## Notation

$\mathbb{N} \quad$ The set of all natural numbers $\{1,2,3, \ldots\}$
$\mathbb{Z} \quad$ The set of all integers
$\mathbb{Q} \quad$ The set of all rational numbers
$\mathbb{R} \quad$ The set of all real numbers
$S_{n} \quad$ The group of permutations of $n$ distinct symbols
$\mathbb{Z}_{\mathrm{n}} \quad\{0,1,2, \ldots, n-1\}$ with addition and multiplication modulo $n$
$\phi \quad$ empty set
$A^{T} \quad$ Transpose of $A$
$i \quad \sqrt{-1}$
$\hat{\imath}, \hat{\jmath}, \hat{k}$ unit vectors having the directions of the positive $x, y$ and $z$ axes of a three dimensional rectangular coordinate system
$\nabla \quad \hat{\imath} \frac{\partial}{\partial x}+\hat{\jmath} \frac{\partial}{\partial y}+\hat{k} \frac{\partial}{\partial z}$
$I_{n} \quad$ Identity matrix of order $n$
$\ln \quad$ logarithm with base $e$

# SECTION - A <br> MULTIPLE CHOICE QUESTIONS (MCQ) 

## Q. 1 - Q. 10 carry one mark each.

Q. 1 The sequence $\left\{s_{n}\right\}$ of real numbers given by

$$
s_{n}=\frac{\sin \frac{\pi}{2}}{1 \cdot 2}+\frac{\sin \frac{\pi}{2^{2}}}{2 \cdot 3}+\cdots+\frac{\sin \frac{\pi}{2^{n}}}{n \cdot(n+1)}
$$

is
(A) a divergent sequence
(B) an oscillatory sequence
(C) not a Cauchy sequence
(D) a Cauchy sequence
Q. $2 \quad$ Let $P$ be the vector space (over $\mathbb{R}$ ) of all polynomials of degree $\leq 3$ with real coefficients. Consider the linear transformation $T: P \rightarrow P$ defined by

$$
T\left(a_{0}+a_{1} x+a_{2} x^{2}+a_{3} x^{3}\right)=a_{3}+a_{2} x+a_{1} x^{2}+a_{0} x^{3}
$$

Then the matrix representation $M$ of $T$ with respect to the ordered basis $\left\{1, x, x^{2}, x^{3}\right\}$ satisfies
(A) $\quad M^{2}+I_{4}=0$
(B) $\quad M^{2}-I_{4}=0$
(C) $\quad M-I_{4}=0$
(D) $\quad M+I_{4}=0$
Q. 3 Let $f:[-1,1] \rightarrow \mathbb{R}$ be a continuous function. Then the integral

$$
\int_{0}^{\pi} x f(\sin x) d x
$$

is equivalent to
(A)

$$
\frac{\pi}{2} \int_{0}^{\pi} f(\sin x) d x
$$

(B)

$$
\frac{\pi}{2} \int_{0}^{\pi} f(\cos x) d x
$$

(C)

$$
\pi \int_{0}^{\pi} f(\cos x) d x
$$

(D)

$$
\pi \int_{0}^{\pi} f(\sin x) d x
$$

Q. 4 Let $\sigma$ be an element of the permutation group $S_{5}$. Then the maximum possible order of $\sigma$ is
(A) 5
(B) 6
(C) 10
(D) 15
Q. 5 Let $f$ be a strictly monotonic continuous real valued function defined on $[a, b]$ such that $f(a)<a$ and $f(b)>b$. Then which one of the following is TRUE?
(A) There exists exactly one $c \in(a, b)$ such that $f(c)=c$
(B) There exist exactly two points $c_{1}, c_{2} \in(a, b)$ such that $f\left(c_{i}\right)=c_{i}, i=1,2$
(C) There exists no $c \in(a, b)$ such that $f(c)=c$
(D) There exist infinitely many points $c \in(a, b)$ such that $f(c)=c$
Q. 6 The value of $\lim _{(x, y) \rightarrow(2,-2)} \frac{\sqrt{(x-y)}-2}{x-y-4}$ is
(A) 0
(B) $\frac{1}{4}$
(C) $\frac{1}{3}$
(D) $\frac{1}{2}$
Q. 7 Let $\vec{r}=(x \hat{\imath}+y \hat{\jmath}+z \hat{k})$ and $r=|\vec{r}|$. If $f(r)=\ln r$ and $g(r)=\frac{1}{r}, r \neq 0$, satisfy $2 \nabla f+h(r) \nabla g=\overrightarrow{0}$, then $h(r)$ is
(A) $r$
(B) $\frac{1}{r}$
(C) $2 r$
(D) $\frac{2}{r}$
Q. 8 The nonzero value of $n$ for which the differential equation

$$
\left(3 x y^{2}+n^{2} x^{2} y\right) d x+\left(n x^{3}+3 x^{2} y\right) d y=0, \quad x \neq 0,
$$

becomes exact is
(A) -3
(B) -2
(C) 2
(D) 3
Q. 9 One of the points which lies on the solution curve of the differential equation

$$
(y-x) d x+(x+y) d y=0
$$

with the given condition $y(0)=1$, is
(A) $(1,-2)$
(B) $(2,-1)$
(C) $(2,1)$
(D) $(-1,2)$
Q. 10 Let $S$ be a closed subset of $\mathbb{R}, T$ a compact subset of $\mathbb{R}$ such that $S \cap T \neq \phi$. Then $S \cap T$ is
(A) closed but not compact
(B) not closed
(C) compact
(D) neither closed nor compact

## Q. 11 - Q. 30 carry two marks each.

Q. 11 Let $S$ be the series

$$
\sum_{k=1}^{\infty} \frac{1}{(2 k-1) 2^{(2 k-1)}}
$$

and $T$ be the series

$$
\sum_{k=2}^{\infty}\left(\frac{3 k-4}{3 k+2}\right)^{\frac{(k+1)}{3}}
$$

of real numbers. Then which one of the following is TRUE?
(A) Both the series $S$ and $T$ are convergent
(B) $S$ is convergent and $T$ is divergent
(C) $S$ is divergent and $T$ is convergent
(D) Both the series $S$ and $T$ are divergent
Q. 12 Let $\left\{a_{n}\right\}$ be a sequence of positive real numbers satisfying

$$
\frac{4}{a_{n+1}}=\frac{3}{a_{n}}+\frac{a_{n}^{3}}{81}, \quad n \geq 1, \quad a_{1}=1
$$

Then all the terms of the sequence lie in
(A) $\left[\frac{1}{2}, \frac{3}{2}\right]$
(B) $[0,1]$
(C) $[1,2]$
(D) $[1,3]$
Q. 13

The largest eigenvalue of the matrix $\left[\begin{array}{ccc}1 & 4 & 16 \\ 4 & 16 & 1 \\ 16 & 1 & 4\end{array}\right]$ is
(A) 16
(B) 21
(C) 48
(D) 64
Q. 14 The value of the integral

$$
\frac{(2 n)!}{2^{2 n}(n!)} \int_{-1}^{1}\left(1-x^{2}\right)^{n} d x, \quad n \in \mathbb{N}
$$

is
(A) $\frac{2}{(2 n+1)!}$
(B) $\frac{2 n}{(2 n+1)!}$
(C) $\frac{2(n!)}{2 n+1}$
(D) $\quad \frac{(n+1)!}{2 n+1}$
Q. 15 If the triple integral over the region bounded by the planes

$$
2 x+y+z=4, \quad x=0, \quad y=0, \quad z=0
$$

is given by

$$
\int_{0}^{2} \int_{0}^{\lambda(x)} \int_{0}^{\mu(x, y)} d z d y d x
$$

then the function $\lambda(x)-\mu(x, y)$ is
(A) $x+y$
(B) $x-y$
(C) $x$
(D) $y$
Q. 16 The surface area of the portion of the plane $y+2 z=2$ within the cylinder $x^{2}+y^{2}=3$ is
(A) $\frac{3 \sqrt{5}}{2} \pi$
(B) $\frac{5 \sqrt{5}}{2} \pi$
(C) $\frac{7 \sqrt{5}}{2} \pi$
(D) $\frac{9 \sqrt{5}}{2} \pi$
Q. 17 Let $f: \mathbb{R}^{2} \rightarrow \mathbb{R}$ be defined by

$$
f(x, y)= \begin{cases}\frac{x y^{2}}{x+y} & \text { if } x+y \neq 0 \\ 0 & \text { if } x+y=0\end{cases}
$$

Then the value of $\left(\frac{\partial^{2} f}{\partial x \partial y}+\frac{\partial^{2} f}{\partial y \partial x}\right)$ at the point $(0,0)$ is
(A) 0
(B) 1
(C) 2
(D) 4
Q. 18 The function $f(x, y)=3 x^{2} y+4 y^{3}-3 x^{2}-12 y^{2}+1$ has a saddle point at
(A) $(0,0)$
(B) $(0,2)$
(C) $(1,1)$
(D) $(-2,1)$
Q. 19 Consider the vector field $\vec{F}=r^{\beta}(y \hat{\imath}-x \hat{\jmath})$, where $\beta \in \mathbb{R}, \vec{r}=x \hat{\imath}+y \hat{\jmath}$ and $r=|\vec{r}|$. If the absolute value of the line integral $\oint_{c} \vec{F} \cdot d \vec{r}$ along the closed curve $C: x^{2}+y^{2}=a^{2}$ (oriented counter clockwise) is $2 \pi$, then $\beta$ is
(A) -2
(B) -1
(C) 1
(D) 2
Q. 20 Let $S$ be the surface of the cone $z=\sqrt{x^{2}+y^{2}}$ bounded by the planes $z=0$ and $z=3$. Further, let $C$ be the closed curve forming the boundary of the surface $S$. A vector field $\vec{F}$ is such that $\nabla \times \vec{F}=-x \hat{\imath}-y \hat{\jmath}$. The absolute value of the line integral $\oint_{c} \vec{F} \cdot d \vec{r}$, where $\vec{r}=x \hat{\imath}+y \hat{\jmath}+z \hat{k}$ and $r=|\vec{r}|$, is
(A) 0
(B) $9 \pi$
(C) $15 \pi$
(D) $18 \pi$
Q. 21 Let $y(x)$ be the solution of the differential equation

$$
\frac{d}{d x}\left(x \frac{d y}{d x}\right)=x ; \quad y(1)=0,\left.\quad \frac{d y}{d x}\right|_{x=1}=0
$$

Then $y(2)$ is
(A) $\frac{3}{4}+\frac{1}{2} \ln 2$
(B) $\frac{3}{4}-\frac{1}{2} \ln 2$
(C) $\frac{3}{4}+\ln 2$
(D) $\frac{3}{4}-\ln 2$
Q. 22 The general solution of the differential equation with constant coefficients

$$
\frac{d^{2} y}{d x^{2}}+b \frac{d y}{d x}+c y=0
$$

approaches zero as $x \rightarrow \infty$, if
(A) $b$ is negative and $c$ is positive
(B) $b$ is positive and $c$ is negative
(C) both $b$ and $c$ are positive
(D) both $b$ and $c$ are negative
Q. 23 Let $S \subset \mathbb{R}$ and $\partial S$ denote the set of points $x$ in $\mathbb{R}$ such that every neighbourhood of $x$ contains some points of $S$ as well as some points of complement of $S$. Further, let $\bar{S}$ denote the closure of $S$. Then which one of the following is FALSE?
(A) $\partial \mathbb{Q}=\mathbb{R}$
(B) $\partial(\mathbb{R} \backslash T)=\partial T, \quad T \subset \mathbb{R}$
(C) $\partial(T \cup V)=\partial T \cup \partial V, \quad T, V \subset \mathbb{R}, T \cap V \neq \phi$
(D) $\partial T=\bar{T} \cap(\overline{\mathbb{R} \backslash T}), \quad T \subset \mathbb{R}$
Q. 24 The sum of the series

$$
\sum_{n=2}^{\infty} \frac{(-1)^{n}}{n^{2}+n-2}
$$

is
(A) $\frac{1}{3} \ln 2-\frac{5}{18}$
(B) $\frac{1}{3} \ln 2-\frac{5}{6}$
(C) $\frac{2}{3} \ln 2-\frac{5}{18}$
(D) $\frac{2}{3} \ln 2-\frac{5}{6}$
Q. 25 Let $f(x)=\frac{1}{1+|x|}+\frac{1}{1+|x-1|}$ for all $x \in[-1,1]$. Then which one of the following is TRUE?
(A) Maximum value of $f(x)$ is $\frac{3}{2}$
(B) Minimum value of $f(x)$ is $\frac{1}{3}$
(C) Maximum of $f(x)$ occurs at $x=\frac{1}{2}$
(D) Minimum of $f(x)$ occurs at $x=1$
Q. 26 The matrix $M=\left[\begin{array}{cc}\cos \alpha & \sin \alpha \\ i \sin \alpha & i \cos \alpha\end{array}\right]$ is a unitary matrix when $\alpha$ is
(A) $(2 n+1) \frac{\pi}{2}, n \in \mathbb{Z}$
(B) $(3 n+1) \frac{\pi}{3}, n \in \mathbb{Z}$
(C) $(4 n+1) \frac{\pi}{4}, n \in \mathbb{Z}$
(D) $(5 n+1) \frac{\pi}{5}, n \in \mathbb{Z}$
Q. 27 Let $M=\left[\begin{array}{rrr}0 & 1 & -2 \\ -1 & 0 & \alpha \\ 2 & -\alpha & 0\end{array}\right], \alpha \in \mathbb{R} \backslash\{0\}$ and $\boldsymbol{b}$ a non-zero vector such that $M \boldsymbol{x}=\boldsymbol{b}$ for some $\boldsymbol{x} \in \mathbb{R}^{3}$. Then the value of $\boldsymbol{x}^{T} \boldsymbol{b}$ is
(A) $-\alpha$
(B) $\alpha$
(C) 0
(D) 1
Q. 28 The number of group homomorphisms from the cyclic group $\mathbb{Z}_{4}$ to the cyclic group $\mathbb{Z}_{7}$ is
(A) 7
(B) 3
(C) 2
(D) 1
Q. 29 In the permutation group $S_{n}(n \geq 5)$, if $H$ is the smallest subgroup containing all the 3-cycles, then which one of the following is TRUE?
(A) Order of $H$ is 2
(B) Index of $H$ in $S_{n}$ is 2
(C) $H$ is abelian
(D) $H=S_{n}$
Q. 30 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$
f(x)=\left\{\begin{array}{lll}
x\left(1+x^{\alpha} \sin \left(\ln x^{2}\right)\right) & \text { if } & x \neq 0 \\
0 & \text { if } & x=0
\end{array} .\right.
$$

Then, at $x=0$, the function $f$ is
(A) continuous and differentiable when $\alpha=0$
(B) continuous and differentiable when $\alpha>0$
(C) continuous and differentiable when $-1<\alpha<0$
(D) continuous and differentiable when $\alpha<-1$

## SECTION - B <br> MULTIPLE SELECT QUESTIONS (MSQ)

## Q. 31 - Q. 40 carry two marks each.

Q. 31 Let $\left\{s_{n}\right\}$ be a sequence of positive real numbers satisfying

$$
2 s_{n+1}=s_{n}^{2}+\frac{3}{4}, \quad n \geq 1 .
$$

If $\alpha$ and $\beta$ are the roots of the equation $x^{2}-2 x+\frac{3}{4}=0$ and $\alpha<s_{1}<\beta$, then which of the following statement(s) is(are) TRUE ?
(A) $\left\{s_{n}\right\}$ is monotonically decreasing
(B) $\left\{s_{n}\right\}$ is monotonically increasing
(C) $\lim _{n \rightarrow \infty} s_{n}=\alpha$
(D) $\lim _{n \rightarrow \infty} s_{n}=\beta$
Q. 32 The value(s) of the integral

$$
\int_{-\pi}^{\pi}|x| \cos n x d x, \quad n \geq 1
$$

is (are)
(A) 0 when $n$ is even
(B) 0 when $n$ is odd
(C) $-\frac{4}{n^{2}}$ when $n$ is even
(D) $-\frac{4}{n^{2}}$ when $n$ is odd
Q. 33 Let $f: \mathbb{R}^{2} \rightarrow \mathbb{R}$ be defined by

$$
f(x, y)=\left\{\begin{array}{cc}
\frac{x y}{|x|} & \text { if } x \neq 0 \\
0 & \text { elsewhere }
\end{array}\right.
$$

Then at the point $(0,0)$, which of the following statement(s) is(are) TRUE ?
(A) $f$ is not continuous
(B) $f$ is continuous
(C) $f$ is differentiable
(D) Both first order partial derivatives of $f$ exist
Q. 34 Consider the vector field $\vec{F}=x \hat{\imath}+y \hat{\jmath}$ on an open connected set $S \subset \mathbb{R}^{2}$. Then which of the following statement(s) is(are) TRUE ?
(A) Divergence of $\vec{F}$ is zero on $S$
(B) The line integral of $\vec{F}$ is independent of path in $S$
(C) $\vec{F}$ can be expressed as a gradient of a scalar function on $S$
(D) The line integral of $\vec{F}$ is zero around any piecewise smooth closed path in $S$
Q. 35 Consider the differential equation

$$
\sin 2 x \frac{d y}{d x}=2 y+2 \cos x, y\left(\frac{\pi}{4}\right)=1-\sqrt{2}
$$

Then which of the following statement(s) is(are) TRUE?
(A) The solution is unbounded when $x \rightarrow 0$
(B) The solution is unbounded when $x \rightarrow \frac{\pi}{2}$
(C) The solution is bounded when $x \rightarrow 0$
(D) The solution is bounded when $x \rightarrow \frac{\pi}{2}$
Q. 36 Which of the following statement(s) is(are) TRUE?
(A) There exists a connected set in $\mathbb{R}$ which is not compact
(B) Arbitrary union of closed intervals in $\mathbb{R}$ need not be compact
(C) Arbitrary union of closed intervals in $\mathbb{R}$ is always closed
(D) Every bounded infinite subset $V$ of $\mathbb{R}$ has a limit point in $V$ itself
Q. 37

Let $P(x)=\left(\frac{5}{13}\right)^{x}+\left(\frac{12}{13}\right)^{x}-1$ for all $x \in \mathbb{R}$. Then which of the following statement(s) is(are) TRUE?
(A) The equation $P(x)=0$ has exactly one solution in $\mathbb{R}$
(B) $P(x)$ is strictly increasing for all $x \in \mathbb{R}$
(C) The equation $P(x)=0$ has exactly two solutions in $\mathbb{R}$
(D) $P(x)$ is strictly decreasing for all $x \in \mathbb{R}$
Q. 38 Let $G$ be a finite group and $o(G)$ denotes its order. Then which of the following statement(s) is(are) TRUE?
(A) $G$ is abelian if $o(G)=p q$ where $p$ and $q$ are distinct primes
(B) $G$ is abelian if every non identity element of $G$ is of order 2
(C) $G$ is abelian if the quotient group $\frac{G}{Z(G)}$ is cyclic, where $Z(G)$ is the center of $G$
(D) $G$ is abelian if $o(G)=p^{3}$, where $p$ is prime
Q. 39 Consider the set $V=\left\{\left.\left[\begin{array}{l}x \\ y \\ z\end{array}\right] \in \mathbb{R}^{3} \right\rvert\, \alpha x+\beta y+z=\gamma, \alpha, \beta, \gamma \in \mathbb{R}\right\}$. For which of the following choice(s) the set $V$ becomes a two dimensional subspace of $\mathbb{R}^{3}$ over $\mathbb{R}$ ?
(A) $\alpha=0, \beta=1, \gamma=0$
(B) $\alpha=0, \beta=1, \gamma=1$
(C) $\alpha=1, \beta=0, \gamma=0$
(D) $\alpha=1, \beta=1, \gamma=0$
Q. 40 Let $S=\left\{\left.\frac{1}{3^{n}}+\frac{1}{7^{m}} \right\rvert\, \quad n, m \in \mathbb{N}\right\}$. Then which of the following statement(s) is(are) TRUE?
(A) $S$ is closed
(B) $S$ is not open
(C) $S$ is connected
(D) 0 is a limit point of $S$

## SECTION - C

## NUMERICAL ANSWER TYPE (NAT)

## Q. 41 - Q. 50 carry one mark each.

Q. 41 Let $\left\{s_{n}\right\}$ be a sequence of real numbers given by

$$
s_{n}=2^{(-1)^{n}}\left(1-\frac{1}{n}\right) \sin \frac{n \pi}{2}, \quad n \in \mathbb{N}
$$

Then the least upper bound of the sequence $\left\{s_{n}\right\}$ is $\qquad$
Q. 42 Let $\left\{s_{k}\right\}$ be a sequence of real numbers, where

$$
s_{k}=k^{\alpha / k}, \quad k \geq 1, \quad \alpha>0
$$

Then

$$
\lim _{n \rightarrow \infty}\left(\begin{array}{llll}
s_{1} & s_{2} & \ldots & s_{n}
\end{array}\right)^{1 / n}
$$

is $\qquad$
Q. 43

Let $\boldsymbol{x}=\left[\begin{array}{l}x_{1} \\ x_{2} \\ x_{3}\end{array}\right] \in \mathbb{R}^{3}$ be a non-zero vector and $A=\frac{\boldsymbol{x} \boldsymbol{x}^{T}}{\boldsymbol{x}^{T} \boldsymbol{x}}$. Then the dimension of the vector space $\left\{\boldsymbol{y} \in \mathbb{R}^{3} \mid \quad A \boldsymbol{y}=\mathbf{0}\right\}$ over $\mathbb{R}$ is $\qquad$
Q. 44 Let $f$ be a real valued function defined by

$$
f(x, y)=2 \ln \left(x^{2} y^{2} e^{\frac{y}{x}}\right), \quad x>0, y>0
$$

Then the value of $x \frac{\partial f}{\partial x}+y \frac{\partial f}{\partial y}$ at any point $(x, y)$, where $x>0, y>0$, is $\qquad$
Q. 45 Let $\vec{F}=\sqrt{x} \hat{\imath}+\left(x+y^{3}\right) \hat{\jmath}$ be a vector field for all $(x, y)$ with $x \geq 0$ and $\vec{r}=x \hat{\imath}+y \hat{\jmath}$. Then the value of the line integral $\int_{C} \vec{F} \cdot d \vec{r}$ from $(0,0)$ to $(1,1)$ along the path $C: x=t^{2}, y=t^{3}, 0 \leq t \leq 1$ is $\qquad$
Q. 46 If $f:(-1, \infty) \rightarrow \mathbb{R}$ defined by $f(x)=\frac{x}{1+x}$ is expressed as

$$
f(x)=\frac{2}{3}+\frac{1}{9}(x-2)+\frac{c(x-2)^{2}}{(1+\xi)^{3}}
$$

where $\xi$ lies between 2 and $x$, then the value of $c$ is $\qquad$
Q. 47 Let $y_{1}(x), y_{2}(x)$ and $y_{3}(x)$ be linearly independent solutions of the differential equation

$$
\frac{d^{3} y}{d x^{3}}-6 \frac{d^{2} y}{d x^{2}}+11 \frac{d y}{d x}-6 y=0
$$

If the Wronskian $W\left(y_{1}, y_{2}, y_{3}\right)$ is of the form $k e^{b x}$ for some constant $k$, then the value of $b$ is $\qquad$
Q. 48 The radius of convergence of the power series

$$
\sum_{n=1}^{\infty} \frac{(-4)^{n}}{n(n+1)}(x+2)^{2 n} \text { is }
$$

$\qquad$
Q. 49 Let $f:(0, \infty) \rightarrow \mathbb{R}$ be a continuous function such that

$$
\int_{0}^{x} f(t) d t=-2+\frac{x^{2}}{2}+4 x \sin 2 x+2 \cos 2 x
$$

Then the value of $\frac{1}{\pi} f\left(\frac{\pi}{4}\right)$ is $\qquad$
Q. 50 Let $G$ be a cyclic group of order 12. Then the number of non-isomorphic subgroups of $G$ is $\qquad$

## Q. $51-$ Q. 60 carry two marks each.

Q. 51

The value of $\lim _{n \rightarrow \infty}\left(8 n-\frac{1}{n}\right)^{\frac{(-1)^{n}}{n^{2}}}$ is equal to $\qquad$
Q. 52 Let $R$ be the region enclosed by $x^{2}+4 y^{2} \geq 1$ and $x^{2}+y^{2} \leq 1$. Then the value of

$$
\iint_{R}|x y| d x d y \quad \text { is }
$$

$\qquad$
Q. 53 Let

$$
M=\left[\begin{array}{ccc}
\alpha & 1 & 1 \\
1 & \beta & 1 \\
1 & 1 & \gamma
\end{array}\right], \alpha \beta \gamma=1, \alpha, \beta, \gamma \in \mathbb{R} \quad \text { and } \quad \boldsymbol{x}=\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right] \in \mathbb{R}^{3}
$$

Then $M \boldsymbol{x}=\mathbf{0}$ has infinitely many solutions if $\operatorname{trace}(M)$ is $\qquad$
Q. 54 Let $C$ be the boundary of the region enclosed by $y=x^{2}, y=x+2$, and $x=0$. Then the value of the line integral

$$
\oint_{C}\left(x y-y^{2}\right) d x-x^{3} d y
$$

where $C$ is traversed in the counter clockwise direction, is $\qquad$
Q. 55 Let S be the closed surface forming the boundary of the region V bounded by $x^{2}+y^{2}=3$, $z=0, \quad z=6$. A vector field $\vec{F}$ is defined over V with $\nabla \cdot \vec{F}=2 y+z+1$. Then the value of

$$
\frac{1}{\pi} \iint_{S} \vec{F} \cdot \hat{n} d s
$$

where $\widehat{n}$ is the unit outward drawn normal to the surface $S$, is $\qquad$ ,
Q. 56 Let $y(x)$ be the solution of the differential equation

$$
\frac{d^{2} y}{d x^{2}}+5 \frac{d y}{d x}+6 y=0, \quad y(0)=1,\left.\quad \frac{d y}{d x}\right|_{x=0}=-1
$$

Then $y(x)$ attains its maximum value at $x=$ $\qquad$
Q. 57 The value of the double integral

$$
\int_{0}^{\pi} \int_{0}^{x} \frac{\sin y}{\pi-y} d y d x
$$

is $\qquad$
Q. 58 Let $H$ denote the group of all $2 \times 2$ invertible matrices over $\mathbb{Z}_{5}$ under usual matrix multiplication. Then the order of the matrix $\left[\begin{array}{ll}2 & 3 \\ 1 & 2\end{array}\right]$ in $H$ is $\qquad$
Q. 59

Let $A=\left[\begin{array}{rrr}1 & 2 & 0 \\ -1 & 5 & 2\end{array}\right], B=\left[\begin{array}{rr}1 & 2 \\ -1 & 0 \\ 3 & 1\end{array}\right], \quad N(A)$ the null space of $A$ and $R(B)$ the range space of $B$. Then the dimension of $N(A) \cap R(B)$ over $\mathbb{R}$ is $\qquad$
Q. 60 The maximum value of $f(x, y)=x^{2}+2 y^{2}$ subject to the constraint $y-x^{2}+1=0$ is $\qquad$

END OF THE QUESTION PAPER

| JAM 2016: Mathematics |  |  |  |
| :---: | :---: | :---: | :---: |
| Qn. No. | Qn. Type | Key(s) | Mark(s) |
| 1 | MCQ | D | 1 |
| 2 | MCQ | B | 1 |
| 3 | MCQ | A | 1 |
| 4 | MCQ | B | 1 |
| 5 | MCQ | A | 1 |
| 6 | MCQ | B | 1 |
| 7 | MCQ | C | 1 |
| 8 | MCQ | D | 1 |
| 9 | MCQ | C | 1 |
| 10 | MCQ | C | 1 |
| 11 | MCQ | B | 2 |
| 12 | MCQ | D | 2 |
| 13 | MCQ | B | 2 |
| 14 | MCQ | C | 2 |
| 15 | MCQ | D | 2 |
| 16 | MCQ | A | 2 |
| 17 | MCQ | B | 2 |
| 18 | MCQ | D | 2 |
| 19 | MCQ | A | 2 |
| 20 | MCQ | MTA | 2 |
| 21 | MCQ | B | 2 |
| 22 | MCQ | C | 2 |
| 23 | MCQ | C | 2 |
| 24 | MCQ | C | 2 |
| 25 | MCQ | A | 2 |
| 26 | MCQ | A | 2 |
| 27 | MCQ | C | 2 |
| 28 | MCQ | D | 2 |
| 29 | MCQ | B | 2 |
| 30 | MCQ | MTA | 2 |


| JAM 2016: Mathematics |  |  |  |
| :---: | :--- | :---: | ---: |
| Qn. No. | Qn. Type | Key(s) | Mark(s) |
| 31 | MSQ | A;C | 2 |
| 32 | MSQ | A;D | 2 |
| 33 | MSQ | B;D | 2 |
| 34 | MSQ | B;C;D | 2 |
| 35 | MSQ | C;D | 2 |
| 36 | MSQ | A;B | 2 |
| 37 | MSQ | A;D | 2 |
| 38 | MSQ | B;C | 2 |
| 39 | MSQ | A;C;D | 2 |
| 40 | MSQ | B;D | 2 |


| JAM 2016: Mathematics |  |  |  |
| :---: | :--- | :--- | ---: |
| Qn. No. | Qn. Type | Key(s) | Mark(s) |
| 41 | NAT | $0.5: 0.5$ | 1 |
| 42 | NAT | $1.0: 1.0$ | 1 |
| 43 | NAT | $2.0: 2.0$ | 1 |
| 44 | NAT | $8.0: 8.0$ | 1 |
| 45 | NAT | $1.49: 1.55$ | 1 |
| 46 | NAT | $-1:-1$ | 1 |
| 47 | NAT | $6.0: 6.0$ | 1 |
| 48 | NAT | $0.5: 0.5$ | 1 |
| 49 | NAT | $0.25: 0.25$ | 1 |
| 50 | NAT | $6.0: 6.0$ | 1 |
| 51 | NAT | $1.0: 1.0$ | 2 |
| 52 | NAT | $0.35: 0.4$ | 2 |
| 53 | NAT | $3.0: 3.0$ | 2 |
| 54 | NAT | $0.8: 1.9$ | 2 |
| 55 | NAT | $72.0: 72.0$ | 2 |
| 56 | NAT | $-0.3:-0.25$ | 2 |
| 57 | NAT | $2.0: 2.0$ | 2 |
| 58 | NAT | $3.0: 3.0$ | 2 |
| 59 | NAT | $1.0: 1.0$ | 2 |
| 60 | NAT | $2.0: 2.0$ | 2 |

