HIGHER SECONDARY II YEAR

MATHEMATICS

Model Question Paper - 3

Time : 2.30 Hours Marks : 90

Part - I

All the questions are compulsory. **20 x 1 = 20**

Choose the correct answer :

1. The given system of equations $x + 2y + 3z = -1$, $2x + 5y - 6z = 3$, $8x + 20y - 24z = 12$ is

   a) consistent and has unique solution

   b) consistent and has infinite no. of solutions

   c) inconsistent and has no solution

   d) consistent and has trivial solution

2. Which of the following statement is false

   a) The rank of a null matrix is zero

   b) The rank of the unit matrix of order n is unity

   c) Rank of an $m \times n$ matrix cannot exceed the minimum of m and n

   d) Equivalent matrices have the same rank

3. Direction ratios of y and z axis are respectively

   a) (1, 0, 0), (0, 1, 0) b) (1, 0, 0), (0, 0, 1)

   c) (0, 0, 1), (0, 1, 0) d) (0, 1, 0), (0, 0, 1)

4. If one end of the chord passing through the centre of the sphere $(x - 2)^2 + (y - 1)^2 + (z + 6)^2 = 18$ is (3, 2, –2) then the other end of the chord is

   a) $\left(\frac{5}{2}, \frac{3}{2}, -4\right)$ b) $\left(\frac{-1}{2}, \frac{-1}{2}, -2\right)$ c) (1, 0, –10) d) (–1, 0, 10)

5. The conjugate of $\overline{Z} + 3i$ is

   a) $Z - 3i$ b) $Z + 3i$ c) $-Z + 3i$ d) $-Z - 3i$

6. The real part of $e^{Z + 2\pi i}$ is

   a) cosy b) siny c) e^x cosy d) e^x siny

7. If a parabolic reflector is 20cm in diameter and 5cm deep, the distance of the focus from the centre of the reflector is

   a) 10 cm b) 6 cm c) 5 cm d) 15 cm

8. The normal to the rectangular hyperbola xy = 9 at $\left(6, \frac{3}{2}\right)$ meets the curve again at

a) $\left(\frac{3}{8}, 24\right)$ b) $\left(-24, \frac{-3}{8}\right)$ c) $\left(\frac{-3}{8}, -24\right)$ d) $\left(24, \frac{3}{8}\right)$

9. The curve $y = ax^3 + bx^2 + cx + d$ has a point of inflection at x = 1 then

a) a + b = 0 b) a + 3b = 0 c) 3a + b = 0 d) 3a + b = 1

10. The characteristic function of irrational numbers is

a) everywhere differentiable and everywhere continuous

b) everywhere differentiable and everywhere discontinuous

c) nowhere differentiable and everywhere continuous

d) nowhere differentiable and everywhere discontinuous

11. The asymptote to the curve $y^2 (1 + x) = x^2 (1 - x)$ is

a) x = 0 b) y = 0 c) x = 1 d) x = – 1

12. The value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ where $u = \log\left(\frac{x^2 + y^2}{x + y}\right)$ is

a) 0 b) e^u c) 1 d) 2

13. Volume of solid obtained by revolving the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about major and minor axes are in the ratio

a) $b^2 : a^2$ b) $a^2 : b^2$ c) a : b d) b : a

14. The value of $\int_0^{\pi} \frac{\sin^2 x}{1 + \cos x} dx$ is

a) $\frac{\pi}{2}$ b) 0 c) $\frac{\pi}{4}$ d) π

15. The particular integral of $(3D^2 + 4D + 1)\, y = 3e^{-x/3}$ is

a) $\frac{2}{3}e^{-x/3}$ b) $\frac{x^2}{2}e^{-x/3}$ c) $\frac{3}{2}xe^{-x/3}$ d) $e^{-x/3}$

16. If $f'(x) = \sqrt{x}$ and f(1) = 2 then, f(x) is

a) $\frac{-2}{3}(x\sqrt{x} + 2)$ b) $\frac{3}{2}(x\sqrt{x} + 2)$ c) $\frac{2}{3}(x\sqrt{x} + 2)$ d) $\frac{2}{3}x(\sqrt{x} + 2)$

17. Which of the following statements is not correct

a) Matrix addition is a binary operation on the set of m × n matrices

b) Matrix addition is a binary operation on the set of n × n singular matrices

c) Matrix multiplication is a binary operation on the set of singular matrices

d) Matrix multiplication is a binary operation on the set of non singular matrices

18. Which of the following is a group

a) (N, +) b) (E, •) c) (R, •) d) (Q, +)

19. If a random variable X follows poisson distribution such that $E(X^2) = 30$ then the variance of the distribution is

a) 6 b) 5 c) 30 d) 25

20. A random vaiable X has the following probability mass function as follows

X	0	1	2
P (X = x)	$\frac{144}{169}$	$\frac{1}{169}$	$\frac{24}{169}$

Then the value of F(1) is

a) 1 b) $\frac{144}{169}$ c) $\frac{145}{169}$ d) $\frac{168}{169}$

Part - II

Answer any seven questions. Question no. 30 is compulsory. **7 × 2 = 14**

21. Show that the lines $\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$ and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$ are not coplanar.

22. Find the multiplicative inverse of the complex number 1 + i.

23. Define Director Circle.

24. Evaluate : $\lim_{x\to\infty} \frac{\sin 1/x}{1/x}$

25. Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 1%.

26. Find the volume of the solid that results when the region enclosed by the given curve $y = x^3$, $x = 0$, $y = 1$ is revolved about y axis.

27. Solve : $xdy - ydx = x^2dx$

28. Prove that (N, *) where * is defined by $a * b = a^b$ is not a semigroup.

29. Prove that the total probability in a poisson distribution is one.

30. Solve the matrix equation XA = B where $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

PART - III

Answer any seven questions. Question no. 40 is compulsory. **(7 × 3 = 21)**

31. Find the rank of the matrix $\begin{pmatrix} 1 & -2 & 3 & 4 \\ 3 & 0 & 1 & 2 \\ -2 & 4 & -1 & -3 \\ 1 & 2 & 7 & 6 \end{pmatrix}$

32. What is the unit vector that is normal to the plane containing $2\vec{i} + \vec{j} + \vec{k}$ and $\vec{i} + 2\vec{j} + \vec{k}$

33. Prove that the product of perpendiculars from any point on the hyperbola to its asymptotes is constant and the value is $\frac{a^2b^2}{a^2+b^2}$.

34. Prove that $e^x > 1 + x$ for all $x > 0$.

35. Discuss the curve $3ay^2 = x(x-a)^2$ for (i) existence (ii) symmetry (iii) asymptote

36. Evaluate the integral $\int_1^5 |x-3|\,dx$.

37. The normal lines to a given curve at each point (x, y) on the curve pass through the point (2, 0). Formualte the Differential equation representing the problem.

38. Write down the negation of the statement $(p \rightarrow q) \vee (q \rightarrow p)$.

39. A continuous random variable X has the p.d.f defined by $f(x) = \begin{cases} \frac{1}{2}\sin x & 0 \le x \le \pi \\ 0 & \text{otherwise} \end{cases}$. Find the mean of the distribution.

40. Find all the values of $(-i)^{1/3}$.

PART - IV

Answer all the questions. **(7 × 5 = 35)**

41. a) Determine the values of K such that the system of linear equations $x-2y=1$, $x-y+Kz=-2$, $Ky + 4z = 6$ has (i) a unique solution (ii) infinite number of solutions (ii) no solution

(or)

b) Prove that the perpendicular bisectors of sides of a triangle are concurrent by vector method.

42. a) If $\arg\left(\frac{Z-1}{Z+1}\right) = \frac{\pi}{2}$ then prove that $|Z| = 1$.

(or)

b) Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 y}{\partial y \partial x}$ for the function $u = \sin xy$.

43. a) A Kho-Kho player in a practice session while running realises that the sum of the distances from the two Kho-Kho poles from him is always 8m. Find the equation of the path traced by him if the distance between the poles is 6m.

(or)

b) Solve the differential equation :

$$\left(D^2+5\right)y=\left(\sqrt{\frac{1+\sin x}{1-\sin x}}+\sqrt{\frac{1-\sin x}{1+\sin x}}\right)\cos^2 x$$

44. a) Find the cartesian equation of the plane passing through the point (–1, 3, 2) and perpendicular to the planes x + 2y + 2z = 5, 3x + y + 2z = 8. Also find the intercepts with the three coordinate axes made by the plane.

(or)

b) Find the condition for the curves ax2 + by2 = 1, a1x2 + b1y2 = 1 such that the tangent lines at their point of intersection are perpendicular.

45. a) Find the surface area of the solid generated by revolving the arc of the parabola y2 = 4ax bounded by its latus rectum about x axis.

(or)

b) Find the equation of the hyperbola whose foci are $(\pm\sqrt{10},0)$ and passing through (3, 2).

46. a) Show that the 6th roots of unity form an abelian group with usual multiplication.

(or)

b) The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued, how many pairs would be expected to need replacement within 12 months.

47. a) Find the point on the parabola $x + y^2 = 0$ that is closest to the point (0, –3) .

(or)

b) A radioactive substance disintegrates at a rate proportional to its mass. When its mass is 10mgm, the rate of disintegration is 0.051 mgm per day. How long will it take for the mass to be reduced from 10mgm to 5mgm ($\log_e 2 = 0.6931$).