sup> term
  - 7<sup>th</sup> term
- If the sum of the roots of the equation  $ax^2 + bx + c = 0$  is equal to the sum of their squares, then which one of the following is correct?
  - $a^2 + b^2 = c^2$
  - $a^2 + b^2 = a + b$
  - $2ac = ab + b^2$
  - $2c + b = 0$
- If  $\cos x / (1 + \operatorname{cosec} x) + \cos x / (\operatorname{cosec} x - 1) = 2$ , then which one of the following is one of the values of  $x$ ?
  - $\pi/2$
  - $\pi/3$
  - $\pi/4$
  - $\pi/6$
- If  $p_1$  and  $p_2$  are two odd prime numbers such that  $p_1 > p_2$ , then  $p_1^2 - p_2^2$  is:
  - An even number
  - An odd number
  - An odd prime number
  - A prime number

9. If the first, second and last terms of an AP are  $a$ ,  $b$  and  $c$ , respectively, then the sum is:
- a.  $[(a + b)(a + c - 2b)]/[2(b - a)]$                       b.  $[(b + c)(a + b - 2c)]/[2(b - a)]$   
c.  $[(a + c)(b + c - 2a)]/[2(b - a)]$                       d. None of these
10. The areas of two similar triangles are  $81 \text{ cm}^2$  and  $49 \text{ cm}^2$ , respectively, then what will be the ratio of their corresponding medians?
- a. 7 : 9    b. 9 : 81  
c. 9 : 7    d. 81 : 7
11. To divide a line segment AB in the ratio 2: 5, first a ray AX is drawn, so that  $\angle BAX$  is an acute angle and then at an equal distance points are marked on the ray AX such that the minimum number of this point is:
- a. 2    b. 5  
c. 4    d. 7
12. Write the general term in the expansion of  $(x^2 - y)^6$ :
- a.  $(-1)^r \cdot {}^6C_r x^{12-2r}$     b.  $(-1)^r {}^6C_r x^{12-2r} \cdot y^r$   
c.  $(1)^r {}^6C_r \cdot y^r$     d.  $(1)^r {}^6C_r x^{12} \cdot y^r$
13. If  $\alpha$ ,  $\beta$  are the roots of the equation  $x^2 - 2x + 3 = 0$ , then find the equation whose roots are  $1/\alpha^2$  and  $1/\beta^2$ .
- a.  $x^2 + 2x + 1 = 0$     b.  $9x^2 + 2x + 1 = 0$   
c.  $9x^2 - 2x + 1 = 0$     d.  $9x^2 + 2x - 1 = 0$
14. The houses of a row are numbered consecutively from 1 to 49. If there is a value of  $x$  such that the sum of the numbers of the houses preceding the house numbered  $x$  is equal to the sum of the numbers of the houses following it. Find the value of  $x$ .
- a.  $x = 33$     b.  $x = 43$   
c.  $x = 39$     d.  $x = 35$
15. Find the values of  $a$  and  $b$  for which  $3x^3 - ax^2 - 74x + b$  is a multiple of  $x^2 + 2x - 24$ .
- a.  $a = -5, b = 24$     b.  $a = 5, b = 24$   
c.  $a = 13, b = 16$     d.  $a = -13, b = 16$
16. What is the value of the expression  $[(a - b)^3 + (b - c)^3 + (c - a)^3] / [(a - b)(b - c)(c - a)]$ ?
- a. 1    b. 0  
c. 2    d. 3





32. Simplify the following expression:

$$[(a - b)^3 - (a + b)^3]/2 + a(a^2 + 3b^2)$$

a.  $a^3 - b^3$

b.  $(a + b)^3$

c.  $a^2 + b^3$

d.  $(a - b)^3$

33. A three-digit number was chosen at random. Find the probability that its hundred's digit, ten's digit and unit's digit are consecutive integers in descending order.

a.  $1/75$

b.  $4/225$

c.  $2/225$

d.  $1/45$

34. In  $\triangle ABC$ ,  $\angle B = 90^\circ$ . P, Q and R are the midpoints of AB, BC and AC, respectively. Which of the following is true?

a. A, P, Q and R

b. B, P, R and Q

c. C, Q, P and R

d. All of these

35. If  $\tan A = 1/2$  and  $\tan B = 1/3$ , then which of the following is true?

a.  $A + B = \pi^c/4$

b.  $A - B = \pi^c/4$

c.  $2(A + B) = \pi^c/4$

d.  $AB = \pi^c/4$

36. If  $\sin A = \sqrt{3}/2$  and A is an acute angle, then find the value of  $(\tan A - \cot A)/(\sqrt{3} + \operatorname{cosec} A)$ :

a.  $-2/5$

b.  $2/5$

c.  $2/(3 + 2\sqrt{3})$

d.  $-2$

37. Ken and Paul can complete a job in 40 days and 50 days, respectively. They worked on alternative days to complete it. Find the minimum possible time in which they could have completed it:

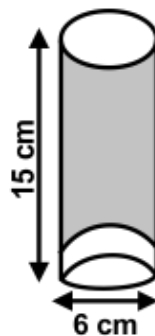
a.  $44 \frac{2}{5}$  days

b.  $44 \frac{1}{2}$  days

c.  $44 \frac{3}{5}$  days

d.  $44 \frac{4}{5}$  days

38. In the adjoining figure, the bottom of the glass has a hemispherical raised portion. If the glass is filled with orange juice, then find the quantity of juice which a person will get:



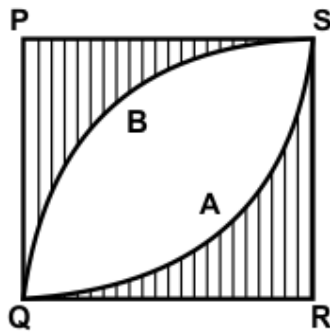
a.  $135 \pi \text{ cm}^3$

b.  $117 \pi \text{ cm}^3$

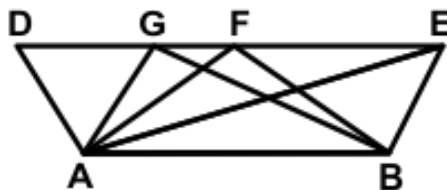
c.  $99 \pi \text{ cm}^3$

d.  $36 \pi \text{ cm}^3$

39. In the given figure, PQRS is a square of side  $7\sqrt{2}$  cm. With P and R as centres and PQ as radius, the arcs QAS and QBS are drawn, respectively. Find the area of the shaded region (in  $\text{cm}^2$ ).



- a. 50  
b. 48  
c. 44  
d. 42
40. In the given figure,  $AB \parallel DE$  and the area of the parallelogram ABFD is  $24 \text{ cm}^2$ . Find the areas of triangles AFB, AGB, and AEB.



- a.  $8 \text{ cm}^2$   
b.  $12 \text{ cm}^2$   
c.  $10 \text{ cm}^2$   
d.  $14 \text{ cm}^2$

### Achiever's Section (Each Question is 2 Marks)

41. Find the quadratic equation whose roots are reciprocal of the roots of the equation  $3x^2 - 20x + 17 = 0$ .
- a.  $17x^2 - 20x + 3 = 0$   
b.  $17x^2 + 20x + 3 = 0$   
c.  $17x^2 - 20x - 3 = 0$   
d.  $17x^2 + 20x - 3 = 0$
42. Two regular polygons are such that the ratio between their number of sides is 1:2 and the ratio of measures of their interior angles is 3:4. Find the number of sides of each polygon.
- a. 5, 10  
b. 6, 12  
c. 4, 8  
d. 2, 3
43.  $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$  is a polynomial such that when it is divided by  $(x - 1)$  and  $(x + 1)$ , the remainders are 5 and 19, respectively. Determine the remainder when  $f(x)$  is divided by  $(x - 2)$ .
- a. 6  
b. 10  
c. 2  
d. 8

44. Perpendiculars are drawn from the vertex of the obtuse angles of a rhombus to its sides. The length of each perpendicular is equal to a unit. The distance between their feet is equal to  $b$  units. Find the area of the rhombus.
- a.  $\sqrt{(a^2 + b^2)} / 2\sqrt{(b^2 - a^2)}$                       b.  $2ab / 2\sqrt{(b^2 - a^2)}$   
c.  $ab^2 / 2\sqrt{(b^2 - a^2)}$                                       d.  $2a^2b^2 / 2\sqrt{(b^2 - a^2)}$
45. Inside a triangular park, there is a flower bed forming a similar triangle. Around the flower bed runs a uniform path of such a width that the sides of the park are exactly double the corresponding sides of the flower bed. Find the ratio of the area of the path to the flower bed:
- a. 1: 1    b. 1: 2  
c. 1: 3    d. 3: 1
46. The angles of elevation of the top of a tower from two points at distances  $m$  and  $n$  metres are complementary. If the two points and the base of the tower are on the same straight line, then what will be the height of the tower?
- a.  $\sqrt{(mn)}$     b.  $mn$   
c.  $m/n$     d.  $n/m$
47. In the binomial expansion of  $(a - b)^n$ ,  $n \geq 5$  the sum of the 5<sup>th</sup> and 6<sup>th</sup> terms is zero. Find the value of  $a/b$ .
- a.  $(n - 5)/6$     b.  $(n - 4)/5$   
c.  $5/(n - 4)$     d.  $6/(n - 5)$
48. A tower stands vertically on the ground. From a point on the ground which is 30 m away from the foot of a tower, the angle of elevation of the top of the tower is found to be  $45^\circ$ . Find the height of the tower.
- a. 28 m    b. 30 m  
c. 32 m    d. 35 m
49. Which among the following is a singleton set?
- a. The set A consisting of all prime numbers less than 10.  
b. The set B consisting of all squares up to 10.  
c. The set C consisting of all numbers up to 10 that are neither prime nor composite.  
d. The set D of all odd numbers up to 10.
50. W borrowed a certain sum of money from X at the rate of 10% per annum under simple interest and lent one-fourth of the amount to Y at 8% per annum under simple interest and the remaining amount to Z at 15% per annum under simple interest. If at the end of 15 years, W made a profit of \$5850 in the deal, then find the sum that W had lent to Z:
- a. \$24,500    b. \$12,000  
c. \$9,000    d. \$18,600



## Answer Key

1.	c	2.	d	3.	b	4.	b	5.	c	6.	c	7.	c
8.	a	9.	c	10.	c	11.	d	12.	b	13.	b	14.	d
15.	a	16.	d	17.	d	18.	c	19.	a	20.	c	21.	a
22.	d	23.	b	24.	c	25.	a	26.	c	27.	b	28.	b
29.	c	30.	d	31.	c	32.	d	33.	c	34.	b	35.	a
36.	b	37.	a	38.	b	39.	d	40.	b	41.	a	42.	a
43.	b	44.	c	45.	d	46.	a	47.	b	48.	b	49.	c
50.	c												