# iOM'13 <br> International Olympiad of Mathematics 

Organized by Mathematics Olympiad Foundation, New Delhi, India

## QUESTION PAPER

LEVEL 1


Read the following instructions carefully before you begin to answer the questions in the OMR Answer Sheets provided along with this question paper. Ask the Examination In-charge/Invigilator how to mark the OMR Answer Sheets, in case you have any doubts.

## INSTRUCTION TO CANDIDATES TO BE EXPLAINED BY THE INVIGILATORS

1. This Booklet contains 40 questions. Each question carries an equal marks of 2.5.
2. All questions are compulsory.
3. This Booklet contains 6 pages. Please check, if any page is misprintied, missing or repeated.
4. Collect your OMR Answer Sheets from the Invigilator/Examination In-Charge to answer the questions given in this Booklet.
5. You must fill all the necessary information's which are required in the space provided in this booklet and OMR Answer Sheet.
6. Correct Answers must be marked by "Darkening" the corresponding circles on the OMR Answer Sheet, against the relevant question number with Pencil or Blue/Black, Ball point Pen only. Answers which are not darkened in circle will not be awarded with any mark.
7. Space for rough work is provided in this booklet. No rough work is to be done in the OMR Answer Sheet.
8. Mobile Phones and other Wireless equipments are banned in the examination halls/rooms.
9. OMR Answer Sheets must be handed over to the Examination In-charge/Invigilator before you leave the examination room/hall and recheck that you have filled all the required informations.
10. The results will be published in our web-site WWW.SILVERZONE.ORG in the month of Jan 2014. You can check it with your 12 digit Roll Number provided in the Enrollment Ticket.

TEACHERS ARE REQUESTED TO CHECK IF THE REQUIRED INFORMATIONS ARE PROPERLY FILLED BYTHE CANDIDATES IN THE QUESTION PAPER \& OMR ANSWER SHEETS, AND ALSO ENSURE THAT OMR ANSWER SHEETS ARE PROPERLY MARKED. PLEASE SEND US BACK THE OMR ANSWER SHEETS ONLY.

Note: Return this question paper along with answer sheet

1. If a set $\mathbf{A}$ has n elements, then the total number of subsets of $A$ is
(A) $n$
(B) $\mathrm{n}^{2}$
(C) $2^{n}$
(D) $2 n$
(E) None of these
2. Let $U=\{1,2,3,4,5,6,7,8,9,10\}, A=\{1,2,5\}$, $B=\{6,7\}$, then $A \cap B^{\prime}$ is
(A) $\mathrm{B}^{\prime}$
(B) A
(C) $\mathrm{A}^{\prime}$
(D) B
(E) None of these
3. The shaded region in the given figure is

(A) $A \cap(B \cup C)$
(B) $A \cup(B \cap C)$
(C) $A \cap(B-C)$
(D) $A-(B \cup C)$
(E) None of these
4. In a group of $\mathbf{1 0 0}$ people, $\mathbf{4 2}$ like coffee, 65 like tea, and each person likes at least one of the two drinks. How many people like both coffee and tea?
(A) 19
(B) 15
(C) 7
(D) 5
(E) None of these
5. If $f(x)=\frac{1-x}{1+x}$, then $f[f(\cos 2 \theta)]=$
(A) $\tan 2 \theta$
(B) $\sec 2 \theta$
(C) $\cos 2 \theta$
(D) $\cot 2 \theta$
(E) None of these
6. If $\sin \theta+\cos \theta=1$, then $\sin \theta \cdot \cos \theta=$
(A) 0
(B) 1
(C) 2
(D) $\frac{1}{2}$
(E) None of these
7. The value of $\cos 105^{\circ}+\sin 105^{\circ}$ is
(A) $\frac{1}{2}$
(B) 1
(C) $\sqrt{2}$
(D) $\frac{1}{\sqrt{2}}$
(E) None of these
8. If $\tan A=\frac{-1}{2}$ and $\tan B=\frac{-1}{3}$, then $A+B=$
(A) $\frac{\pi}{4}$
(B) $\frac{3 \pi}{4}$
(C) $\frac{5 \pi}{4}$
(D) $\frac{7 \pi}{4}$
(E) None of these
9. If the arcs of same length in two circles subtend angles of $60^{\circ}$ and $75^{\circ}$ at their centres. Find the ratio of their radii.
(A) $5: 4$
(B) $4: 5$
(C) $5: 3$
(D) $3: 5$
(E) None of these
10. The probability that an ordinary or a nonleap year has 53 Sunday is
(A) $\frac{2}{7}$
(B) $\frac{3}{7}$
(C) $\frac{1}{7}$
(D) $\frac{5}{7}$
(E) None of these
11. $\lim _{x \rightarrow 1} \frac{\log x}{x-1}=$
(A) 1
(B) -1
(C) 0
(D) $\infty$
(E) None of these
12. Match the column in the table given below:

| S.No. | Sets |  | Roster <br> Form |
| :--- | :--- | :--- | :--- |
| 1. | $\left\{x: x^{2}-3=0\right.$ and $x$ <br> is a rational <br> number $\}$ | a. | $\{4\}$ |
| 2. | $\{x: x$ is an even <br> prime number $\}$ | b. | $\{-5,5\}$ |
| 3. | $\{x: 3<x<5, x$ is a <br> natural number $\}$ | c. | $\{*\}$ |
| 4. | $\left\{x: x^{2}=25\right.$, and $x$ <br> is an odd <br> integer $\}$ | d. | $\{2\}$ |

(A) $1-\mathrm{c}, 2-\mathrm{d}, 3-\mathrm{a}, 4-\mathrm{b}$
(B) $1-\mathrm{d}, 2-\mathrm{c}, 3-\mathrm{a}, 4-\mathrm{b}$
(C) $1-\mathrm{c}, 2-\mathrm{a}, 3-\mathrm{d}, 4-\mathrm{b}$
(D) 1-b, 2-d, 3-a, 4-c
(E) None of these
13. Conjugate of $\mathbf{1 + i}$ is
(A) i
(B) 1
(C) $1-i$
(D) $1+i$
(E) None of these
14. If $\left(\frac{1+i}{1-i}\right)^{m}=1$, then the least integral value of $m$ is
(A) 2
(B) 4
(C) 8
(D) 10
(E) None of these
15. The amplitude of $\frac{1+\sqrt{3} i}{\sqrt{3}+i}$ is
(A) $\frac{\pi}{6}$
(B) $-\frac{\pi}{6}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{2}$
(E) None of these
16. If one root of $5 x^{2}+13 x+k=0$ is reciprocal of the other, then $k=$
(A) 0
(B) 5
(C) $\frac{1}{6}$
(D) 6
(E) None of these
17. If the $9^{\text {th }}$ term of an A.P be zero, then the ratio of its $29^{\text {th }}$ and $19^{\text {th }}$ term is
(A) $1: 2$
(B) $2: 1$
(C) $1: 3$
(D) $3: 1$
(E) None of these
18. The sum of the roots of a equation is 2 and sum of their cubes is 98 , then the equation is
(A) $x^{2}+2 x+15=0$
(B) $x^{2}+15 x+2=0$
(C) $2 x^{2}-2 x+15=0$
(D) $x^{2}-2 x-15=0$
(E) None of these
19. The number of ways in which 6 rings can be worn on the four fingers of one hand is
(A) $4^{6}$
(B) ${ }^{6} \mathrm{C}_{4}$
(C) $6^{4}$
(D) 24
(E) None of these
20. The sum of $n$ arithmetic mean between a and $b$, is
(A) $(n+1)(a+b)$
(B) $\frac{(n+1)(a+b)}{2}$
(C) $n(a+b)$
(D) $\frac{n(a+b)}{2}$
(E) None of these
21. The angle between the lines $y-2 x=9$ and $x$ $+2 y=-7$, is
(A) $60^{\circ}$
(B) $45^{\circ}$
(C) $30^{\circ}$
(D) $90^{\circ}$
(E) None of these
22. If a man and his wife enter in a bus, in which five seats are vacant, then the number of different ways in which they can be seated is
(A) 2
(B) 5
(C) 20
(D) 40
(E) None of these
23. The greatest number among $\sqrt[3]{9}, \sqrt[4]{11}, \sqrt[6]{17}$ is
(A) $\sqrt[3]{9}$
(B) $\sqrt[4]{11}$
(C) $\sqrt[6]{17}$
(D) Can not be determine
(E) None of these
24. How many words can be formed using the letter A thrice, the letter B twice and the letter $C$ thrice?
(A) 500
(B) 560
(C) 580
(D) 520
(E) None of these
25. The middle term in the expansion of $\left(x-\frac{1}{x}\right)^{18}$ is
(A) ${ }^{18} \mathrm{C}_{9}$
(B) ${ }^{-18} \mathrm{C}_{9}$
(C) ${ }^{18} \mathrm{C}_{0}$
(D) ${ }^{-18} \mathrm{C}_{10}$
(E) None of these
26. If $\mathbf{n}^{\text {th }}$ terms of two A.P.'s are $3 n+8$ and $7 n+$ 15 , then the ratio of their $12^{\text {th }}$ terms will be
(A) $\frac{4}{9}$
(B) $\frac{7}{16}$
(C) $\frac{3}{7}$
(D) $\frac{8}{15}$
(E) None of these
27. If the first term of an A.P is 2 and common difference is 4 . The sum of its 40 terms will be
(A) 3200
(B) 2800
(C) 1600
(D) 200
(E) None of these
28. Distance between the points $(1,3,2)$ and $(2,1,3)$ is
(A) 12
(B) $\sqrt{12}$
(C) $\sqrt{6}$
(D) 6
(E) None of these
29. The length of the latus rectum of the ellipse $5 x^{2}+9 y^{2}=45$ is
(A) $\frac{\sqrt{5}}{4}$
(B) $\frac{\sqrt{5}}{2}$
(C) $\frac{5}{3}$
(D) $\frac{10}{3}$
(E) None of these
30. $\lim _{x \rightarrow \infty} \frac{\sin x}{x}$
(A) 0
(B) $\infty$
(C) 1
(D) 2
(E) None of these
31. The distance between two parallel lines $3 x+4 y-8=0$ and $3 x+4 y-3=0$, is given by
(A) 4
(B) 5
(C) 3
(D) 1
(E) None of these
32. The eccentricity of the conic $x^{2}-4 y^{2}=1$ is
(A) $\frac{2}{\sqrt{3}}$
(B) $\frac{\sqrt{3}}{2}$
(C) $\frac{2}{\sqrt{5}}$
(D) $\frac{\sqrt{5}}{2}$
(E) None of these
33. If $A$ and $B$ are two events such that $P(A \cup B)$ $+P(A \cap B)=\frac{7}{8}$ and $P(A)=2 P(B)$, then $P(A)=$
(A) $\frac{17}{24}$
(B) $\frac{5}{12}$
(C) $\frac{7}{12}$
(D) $\frac{7}{24}$
(E) None of these
34. Find the value of:
$\sin 20^{\circ} \sin 40^{\circ} \sin 60^{\circ} \sin 80^{\circ}$
(A) $\frac{5}{16}$
(B) $\frac{3}{16}$
(C) $\frac{1}{4}$
(D) 1
(E) None of these
35. The triangle formed by the points
$(0,7,10),(-1,6,6),(-4,9,6)$ is
(A) Equilateral
(B) Isosceles
(C) Right angled
(D) Right angled isosceles
(E) None of these
36. Argument and modulus of $\frac{1+i}{1-i}$ are respectively
(A) $\frac{-\pi}{2}$ and 1
(B) $\frac{\pi}{2}$ and $\sqrt{2}$
(C) $\frac{\pi}{2}$ and 1
(D) 0 and $\sqrt{2}$
(E) None of these
37. $\lim _{x \rightarrow 0} \frac{1-\cos 2 x}{x}$ is
(A) 0
(B) 1
(C) 2
(D) 4
(E) None of these
38. If $P(A)=\frac{2}{3}, P(B)=\frac{1}{2}$ and $P(A \cup B)=\frac{5}{6}$, then events $A$ and $B$ are
(A) Mutually exclusive
(B) Independent as well as mutually exhaustive
(C) Independent
(D) Dependent only on A
(E) None of these
39. From the following figures, find the figure which does not belong to the series.

A

B

C

D
40. If the numerator of a certain fraction is increased by $100 \%$ and the denominator is increased by 200\%, the new fraction thus formed is $\frac{4}{21}$. What is the original fraction?
(A) $\frac{2}{5}$
(B) $\frac{3}{7}$
(C) $\frac{2}{7}$
(D) $\frac{4}{7}$
(E) None of these

