

DO NOT START ATTEMPTING THE TEST PAPER UNTIL YOU ARE ASKED TO DO SO

CLASS

12th

iOM'13

International Olympiad of Mathematics

Organized by Mathematics Olympiad Foundation, New Delhi, India



SILVER
ZONE
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TEST PAPER TYPE

DD

LEVEL 1

QUESTION PAPER

Duration : 60 Minutes

Maximum Marks : 100

Enrollment No.

Student Name

School Name

Read the following instructions carefully before you begin to answer the questions in the OMR Answer Sheets provided along with this question paper. Ask the Examination In-charge/Invigilator how to mark the OMR Answer Sheets, in case you have any doubts.

INSTRUCTION TO CANDIDATES TO BE EXPLAINED BY THE INVIGILATORS

1. This Booklet contains 40 questions. Each question carries an equal marks of 2.5.
2. All questions are compulsory.
3. This Booklet contains 6 pages. Please check, if any page is misprinted, missing or repeated.
4. Collect your OMR Answer Sheets from the Invigilator/Examination In-Charge to answer the questions given in this Booklet.
5. You must fill all the necessary information's which are required in the space provided in this booklet and OMR Answer Sheet.
6. Correct Answers must be marked by "**Darkening**" the corresponding circles on the OMR Answer Sheet, against the relevant question number with **Pencil or Blue/Black, Ball point Pen** only. Answers which are not darkened in circle will not be awarded with any mark.
7. Space for rough work is provided in this booklet. No rough work is to be done in the OMR Answer Sheet.
8. Mobile Phones and other Wireless equipments are banned in the examination halls/rooms.
9. OMR Answer Sheets must be handed over to the Examination In-charge/Invigilator before you leave the examination room/hall and recheck that you have filled all the required informations.
10. The results will be published in our web-site WWW.SILVERZONE.ORG in the month of Jan 2014. You can check it with your 12 digit Roll Number provided in the Enrollment Ticket.

TEACHERS ARE REQUESTED TO CHECK IF THE REQUIRED INFORMATIONS ARE PROPERLY FILLED BY THE CANDIDATES IN THE QUESTION PAPER & OMR ANSWER SHEETS, AND ALSO ENSURE THAT OMR ANSWER SHEETS ARE PROPERLY MARKED. PLEASE SEND US BACK THE OMR ANSWER SHEETS ONLY.

Note: Return this question paper along with answer sheet

1. Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$.

- (A) 6 (B) 3
(C) 18 (D) 9
(E) None of these

2. Let R be a relation on a set A such that $R = R^{-1}$, then R is

- (A) Reflexive (B) Symmetric
(C) Transitive (D) All of these
(E) None of these

3. In a collage of 300 students, every student reads 5 newspaper and every newspaper is read by 60 students. The number of newspaper is

- (A) At least 30 (B) At most 20
(C) Exactly 25 (D) 10
(E) None of these

4. If for two functions g and f, gof of both injective and surjective, then which of the following is true

- (A) g and f should be injective and surjective
(B) g should be injective and surjective
(C) f should be injective and surjective
(D) None of them may be surjective and injective
(E) None of these

5. If * is defined on the set R of real numbers by $a * b = \frac{3ab}{7}$. Find the identity element in R for the binary operations.

- (A) $\frac{7}{3}$ (B) $\frac{5}{3}$
(C) $\frac{3}{7}$ (D) $\frac{8}{7}$
(E) None of these

6. The principal value of $\sin^{-1} \left[\sin \left(\frac{2\pi}{3} \right) \right]$ is

- (A) $\frac{-2\pi}{3}$ (B) $\frac{2\pi}{3}$

- (C) $\frac{4\pi}{3}$ (D) $\frac{\pi}{3}$

(E) None of these

7. If the function $f(x) = \begin{cases} \frac{1 - \cos x}{x^2} & \text{for } x \neq 0 \\ k & \text{for } x = 0 \end{cases}$

is continuous at $x = 0$, then the value of k is

- (A) 1 (B) 0

- (C) -1 (D) $\frac{1}{2}$

(E) None of these

8. The solution set of the equation

$\sin^{-1} x = 2 \tan^{-1} x$ is

- (A) {1, 2} (B) {-1, 2}

- (C) {-1, 1, 0} (D) $\left\{ 1, \frac{1}{2}, 0 \right\}$

(E) None of these

9. If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, then $A^n =$

- (A) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, if $n \in \mathbb{N}$

- (B) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, if n is an odd natural number

- (C) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, if n is an even natural number

(D) All of these

(E) None of these

10. Which of the following is incorrect

(A) $A^2 - B^2 = (A + B)(A - B)$

(B) $(A^T)^T = A$

(C) $(AB)^n = A^n B^n$, where A, B commute

(D) $(A - I)(I + A) = 0 \Leftrightarrow A^2 = I$

(E) None of these

11. Which of the following is not correct?

- (A) $\begin{vmatrix} a+b & c+d \\ e+f & g+h \end{vmatrix} = \begin{vmatrix} a & c \\ e & g \end{vmatrix} + \begin{vmatrix} b & d \\ f & h \end{vmatrix}$
- (B) $|kA| = k^3 |A^T|$, where $A = [a_{ij}]_{3 \times 3}$
- (C) If A is a skew-symmetric matrix of odd order, then $|A| = 0$
- (D) $|A| = |A^T|$, where $A = [a_{ij}]_{3 \times 3}$
- (E) None of these

12. A pair has two children. If one of them is boy, then the probability that other is also a boy is

- (A) $\frac{1}{2}$ (B) $\frac{1}{4}$
- (C) $\frac{1}{3}$ (D) $\frac{4}{3}$
- (E) None of these

13. If ${}^n P_r = 840$, ${}^n C_r = 35$, then n is equal to

- (A) 1 (B) 3
- (C) 5 (D) 7
- (E) None of these

14. $\frac{d}{dx} (\log \tan x)$

- (A) $2 \sec 2x$ (B) $2 \operatorname{cosec} 2x$
- (C) $\sec 2x$ (D) $\operatorname{cosec} 2x$
- (E) None of these

15. If $y = \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$, then $\frac{dy}{dx} =$

- (A) $\frac{-8}{(e^{2x} - e^{-2x})^2}$ (B) $\frac{8}{(e^{2x} - e^{-2x})^2}$
- (C) $\frac{-4}{(e^{2x} - e^{-2x})^2}$ (D) $\frac{4}{(e^{2x} - e^{-2x})^2}$
- (E) None of these

16. If $x = a \sin \theta$ and $y = b \cos \theta$, then $\frac{d^2y}{dx^2}$ is

- (A) $\frac{a}{b^2} \sec^2 \theta$ (B) $\frac{-b}{a} \sec^2 \theta$
- (C) $\frac{-b}{a^2} \sec^3 \theta$ (D) $\frac{-b^2}{a^2} \sec^2 \theta$
- (E) None of these

17. If normal to the curve $y = f(x)$ is parallel to x-axis, then correct statement is

- (A) $\frac{dy}{dx} = 0$ (B) $\frac{dy}{dx} = 1$
- (C) $\frac{dx}{dy} = 0$ (D) All of these
- (E) None of these

18. The tangent to the curve $y = ax^2 + bx$ at $(2, -8)$ is parallel to x-axis. Then

- (A) $a = 4, b = -4$ (B) $a = 2, b = -8$
- (C) $a = 2, b = -4$ (D) $a = 2, b = -2$
- (E) None of these

19. Local maximum value of the function

$$\frac{\log x}{x} \text{ is}$$

- (A) e (B) 1
- (C) $\frac{1}{e}$ (D) $2e$
- (E) None of these

20. $\int \frac{x}{4+x^4} dx$ is equal to:

- (A) $\frac{1}{2} \tan^{-1} \left(\frac{x^2}{2} \right)$ (B) $\frac{1}{4} \tan^{-1} \left(\frac{x^2}{2} \right)$
- (C) $\frac{1}{4} \tan^{-1} x^2 + X$ (D) $\frac{1}{4} \tan^{-1} \left(\frac{x}{2} \right)$
- (E) None of these

21. $\int \frac{dx}{\sqrt{x} + \sqrt{x-2}} =$

(A) $\frac{1}{3} \left[x^{\frac{3}{2}} - (x-2)^{\frac{3}{2}} \right] + C$

(B) $\frac{2}{3} \left[x^{\frac{3}{2}} - (x-2)^{\frac{3}{2}} \right] + C$

(C) $\frac{2}{3} \left[(x-2)^{\frac{3}{2}} - x^{\frac{3}{2}} \right] + C$

(D) $\frac{1}{3} \left[(x-2)^{\frac{3}{2}} - x^{\frac{3}{2}} \right] + C$

(E) None of these

22. $\int \frac{1 + \cos^2 x}{\sin^2 x} dx =$

(A) $-\cot x - 2x + c$

(B) $-2 \cot x - 2x + c$

(C) $-2 \cot x - x + c$

(D) $-2 \cot x + x + c$

(E) None of these

23. $\int (e^{a \log x} + e^{x \log a}) dx =$

(A) $x^{a+1} + \frac{a^x}{\log a} + c$ (B) $\frac{x^{a+1}}{a+1} + a^x \log a + c$

(C) $\frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c$ (D) All of these

(E) None of these

24. $\int_0^{\pi} \frac{dx}{1 + \sin x} =$

(A) 0 (B) $\frac{1}{2}$

(C) 2 (D) $\frac{3}{2}$

(E) None of these

25. $\int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} =$

(A) πab (B) $\pi^2 ab$

(C) $\frac{\pi}{ab}$ (D) $\frac{\pi}{2ab}$

(E) None of these

26. The order and the degree of differential

equation $\frac{d^4 y}{dx^4} - 4 \frac{d^3 y}{dx^3} + 8 \frac{d^2 y}{dx^2} - 8 \frac{dy}{dx} + 4y = 0$

are respectively

(A) (1, 4) (B) (3, 4)

(C) (4, 1) (D) (1, 1)

(E) None of these

27. $y = 4 \sin 3x$ is a solution of the differential equation

(A) $\frac{dy}{dx} + 8y = 0$ (B) $\frac{dy}{dx} - 8y = 0$

(C) $\frac{d^2 y}{dx^2} + 9y = 0$ (D) $\frac{d^2 y}{dx^2} - 9y = 0$

(E) None of these

28. The solution of the differential equation

$\frac{dy}{dx} + y = \cos x$ is

(A) $y = \frac{1}{2} (\cos x + \sin x) + ce^{-x}$

(B) $y = \frac{1}{2} (\cos x - \sin x) + ce^{-x}$

(C) $y = \cos x + \sin x + ce^{-x}$

(D) All of these

(E) None of these

29. The maximum and minimum values of the function $|\sin 4x + 3|$ are:

(A) (1, 2) (B) (-1, 1)

(C) (2, 4) (D) (4, 2)

(E) None of these

30. The perimeter of a triangle with sides $3i + 4j + 5k$, $4i - 3j - 5k$ and $7i + j$ is

- (A) $\sqrt{200}$ (B) $\sqrt{50}$
 (C) $\sqrt{450}$ (D) $\sqrt{250}$
 (E) None of these

31. If three vectors a, b, c satisfy $a + b + c = 0$ and $|a| = 3$, $|b| = 5$, $|c| = 7$, then the angle between a and b is

- (A) 90° (B) 60°
 (C) 45° (D) 30°
 (E) None of these

32. An orthogonal matrix is

- (A) $\begin{bmatrix} \cos x & 2\sin x \\ -2\sin x & \cos x \end{bmatrix}$ (B) $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$
 (C) $\begin{bmatrix} \cos x & \sin x \\ \sin x & \cos x \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
 (E) None of these

33. The shortest distance between lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} \text{ and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4} \text{ is.}$$

- (A) $\sqrt{30}$
 (B) $2\sqrt{30}$
 (C) $5\sqrt{30}$
 (D) $3\sqrt{30}$
 (E) None of these

34. Two lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ ($i = 1, 2$)

are perpendicular to each other if their direction ratios satisfy

- (A) $l_1 = m_1 = n_1$
 (B) $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$
 (C) $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$
 (D) All of these
 (E) None of these

35. The equation of the plane through $(1, 2, 3)$ and parallel to the plane $2x + 3y - 4z = 0$ is

- (A) $2x + 3y + 4z + 4 = 4$
 (B) $2x + 3y + 4z + 4 = 0$
 (C) $2x - 3y + 4z + 4 = 0$
 (D) $2x + 3y - 4z + 4 = 0$
 (E) None of these

36. If $p = 7i - 2j + 3k$ and $q = 3i + j + 5k$, then the magnitude of $p - 2q$ is

- (A) $\sqrt{29}$
 (B) 4
 (C) $\sqrt{62} - 2\sqrt{35}$
 (D) $\sqrt{66}$
 (E) None of these

37. If $6P(A) = 8P(B) = 14P(A \cap B) = 1$, then

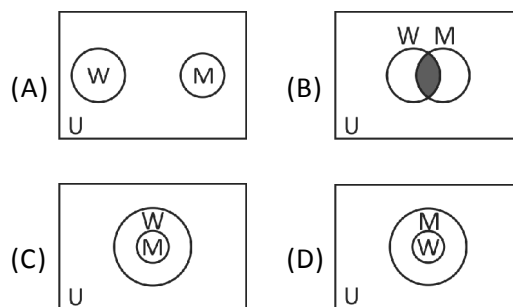
$$P\left(\frac{B}{A}\right) =$$

- (A) $\frac{4}{7}$ (B) $\frac{3}{5}$
 (C) $\frac{3}{7}$ (D) $\frac{3}{4}$
 (E) None of these

38. Which of the following Venn diagram corresponds to the statement

“All mothers are women”

(M is the set of all mothers, W is the set of all women)



- (E) None of these

