North Maharashtra University, Jalgaon
Question Bank

(New syllabus w.e.f. June 2007)

Class: F. Y. B. Sc.

Subject: Mathematics

Paper I

(ALGEBRA AND TRIGNOMETRY)

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Unit – 01

Adjoint and Inverse of Matrix, Rank of a Matrix and

Eigen Values and Eigen Vectors

Marks – 02

1) If $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 1 \\ 3 & 1 & 0 \end{bmatrix}$, find minor and cofactor of a_{11}, a_{23} and a_{32}

2) If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, find adj A

3) If $A = \begin{bmatrix} -1 & 3 \\ 7 & 2 \end{bmatrix}$, find A^{-1}

4) If $A = \begin{bmatrix} -1 & 2 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$, find $\rho(AB)$

5) If $A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 5 & 7 \\ 1 & 2 & 3 \end{bmatrix}$, find $\rho(A)$

6) Find rank of $A = \begin{bmatrix} 2 & 6 \\ 1 & 3 \\ 3 & 9 \end{bmatrix}$

7) Find the characteristic equation and eigen values of $A = \begin{bmatrix} 9 & -7 \\ 3 & -1 \end{bmatrix}$

8) Define characteristic equation of a matrix A and state Cayley-Hamilton Theorem.

9) Define adjoint of a matrix A and give the formula for A^{-1} if it exist.

10) Define inverse of a matrix and state the necessary and sufficient condition for existence of a matrix.

11) Compute $E_{12}(3)$, $E'_2(3)$ of order 3

12) If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ -3 & 2 \end{bmatrix}$, find $(AB)^{-1}$

13) If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ then ρ (A) is ---

a) 0 b) 1 c) 2 d) 4

14) If $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ then which of the following is true ?

a) adjA is nonsingular b) adjA has a zero row

c) adjA is symmetric d) adjA is not symmetric

15) If $A = \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$ then which of the following is true ?

a) $A^2 = A$ b) A^2 is identity matrix

c) A^2 is non-singular d) A^2 is singular

16) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

Statement I : AB singular

Statement II : adj(AB) = adjB adjA

then which of the following is true

a) Statement I is true b) Statement II is true

c) Both Statements are true d) both statements are false

17) If A is a square matrix, then A^{-1} exists iff

a) $|A| > 0$ b) $|A| < 0$

c) $|A| = 0$ d) $|A| \neq 0$

18) If $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ then A(adj A) is

a) $\begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$

c) $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ d) $\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$

19) If A is a square matrix of order n then $|KA|$ is

a) $K|A|$ b) $\left(\frac{1}{K}\right)^n |A|$

c) $K^n|A|$ d) None of these

20) Let I be identity matrix of order n then

a) adj A = I b) adj A = 0

c) adj A = n I d) None of these

21) Let A be a matrix of order m x n then $|A|$ exists iff

a) $m > n$ b) $m < n$

c) $m = n$ d) $m \neq n$

22) If $AB = \begin{bmatrix} 4 & 11 \\ 4 & 5 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$ then det. B is equal to

a) 4 b) -6 c) - ¼ d) -28

23) If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ then x = ---

a) -1/2 b) -1/2 c) 1 d) 2

24) If $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ and n∈N then A^n is ----

a) $\begin{bmatrix} 2^n & 2^n \\ 2^n & 2^n \end{bmatrix}$ b) $\begin{bmatrix} 2n & 2n \\ 2n & 2n \end{bmatrix}$

c) $\begin{bmatrix} 2^{2n-1} & 2^{2n-1} \\ 2^{2n-1} & 2^{2n-1} \end{bmatrix}$ d) $\begin{bmatrix} 2^{2n+1} & 2^{2n+1} \\ 2^{2n+1} & 2^{2n+1} \end{bmatrix}$

25) If $A = \begin{bmatrix} -1 & -3 \\ 4 & 2 \end{bmatrix}$ then $|adjA|$ is

a) 10 b) 1000 c) 100 d) 110

26) If a square matrix A of order n has inverses B and C then

a) $B \neq C$ b) $B = C^n$ c) $B = C$ d) None of these

27) If A is symmetric matrix then

a) adjA is non-singular matrix   b) adjA is symmetric matrix

c) adjA does not exist   d) None of these

28) If $AB = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & -7 \\ 4 & -2 \end{bmatrix}$ then

a) $(AB)^{-1} = AB$ b) $(AB)^{-1} = A^{-1} B^{-1}$

c) $(AB)^{-1} = B^{-1} A^{-1}$ d) None of these

29) If $|A| \neq 0$ and B, C are matrices such that AB = AC then

a) $B \neq C$ b) $B \neq A$ c) $B = C$ d) $C \neq A$

30) If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ then

a) $A^2 = I$ b) $A^2 = 0$ c) $A^2 = A$ d) None of these

31) If matrix A is equivalent to matrix B then

a) $\rho(A) \neq \rho(B)$ b) $\rho(A) > \rho(B)$

c) $\rho(A) = \rho(B)$ d) None of these

32) If $A = \begin{bmatrix} 1 & -4 & 0 \end{bmatrix}$ then $\rho(A)$ is

a) 0   b) 1   c) 3   d) None of these

33) If $A = \begin{bmatrix} 1 & 9 & 2 & 0 \\ 0 & -3 & 4 & 1 \\ 1 & 9 & 2 & 0 \end{bmatrix}$ then $\rho(A)$ is

a) 0   b) 1   c) 2   d) 3

34) If A is a matrix of order m x n then

a) $\rho(A) \nleq \min\{m,n\}$ b) $\rho(A) \leq \min\{m,n\}$

c) $\rho(A) \geq \max\{m,n\}$ d) None of these

35) The eigen values of $A = \begin{bmatrix} -2 & 7 \\ 2 & 3 \end{bmatrix}$ are

a) -5, -4   b) 5, 4   c) 5, -4   d) None of these

36) If $A = \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$ then A satisfies

a) $A^2 + 3A + 17I = 0$ b) $A^2 - 3A - 17I = 0$

c) $A^2 - 3A + 17I = 0$ d) $A^2 + 3A - 17I = 0$

37) If A is a matrix and λ is some scalar such that $A - \lambda I$ is singular then

a) λ is eigen value of A b) λ is not an eigen value of A

c) $\lambda = 0$ d) None of these

38) If $A = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ then A^{-1} exists if

a) $\rho(A) = 0$ b) $\rho(A) = 3$ c) $\rho(A) = 1$ d) None of these

39) If $A = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$ then which of the following is incorrect ?

a) $A = A^{-1}$ b) $A^2 = I$ c) $A^2 = 0$ d) None of these

Marks : 04

1) If A is a square matrix of order n then prove that $(\mathrm{adj}A)' = \mathrm{adj}\ A'$

and verify it for $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$

2) For the following matrix, verify that $(\mathrm{adj}A)' = \mathrm{adj}\ A'$

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

3) If $A = \begin{bmatrix} 4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ then show that adj A = A

4) If $A = \begin{bmatrix} -3 & 1 & 0 \\ 2 & -2 & 1 \\ -1 & -1 & 1 \end{bmatrix}$, show that A(adj A) is null matrix.

5) Show that the adjoint of a symmetric matrix is symmetric and verify it for

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

6) Verify that $(\mathrm{adj}A)A = |A|\, I$ for the matrix $A = \begin{bmatrix} 4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

7) Verify that $A(\mathrm{adj}A) = (\mathrm{adj}A)A = |A|\, I$ for the matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

8) Verify that $A(\mathrm{adj}A) = |A|\, I$ for the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$

9) Find the inverse of $A = \begin{bmatrix} -1 & -2 & -1 \\ 2 & 1 & 0 \\ -3 & 1 & -1 \end{bmatrix}$

10) Find the inverse of A = $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ -3 & -1 & 1 \end{bmatrix}$

11) Show that the matrix A = $\begin{bmatrix} 4 & -1 \\ -3 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 6A + 5I = 0$.

Hence find A^{-1}

12) If A = $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, B = $\begin{bmatrix} -1 & 3 \\ 7 & 2 \end{bmatrix}$, show that adj(AB) = adjB adjA

13) If A = $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, show that A(adjA) = (adjA)A = $|A|$ I

14) If A = $\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ then show that adjA = $3A'$

15) If A = $\begin{bmatrix} -2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, show that $A^2 = A$, but A^{-1} does not exist.

16) If A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$, show that $A^3 = A^{-1}$

17) What is the reciprocal of the following matrix ?

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

18) If A = $\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1} A^{-1}$

19) Using adjoint method find the inverse of the matrix A = $\begin{bmatrix} -1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$

20) If A is a non-singular matrix of order n then prove that adj (adjA) = $|A|^{n-2}$ A

21) For a non-singular square matrix A of order n , prove that

$$|\operatorname{adj}(\operatorname{adj} A)| = |A|^{(n-1)^2}$$

22) For a non-singular square matrix A of order n , prove that

$$\operatorname{adj}\{\operatorname{adj}(\operatorname{adj} A)\} = |A|^{n^2-3n+3} A^{-1}$$

23) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, show that $A^3 = A^{-1}$

24) Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 2 & 3 \\ 1 & 4 & 1 \end{bmatrix}$

25) Find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 4 \\ 3 & 2 & 4 \end{bmatrix}$

26) Compute the elementary matrix $[E_2(-3)]^{-1} . E_{31}(2) . E'_{21}(1/2)$ of order 3

27) Compute the matrix $E'_2(1/3) . E_{31} . [E_2(-4)]^{-1}$ for E-matrices of order 3

28) Determine the values of x so that the matrix $\begin{bmatrix} x & x & 2 \\ 2 & x & x \\ x & 2 & x \end{bmatrix}$ is of

i) rank 3 ii) rank 2 iii) rank 1

29) Determine the values of x so that the matrix $\begin{bmatrix} x & x & 1 \\ 1 & x & x \\ x & 1 & x \end{bmatrix}$ is of

i) rank 3 ii) rank 2 iii) rank 1

30) Reduce the matrix A to the normal form. Hence determine its rank,

where $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 8 & 10 \end{bmatrix}$

31) Reduce the matrix A to the normal form. Hence determine its rank,

$$\text{where } A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 2 & 6 & 7 & 5 \end{bmatrix}$$

32) Reduce the matrix A to the normal form. Hence determine its rank,

$$\text{where } A = \begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

33) Find non-singular matrices P and Q such that PAQ is in normal form,

$$\text{where } A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

34) Find non-singular matrices P and Q such that PAQ is in normal form,

$$\text{where } A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$$

35) Find non-singular matrices P and Q such that PAQ is in normal form,

$$\text{where } A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \text{ Also find } \rho(A)$$

36) Show that the matrix $A = \begin{bmatrix} x-1 & 1 & 2 \\ 0 & x & 4 \\ -3 & 2 & x \end{bmatrix}$

has rank 3 when $x \neq 2$ and $x \neq \pm\sqrt{2}$, find its rank when $x = 2$.

37) Find a non-singular matrix P such that $PA = \begin{bmatrix} G \\ 0 \end{bmatrix}$ for the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 4 \\ 3 & 2 & 4 \end{bmatrix}. \text{ Hence find } \rho(A).$$

38) Given $A = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$,

verify that $\rho(AB) \leq \min\{\rho(A), \rho(B)\}$

39) Find all values of θ in $[-\pi/2, \pi/2]$ such that the matrix

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$ is of rank 2.

40) Express the following non-singular matrix A as a product of E – matrices,

where $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

41) Express the following non-singular matrix A as a product of E – matrices,

where $A = \begin{bmatrix} 7 & 0 & 3 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$

42) Express the following non-singular matrix A as a product of E – matrices,

where $A = \begin{bmatrix} 13 & 3 & 3 \\ 4 & 1 & 1 \\ 4 & 0 & 1 \end{bmatrix}$

43) State Cayley- Hamilton Theorem. Verify it for $A = \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$

44) State Cayley- Hamilton Theorem. Verify it for $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$

45) Verify Cayley- Hamilton Theorem for $A = \begin{bmatrix} 1 & 2 & 0 \\ -3 & -2 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

46) Find the characteristics equation of $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 3 & 0 \\ 2 & -1 & 2 \end{bmatrix}$

47) Find eigen values of $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$

48) If λ is a non-zero eigen value of a non-singular matrix A, show that $1/\lambda$ is an eigen value of A^{-1}

49) If $\lambda \neq 0$ is an eigen value of a non-singular matrix A, show that $|A|/\lambda$ is an eigen value of adj A.

50) Let k be a non-zero scalar and A be a non-zero square matrix, show that if λ is an eigen value of A then λk is an eigen value of kA.

51) Let A be a square matrix. Show that 0 is an eigen value of A iff A is singular.

52) Show that $A = \begin{bmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & b & a \\ b & 0 & c \\ a & c & 0 \end{bmatrix}$

have the same characteristic equation.

53) Find eigen values and corresponding eigen vectors of $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

54) Find eigen values and corresponding eigen vectors of $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$

55) Find characteristic equation of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

Also find A^{-1} by using Cayley Hamilton theorem.

56) Verify Cayley Hamilton theorem for A and hence find A^{-1}

where $A = \begin{bmatrix} -2 & 7 \\ 3 & 4 \end{bmatrix}$

Marks – 04 / 06

1) If A, B are matrices such that product AB is defined then prove that $(AB)' = B'A'$

2) If $A = [a_{ij}]$ is a square matrix of order n then show that

$A(\text{adj}A) = (\text{adj}A)A = |A|\, I$

3) Show that a square matrix A is invertible if and only if $|A| \neq 0$

4) If A, B are non-singular matrices of order n then prove that AB is non-singular and $(AB)^{-1} = B^{-1}A^{-1}$

5) If A, B are non-singular matrices of same order then prove that

$\text{adj}(AB) = (\text{adj}B)(\text{adj}A)$

6) If A is a non-singular matrix then prove that $(A^n)^{-1} = (A^{-1})^n,\ \forall n \in N$

7) If A is a non-singular matrix and $k \neq 0$ then prove that $(kA)^{-1} = \frac{1}{k}A^{-1}$

8) If A is a non-singular matrix then prove that $(\text{adj } A)^{-1} = \text{adj } A^{-1} = \frac{A}{|A|}$

9) State and prove the necessary and sufficient condition for a square matrix A to have an inverse.

10) If A is a non-singular matrix then show that AB = AC implies B = C

Is the result true when A is singular ? Justify.

11) When does the inverse of a matrix exist ? Prove that the inverse of a matrix, if it exists, is unique.

12) If a non-singular matrix A is symmetric prove that A^{-1} is also symmetric.

13) Prove that inverse of an elementary matrix is an elementary matrix of the same type.

14) If A is a m x n matrix of rank r, prove that their exist non-singular matrices P and Q such that $PAQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$

15) Prove that every non-singular matrix can be expressed as a product of finite number of elementary matrices.

16) If A is an mxn matrix of rank r, then show that there exists a non-singular matrix P such that PA = $\begin{bmatrix} G \\ 0 \end{bmatrix}$, where G is rxn matrix of rank r and 0 is null matrix of order (m-r)xn.

17) Prove that the rank of the product of two matrices can not exceed the rank of either matrix.

18) If A is an mxn matrix of rank r then show that there exists a non-singular matrix Q such that AQ = $\begin{bmatrix} H & 0 \end{bmatrix}$ Where H is mxr matrix of rank r and 0 is null matrix of order mx(n-r).

Unit - 02

System of Linear Equations and Theory of Equations

Marks - 02

1) Examine for non-trivial solutions

$x + y + z = 0$

$4x + y = 0$

$2x + 2y + 3z = 0$

2) Define i) Consistent and inconsistent system ii) Equivalent system

3) Define homogeneous, non-homogeneous system of equations.

4) The equation $x^4+4x^3-2x^2-12x+9 = 0$ has two pairs of equal roots, find them.

5) Change the signs of the roots of the equation $x^7 + 5x^5 - x^3 + x^2 + 7x + 3 = 0$

6) Transform the equation $x^7 - 7x^6 - 3x^4 + 4x^2 - 3x - 2 = 0$ into another whose roots shall be equal in magnitude but opposite in sign to those of this equation.

7) Change of the equation $3x^4 - 4x^3 + 4x^2 - 2x + 1 = 0$ into another the coefficient of whose highest term will be unity.

8) A system AX = B, of m linear equations in n unknowns, is consistent iff

A) rankA $\neq$ rank [A, B] B) rankA = rank [A, B]

C) rankA $\geq$ rank [A, B] D) rankA $\leq$ rank [A, B]

9) For the equation $x^4 + x^2 + x + 1 = 0$, sum of roots taken one, two, three and four at time is respectively.

A) 1, 1, 1, 1 B) 0, 1, -1, 1

C) 1, 0, -1, 1 D) -1, 1, -1, 1

10) For the equation $x^4 + x^3 + x^2 + x + 1 = 0$, sum of roots taken one , two, three and four at a time is respectively.

A) 1, 1, 1, 1 B) -1, 1, -1, 1

C) 1, -1, 1, -1 D) -1, -1, -1, 1

11) If sum and product of roots of a quadratic equation are 1 and –1 respectively the required quadratic equation is

A) $x^2 + x + 1 = 0$ B) $x^2 - x + 1 = 0$

C) $x^2 + x - 1 = 0$ D) $-x^2 + x + 1 = 0$

12) The quadratic equation having roots α and β is

A) $x^2 - (\alpha + \beta) x + \alpha\beta = 0$ B) $x^2 + (\alpha + \beta) x + \alpha\beta = 0$

C) $x^2 + (\alpha + \beta) x - \alpha\beta = 0$ D) $- x^2 + (\alpha + \beta) x + \alpha\beta = 0$

13) The equation having roots 2, 2, -1 is

A) $x^3 + x^2 + x + 4 = 0$ B) $x^3 + 3x^2 + 4 = 0$

C) $x^3 - 3x^2 + 4 = 0$ D) $x^3 - 3x^2 + x - 4 = 0$

14) The equation having roots 1, 1, 1 is

A) $x^3 + 3x^2 + 3x + 1 = 0$ B) $x^3 - 3x^2 + 3x - 1 = 0$

C) $x^3 + 3x^2 - x - 1 = 0$ D) $x^3 + 3x^2 - 3x + 1 = 0$

15) Roots of equation $x^3 - 3x^2 + 4 = 0$ are 2, 2, -1,

so the roots of equation $x^3 - 6x^2 + 32 = 0$ are

A) 4, 2, -1 B) 4, -4, -1

C) 4, 4, -2 D) 4, -4, -2

16) Roots of equation $x^2 + 2x + 1 = 0$ are -1, -1 so the roots of equation

$x^3 + 6x + 9 = 0$ are

A) -3, 3 B) 3, 3

C) -3, -3 D) 3, -3

17) Roots of equation $x^2–2x+4=0$ are 2, 2 so the roots of equation $4x^2–2x+1=0$ are

A) 2, -2 B) 2, 2

C) 1/2, 1/2 D) -1/2, 1/2

18) Roots of equation $x^2 - 5x + 6 = 0$ are 2, 3 so the roots of equation

$6x^2 - 5x + 1 = 0$ are

A) 2, -3 B) 2, 3

C) 1/2, 1/3 D) -1/2, 1/3

19) Find the equation whose roots are the roots of $x^2 - 4x + 4 = 0$ each diminished by 1.

A) $x^2 - 4x + 4 = 0$ B) $x^2 - 2x + 1 = 0$

C) $x^2 + 2x + 1 = 0$ D) $x^2 - 2x - 1 = 0$

20) Find the equation whose roots are the roots of $x^3 - 6x^2 + 12x - 8 = 0$ each diminished by 1.

A) $x^3 - 3x^2 + 3x - 1 = 0$ B) $x^3 + 3x^2 + 3x + 1 = 0$

C) $x^3 - 3x^2 - 3x - 1 = 0$ D) $x^3 - 3x^2 - 3x + 1 = 0$

21) To remove the second term from equation $x^4 - 8x^3 + x^2 - x - 3 = 0$ the roots diminished by

A) 3 B) 2 C) 1 D) -2

22) To remove the second term from equation $x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$ the roots diminished by

A) 1 B) -1

C) 2 D) -2

Marks – 04

1. Examine for consistency the following system of equations

   $x + z = 2$

   $-2x + y + 3z = 3$

   $-3x + 2y + 7z = 4$

2. Solve the following system of equations

   $x + y + z = 6$

   $2x + y + 3z = 13$

   $5x + 2y + z = 12$

   $2x - 3y - 2z = -10$

3. If $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & -2 & 3 \\ -1 & 3 & -4 \end{bmatrix}$, find A^{-1}. Hence solve the following system of linear equations $2x + y - z = 1$ $x - 2y + 3z = 9$ $-x + 3y - 4z = -12$

4. Test the following equations for consistency and if consistent solve them

   $2x - y - 5z + 4w = 1$

   $x + 3y + z - 5w = 18$

   $3x - 2y - 8z + 7w = -1$

5. Solve the following system of equations

   $x_1 + 3x_2 + 4x_3 - 6x_4 = 0$

   $x_2 + 6x_3 = 0$

   $2x_1 + 2x_2 + 2x_3 - 3x_4 = 0$

   $x_1 + x_2 - 4x_3 - 4x_4 = 0$

6. Examine for non-trivial solutions the following homogeneous system of linear equations

   $x + y + 3z = 0$

   $x - y + z = 0$

   $-x + 2y = 0$

   $x - y + z = 0$

7. Solve the system of equations

$x + 3y + 3z = 14$

$x + 4y + 3z = 16$

$x + 3y + 4z = 17$

by i) method of inversion ii) method of reduction.

8. Examine the following systems of equation for consistency

$x - 2y + z - u = 1$

$x + y - 2z + 3u = -2$

$4x + y - 5z + 8u = -5$

$5x - 7y + 2z - u = 3$

9. Test the following equations for consistency and solve them

$x + 2y + z = 2$

$3x + y - 2z = 1$

$4x - 3y - z = 3$

$x + 2y + z = 2$

10. Solve the following equations

$4u + 2v + w + 3t = 0$

$2u + v + t = 0$

$6u + 3v + 4w + 7t = 0$

11. Solve the equation $x^3 - 3x^2 - 6x + 8 = 0$ if the roots are in A.P.

12. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if two of its roots are in the ratio 3:2.

13. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if the roots are in G.P.

14. Solve the equation $x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$ whose roots are in A.P.

15. If α, β and γ are roots of the equation $x^3 - 5x^2 - 2x + 24 = 0$ find the value of

i) $\sum \alpha^2\beta$ ii) $\sum \alpha^2$ iii) $\sum \alpha^3$ iv) $\sum \alpha^2\beta^2$

16. Remove the fractional coefficients from the equation $x^3 - \frac{1}{2}x^2 + \frac{2}{3}x - 1 = 0$

17. Remove the fractional coefficients from the equation $x^3 - \frac{5}{2}x^2 - \frac{7}{18}x + \frac{1}{108} = 0$

18. Transform the equation $5x^3 - \frac{3}{2}x^2 - \frac{3}{4}x + 1 = 0$ to another with integral coefficients and unity for the coefficient of the first term.

19. Remove the fractional coefficients from the equation

$x^4 + \frac{3}{10}x^2 + \frac{13}{25}x + \frac{77}{1000} = 0$

20. Find the equation whose roots are reciprocals of the roots of $x^4 - 5x^3 + 7x^2 + 3x - 7 = 0$

21. Find the equation whose roots are the roots of $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ each diminished by 4.

22. Find the equation whose roots are those of $3x^3 - 2x^2 + x - 9 = 0$ each diminished by 5.

23. Remove the second term from equation $x^4 - 8x^3 + x^2 - x + 3 = 0$

24. Remove the third term of equation $x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$, hence obtain the transformed equation in case h =3.

25. Transform the equation $x^4 + 8x^3 + x - 5 = 0$ into one in which the second term is vanishing.

26. Solve the equation $x^4+16x^3+83x^2+152x+84 = 0$ by removing the second term.

27. Solve the equation $x^3 + 6x^2 + 9x + 4 = 0$ by Carden's method.

28. Solve the equation $x^3 - 15x^2 - 33x + 847 = 0$ by Carden's method.

29. Solve the equation $z^3 - 6z^2 - 9 = 0$ by Carden's method.

30. Solve the equation $x^3 - 21x - 344 = 0$ by Carden's method.

31. Solve $x^3 - 15x - 126 = 0$ by Carden's method

32. Solve $27x^3 - 54x^2 + 198x - 73 = 0$ by Carden's method

33. Solve $x^3 + 3x^2 - 27x + 104 = 0$ by Carden's method

34. Solve $x^3 - 3x^2 + 12x + 16 = 0$ by Carden's method

35. Solve $x^4 - 5x^2 - 6x - 5 = 0$ by Descarte's method.

36. Solve the biquadratic $x^4 + 12x - 5 = 0$ by Descarte's method.

37. Solve $x^4 - 8x^2 - 24x + 7 = 0$ by Descarte's method.

Marks – 04 / 06

1. For what values of a , the equations

   $x + y + z = 1$

   $2x + 3y + z = a$

   $4x + 9y - z = a^2$ have a solution and solve then completely in each case.

2. Investigate for what values of λ and μ the following system of equations

   $x + 3y + 2z = 2$

   $2x + 7y - 3z = -11$

   $x + y + \lambda z = \mu$ have

   i) No solution ii) A unique solution iii) Infinite number of solutions.

3. Show that the system of equations

   $ax + by + cz = 0$

   $bx + cy + az = 0$

   $cx + ay + bz = 0$ has a non-trivial solution iff $a + b + c = 0$ or $a = b = c$

4. Find the value of λ for which the following system have a non-trivial solution

   $x + 2y + 3z = 0$

   $2x + 3y + 4z = 0$

   $3x + 4y + \lambda z = 0$

5. Discuss the solutions of system of equations

   $(5 - \lambda)x + 4y = 0$

   $x + (2 - \lambda)y = 0$ for all values of λ.

6. Obtain the relation between the roots and coefficients of general polynomial equation $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \text{-----} + a_{n-1}x + a_n = 0$

7. Solve the equation $x^3 - 5x^2 - 16x + 80 = 0$ if the sum of two of its roots being equal to zero.

8. Solve the equation $x^3 - 3x^2 + 4 = 0$ if the two of its roots are equal.

9. Solve the equation $x^3 - 5x^2 - 2x + 24 = 0$ if the product of two of the roots is 12.

10. Solve the equation $x^3 - 7x^2 + 36 = 0$ if one root is double of another.

11. Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in A.P.

12. Find the condition that the cubic equation $x^3 + px^2 + qx + r = 0$ should have two roots α and β connected by the relation $\alpha\beta + 1 = 0$

13. If α, β and γ are roots of the cubic equation $x^3 + px^2 + qx + r = 0$ find the value of i) $\sum \alpha^2\beta$ ii) $\sum \alpha^2$ iii) $\sum \alpha^3$ iv) $\sum \alpha^2\beta^2$

14. If α, β and γ are roots of the cubic equation $x^3 + px^2 + qx + r = 0$ find the value of $(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)$

15. If α, β and γ are roots of the cubic equation $x^3 - px^2 + qx - r = 0$ find the value of $\frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2} + \frac{1}{\alpha^2\beta^2}$

16. If α, β, γ and δ are roots of biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, find the value of the following symmetric functions

i) $\sum \alpha^2\beta$ ii) $\sum \alpha^2$ iii) $\sum \alpha^3$

17. If α, β, γ and δ are roots of biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, find the value of the following symmetric functions

i) $\sum \alpha^2\beta\gamma$ ii) $\sum \alpha^2\beta^2$ iii) $\sum \alpha^4$

18. Remove the fractional coefficients from the equation

$$x^4 - \frac{5}{6}x^3 + \frac{5}{12}x^2 - \frac{13}{900} = 0$$

19. Find the equation whose roots are the reciprocals of the roots of

$$x^4 - 3x^3 + 7x^2 + 5x - 2 = 0$$

20. Transform an equation $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \text{-----} + a_{n-1}x + a_n = 0$ into another whose roots are the roots of given equation diminished by given quantity h.

21. If α, β, γ are the roots of $8x^3 - 4x^2 + 6x - 1 = 0$ find the equation whose roots are $\alpha + 1/2$, $\beta + ½$, $\gamma + 1/2$

22. Solve the equation $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$ by removing its second term.

23. Reduce the cubic $2x^3 - 3x^2 + 6x - 1 = 0$ to the form $Z^3 + 3HZ + G = 0$

24. Explain Carden's method of solving equation $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$

Unit – 03

Relations, Congruence Classes and Groups

Marks - 02

1) Let A = { 1, 2, 3,4, 5, 6, 7, 8, 9,10 }

A_1 = { 1, 2, 3, 4 }, A_2 = { 5, 6, 7 }, A_3 = { 4, 5, 7, 9 }, A_4 = { 4, 8, 10 }, A_5 = { 8, 9, 10 } , A_6 = { 1, 2, 3, 6, 8, 10 } Which of the following is the partition of A.

A) { A_1, A_2, A_5 } B) { A_1, A_3, A_5 }

C) { A_2, A_3, A_6 } D) { A_2, A_3, A_6 }

2) Let A = Z^+, the set of all positive integers. Define a relation on A as "aRb iff a divides b" then this relation is not ---

A) Reflexive B) Symmetric

C) Transitive D) Antisymmetric

3) For $n \in N$, $a, b \in Z$ and $d = (a, n)$, linear congruence $ax \equiv b \pmod{n}$ has a solution iff ----

A) $d \mid b$ B) $x \mid b$

C) $n \mid d$ D) $a \mid b$

4) If the Linear congruence $ax \equiv b \pmod{n}$ has a solution then it has exactly ---- non-congruent modulo n solutions

A) a B) b

C) n D) (a, n)

5) If $a^2 \equiv b^2 \pmod{p}$ then $p \mid a+b$ or $p \mid a-b$ only when p is ---

A) Even B) Odd

C) Prime D) Composite

6) G = { 1, -1 } is a group w.r.t. usual

A) Addition B) Subtraction

C) Multiplication D) None of these

7) In the group (Z_6, $+_6$), $o(\bar{5})$ is

A) 2 B) 5 C) 6 D) 1

8) Linear congruence $207x \equiv 6 \pmod{18}$ has

A) No solution B) Nine solutions

C) Three solutions D) One solution

9) The number of residue classes of integers modulo 7 are

A) one B) five C) six D) seven

10) The solution of the linear congruence $5x \equiv 2 \pmod{7}$ is

A) x = 2 B) x = 4 C) x = 6 D) x = 3

11) The set of positive integers under usual multiplication is not a group as following does not exist

A) identity B) inverse

C) associativity D) commutativity

12) Define an equivalence relation and show that '>' on set of naturals is not an equivalence relation.

13) Define a partition of a set and find any two partitions of A = { a, b, c, d }

14) Define equivalence class of an element. Find equivalence classe of '2' if R ={ (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1), (2, 3), (3, 3), (4, 4),(3, 2), (5, 5)} is an equivalence relation on A = {1, 2, 3, 4, 5}

15) Define residue classes of integers modulo n. Find the residue class of $\bar{2}$ for the relation "congruence modulo 5".

16) Define prime residue class modulo n. Find the prime residue class modulo 7

17) Define a group and show that set of integers with respect to usual multiplication is not a group.

18) Define Abelian group and show that group

$$G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : ad - bc \neq 0, a, b, c, d \in R \right\}$$ is not abelian.

19) Define finite and infinite group. Illustrate by an example.

20) Define order of an element and find order of each element in a group

$G = \{1, -1, i, -i\}$ under multiplication.

21) Find any four partitions of the set $S = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

22) Show that $A \times B \neq B \times A$ if $A = \{2, 4, 6\}$, $B = \{7, 9, 11\}$

23) In the group (Z'_8, X_8), find order of $\bar{3}, \bar{4}, \bar{5}, \bar{6}$

24) Let $Z'_8 = \{\bar{1}, \bar{3}, \bar{5}, \bar{7}\}$, find $(\bar{3})^4, (\bar{3})^0, (\bar{3})^{-4}$ in a group (Z'_8, X_8)

25) In the group (Z'_7, X_7), find $(\bar{3})^2, (\bar{4})^{-3}, o(\bar{3}), o(\bar{4})$

26) Find domain and range of a relation $R = \{(x, y) : x \mid y \text{ for } x \in A, y \in B\}$

where $A = \{2, 3, 7, 8\}$, $B = \{4, 6, 9, 14\}$

27) Solve the linear congruence $2x + 1 \equiv 4 \pmod 5$

28) Let $X = \{1, 2, 3\}$ and $R = \{(1,1), (1, 2), (2, 1), (2, 2), (3, 3), (1, 3), (3, 1), (2, 3), (3, 2)\}$ Is the relation R reflexive, symmetric and transitive ?

29) Prepare the multiplication table for the set of prime residue classes modulo 12.

30) Show that in a group G every element has unique inverse.

31) Show that the linear congruence $13x \equiv 9 \pmod{25}$ has only one solution.

32) Show that the linear congruence $4x \equiv 11 \pmod 6$ has no solution.

33) If $a \equiv b \pmod n$ and $c \equiv d \pmod n$ then show that $ac \equiv bd \pmod n$

34) A relation R is defined in the set Z of all integers as "aRb iff $7a - 4b$ is divisible by 3". Prove that R is symmetric.

35) Let $\sim$ be an equivalence relation on a set A and $a, b \in A$. Show that $b \in [a]$ iff $[a] = [b]$

36) If in a group G, every element is its own inverse then prove that G is abelian.

37) In a group every element except identity element is of order two. Show that G is abelian.

38) If R and S are equivalence relations in set X. Prove that $R \cap S$ is an equivalence relation.

39) In the set R of all real numbers, a relation $\sim$ is defined by $a \sim b$ if $2 + ab > 0$. Show that $\sim$ is reflexive, symmetric and not transitive.

Marks - 04

1. Let Z be the set of all integers. Define a relation R on Z by xRy iff x-y is an even integer. Show that R is an equivalence relation.

2. Let P be the set of all people living in a Jalgaon city. Show that the relation "has the same surname as" on P is an equivalence relation.

3. Let P(X) be the collection of all subsets of X (power set of X). Show that the relation "is a proper subset of" in P(X) is not an equivalence relation.

4. Show that in the set of integers $x \sim y$ iff $x^2 = y^2$ is an equivalence relation and find the equivalence classes.

5. Let S be the set of points in the plane. For any two points $x, y \in S$, define $x \sim y$ if distances of x and y is same from origin. Show that $\sim$ is an equivalence relation. What are the equivalence classes?

6. Consider the set NxN. Define $(a, b) \sim (c, d)$ iff $ad = bc$. Show that $\sim$ is an equivalence relation. What are the equivalence classes?

7. Find the composition table for

   i) $(Z_5, +_5)$ and (Z_5, x_5) ii) $(Z_7, +_7)$ and (Z_7, x_7)

8. Prepare the composition tables for addition and multiplication of

   $Z_6 = \{ \bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5} \}$

9. Show that $\bar{a} \in Z_n$ has a multiplicative inverse in Z_n iff $(a, n) = 1$

10. Find the remainder when 8^{103} is divided by 13.

11. Show that G = { 1, -1, i, -i }, where $i = \sqrt{-1}$, is an abelian group w.r.t. usual multiplication of complex numbers.

12. Show that the set of all 2 x 2 matrices with real numbers w.r.t multiplication of matrices is not a group.

13. Show that G = { A : A is non-singular matrix of order n over R } is a group w.r.t. usual multiplication of matrices.

14. Let Q^+ denote the set of all positive rationals. For $a, b \in Q^+$ define $a * b = \frac{ab}{2}$

    Show that $(Q^+, *)$ is a group.

15. Show that G = $\left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : ad - bc \neq 0 \ \& \ a, b, c, d \in R \right\}$ w.r.t. matrix multiplication is a group but it is not an abelian group.

16. Let Z_n be the set of residue classes modulo n with a binary operation $a +_n b = \overline{a+b} = \bar{r}$ where $\bar{r}$ is the remainder when a + b is divided by n Show that $(Z_n, +_n)$ is a finite abelian group.

17. Let Z'_n denote the set of all prime residue classes modulo n. Show that Z'_n is an abelian group of order $\phi(n)$ w.r.t. x_n

18. Let G be a group and a, b $\in$ R be such that ab = ba. Prove that

$(ab)^n = a^n b^n$, $n \in Z$

19. If in a group G every element is its own inverse then prove that G is an abelian group.

20. Let G = { (a, b) / a, b $\in$ R , a $\neq$ 0 } Define $\odot$ on G as

(a, b) $\odot$ (c, d) = (ac, bc + d). Show that (G, $\odot$) is a non-abelian group.

21. Let f_1, f_2 be real valued functions defined by $f_1(x) = x$ and $f_2(x) = 1 - x$, $\forall x \in R$. Show that G = { f_1, f_2 }is group w.r.t. composition of mappings.

22. Let G be a group and $\forall$ a, b $\in$ G, $(ab)^n = a^n b^n$ for three consecutive integers n. Show that G is an abelian group.

23. Show that a group G is abelian iff $(ab)^2 = a^2 b^2$, $\forall$ a, b $\in$ G

24. Prove that a group having 4 elements must be abelian.

25. Using Fermat's Theorem, Show that $5^{10} - 3^{10}$ is divisible by 11.

26. Using Fermat's Theorem find the remainder when 2^{105} is divided by 11.

27. Solve

i) $8x \equiv 6 \pmod{14}$ ii) $13x \equiv 9 \pmod{25}$

28. Let $*$ be an operation defined by $a * b = a + b + 1$ $\forall$ a, b $\in$ Z where Z is the set of integers. Show that < Z, $*$> is an abelian group.

29. Let $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, where $\alpha \in R$ and G = { $A\alpha$: $\alpha \in R$. Prove that G is an abelian group under multiplication of matrices.

30. Let Q^+ be the set of all positive real numbers and define $*$ on Q^+ by $a*b = \frac{ab}{3}$.

Show that $(Q^+, *)$ is an abelian group.

31. Find the remainder when $2^{73} + 14^3$ is divided by11.

32. Show that the set $G = \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}, \begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}\right\}$ is a group w.r.t. multiplication.

33. Show that $G = R - \{1\}$ is an abelian group under the binary operation

$a * b = a + b - ab, \forall\, a, b \in G$

34. If the elements a, b and ab of a finite group G are each of order 2 then show that ab = ba.

35. A relation R is defined in the set of integers Z by xRy iff 7x – 3y is divisible by 4. Show that R is an equivalence relation in Z.

36. A relation R is defined in the set of integers Z by xRy iff 3x + 4y is a multiple of 7. Show that R is an equivalence relation in Z.

37. Consider the set NxN, the set of ordered pairs of natural numbers. Let ~ be a relation in NxN defined by (x, y) ~ (z, u) if x + u = y + z. Prove that ~ is an equivalence relation. Determine the equivalence class of (1, 4).

38. Define congruence modulo n relation and prove that congruence modulo n is an equivalence relation in Z.

39. Show that the set of all 2x2 matrices with real numbers w.r.t. addition of matrices is a group.

Marks – 04 / 06

1. Let ~ be an equivalence relation on set A. Prove that any two equivalence classes are either disjoint or identical.

2. Prove that every equivalence relation on a non-empty set S induces a partition on S and conversely every partition of S defines an equivalence relation on S.

3. If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ for a, b, c, d $\in$ Z and n $\in$ N then prove that

   i) $(a + c) \equiv (b + d) \pmod{n}$

   ii) $(a - c) \equiv (b - d) \pmod{n}$

   iii) $ac \equiv bd \pmod{n}$

4. Write the algorithm to find solution of linear congruence,

   $ax \equiv b \pmod{n}$, for a, b$\in$ Z and n $\in$ N

5. State and prove Fermat's Theorem.

6. If G is a group then prove that

   i) identity of G is unique

   ii) Every element of G has unique inverse in G.

   iii) $(a^{-1})^{-1} = a, \quad \forall \ a \in G$

7. If G is a group then prove that

   i) identity of G is unique

   ii) $(a^{-1})^{-1} = a, \quad \forall \ a \in G$

   iii) $(ab)^{-1} = b^{-1}a^{-1}, \quad \forall \ a, b \in G$.

8. Let G be a group and a,b,c $\in$ G. Prove that

   i) $ab = ac \Rightarrow b = c$ left cancellation law.

   ii) $ba = ca \Rightarrow b = c$ Right cancellation law.

9. Let G be a group and a, b $\in$ G. Prove that the equations i) $ax = b$ ii) $ya = b$ have unique solutions in G

10. Let G be a group and a $\in$ G. Prove that $(a^n)^{-1} = (a^{-1})^n \ , \forall \ n \in N$

11. Let G be a group and $a \in G$. For $m, n \in N$,

Prove that i) $a^m a^n = a^{m+n}$ ii) $(a^m)^n = a^{mn}$

12. Define an abelian group. If in a group G the order of every element (except identity element) is two then prove that G is an abelian group.

13. Solve the following linear congruence equations

i) $3x \equiv 2 \pmod{8}$ ii) $6x \equiv 5 \pmod{9}$

14. Define a group. Show that any element $a \in G$ has a unique inverse in G. Further show that $(a*b)^{-1} = b^{-1}*a^{-1}$, $\forall\ a, b \in G$.

15. If R is an equivalence relation on a set A then for any $a, b \in A$, prove that

i) $[a] = [b]$ or $[a] \cap [b] = \phi$

ii) $\cup \{ [a] / a \in A \} = A$

16. If $\sim$ is an equivalence relation on set A and A and $a, b \in A$ then show that

i) $a \in [a]$ for all $a \in A$

ii) $b \in [a]$ iff $[a] = [b]$

iii) $a \sim b$ iff $[a] = [b]$

17. Define residue classes of integers modulo n. Show that the number of residue classes of integers modulo n are exactly n.

Unit – 04

Subgroups and Cyclic Groups

Marks – 02

1) Define subgroup. Give example
2) Define proper and improper subgroups. Give example.
3) Define a cyclic group. Give example.
4) Define left coset and right coset..
5) State Lagrange's Theorem.
6) State Fermat's Theorem.
7) State Euler's Theorem.
8) Show that nZ = { nr / r∈Z) is a subgroup of (Z, +), where n∈N.
9) Show that (5Z, +) is a subgroup of (Z, +)
10) Is group (Q+, ·) a subgroup of (R, +) ? Justify.
11) Determine whether or not H = {ix : x∈R} under addition is a subgroup of G = group of complex numbers under addition.
12) Find all possible subgroups of G = { 1, -1, i, -i) under multiplication.
13) Find proper subgroups of (Z, +)
14) Write all subgroups of the multiplicative group of 6^{th} roots of unity.
15) Find all proper subgroups of the group of non-zero reals under multiplication.
16) Give an example of a proper subgroup of a finite group.
17) Give an example of a proper finite subgroup of an infinite group.
18) Give an example of a proper infinite subgroup of an infinite group.
19) Is union of two subgroups a subgroup ? Justify.
20) Prove that cyclic group ia abelian.
21) Show by an example that abelian group need not be cyclic.
22) Let G = { 1, -1, i, -i) be a group under multiplication and H = { 1, -1) be its subgroup. Find all right cosets of H in G.
23) Find the order of each proper subgroup of a group of order 15. Are they cyclic.
24) Find generators of Z_6 under addition modulo 6.
25) Verify Euler's theorem by taking m = 12, a = 7.
26) Verify Lagrange's theorem for Z_9 under addition modulo 9.

27) Let G = Z be a additive group of integers and H = 3Z a subgroup of G then H+2 is

A) {3. 6. 9. 12, ----}
B) {2, 5, -1, 8, -4, ---}
C) {1, 2, 3, 4, 5, ----}
D) {1, 4, -2, 7, -5, ---}

28) Let G = { 1, -1, i, -i) be a group under multiplication and H = { 1, -1) is a subgroup of G then H(-i) is

A) {-1, 1}
B) {-i, i}
C) {i, -1}
D) {1, 1}

29) If n = 12 then $\phi(12)$ is

A) 5
B) 4
C) 7
D) 12

30) If p is prime then generators of a cyclic group of order p ---

A) p
B) p-1
C) p^2
D) p+1

31) A cyclic group having only one generator can have at the most --- element

A) 1
B) 3
C) 2
D) None of these

32) In additive group Z_{12}, ($\bar{4}$) = ---

A) $\{\bar{4}, \bar{8}\}$
B) $\{\bar{4}, \bar{8}, \bar{0}\}$
C) $\{\bar{2}, \bar{4}, \bar{0}\}$
D) Z_{12}

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Marks – 04

1) If H is a subgroup of G and $x \in G$, show that $xHx^{-1} = \{ xhx^{-1} / h \in H \}$ is a subgroup of G

2) Let G be a group. Show that $H = Z(G) = \{ x \in G / xa = ax, \forall a \in G \}$ is a subgroup of G.

3) Let G be an abelian group with identity e and $H = \{ x \in G / x^2 = e \}$. Show that H is a subgroup of G.

4) Let G be the group of all non-zero complex numbers under multiplication. Show that $H = \{ a+ib \in G / a^2+b^2 = 1 \}$ is a subgroup of G

5) Show that $H = \{ x \in G : xb^2 = b^2x , \forall b \in G \}$ is subgroup of G.

6) Write all subgroups of the multiplicative group of non-zero residue classes modulo 7.

7) Determine whether $H_1 = \{ \overline{0}, \overline{4}, \overline{8} \}$ and $H_2 = \{ \overline{0}, \overline{5}, \overline{10} \}$ are subgroups of $(Z_{12} , +_{12})$

8) Let G be a finite cyclic group of order n, and $G = \langle a \rangle$. Show that $G = \langle a^m \rangle \Leftrightarrow (m, n) = 1$, where $0 < m < n$

9) Find all subgroups of $(Z_{12} , +_{12})$.

10) Find all subgroups of (Z^1_7 , X_7)

11) Find all generators of additive group Z_{20}

12) Let G = { 1, -1, i, -i } be a group under multiplication and H = { 1, -1 } be it's subgroup. Find all right coset of H in G

13) Compute the right cosets of 4Z in (Z, +).

14) Let Q = { 1, -1, i, -i , j , -j , k, -k } be a group under multiplication and H = { 1, -1, i, -i } be its subgroup. Find all the right and left cosets of H in G

15) Let $G \equiv (Z_8 , +_8)$ and $H = \{ \overline{0} , \overline{4} \}$. Find all right cosets of H in G

16) Let H be a subgroup of a group G and $a \in G$. Show that $Ha = \{ x \in G / xa^{-1} \in H \}$

17) Let G = { 1,2,3,4,5,6,7,8,9,10 }. Show that G is a cyclic group under multiplication modulo 11. Find all its generators, all its subgroups and order of every element. Also verify the Lagrange's theorem.

18) List all the subgroups of a cyclic group of order 12.

19) Find order of each element in (Z_7 , $+_7$)

20) If Z_8 is a group w.r.t. addition modulo 8

i) Show that Z_8 is cyclic.

ii) Find all generators of Z_8

iii) Find all proper subgroups of Z_8

21) Show that every proper subgroup of a group of order 35 is cyclic.

22) Show that every proper subgroup of a group of order 77 is cyclic.

23) Let G be a group of order 17. Show that for any $a \in G$ either o(a) =1 or o(a) = 17.

24) Let A, B be subgroups of a finite group G , whose orders are relatively prime. Show that $A \cap B = \{ e \}$.

25) Find the order of each element in the group G = { 1, w, w^2 }, where w is complex cube root of unity, under usual multiplication.

26) Find all subgroups of group of order 41. How many of them are proper ?

27) Find the remainder obtained when 3^{54} is divided by 11.

28) Find the remainder obtained when 33^{19} is divided by 7.

29) Using Fermat's theorem, find the remainder when

i) 9^{87} is divided by 13 .

ii) $5^{41} + 41^{12}$ is divided by 13

30) Find the remainder obtained when 15^{27} is divided by 8.

Marks – 04 / 06

1) A non-empty subset H of a group G is a subgroup of G iff
$a, b \in H \Rightarrow .ab^{-1} \in H$.

2) A non-empty subset H of a group G is a subgroup of G iff

i) $a, b \in H \Rightarrow .ab^{-1} \in H$ ii) $a \in H \Rightarrow a^{-1} \in H$.

3) Prove that Intersection of two subgroups of a group is a subgroup

4) Let H, K be subgroups of a group G. Prove that $H \cup K$ is a subgroup of G, iff either $H \subseteq K$ or $K \subseteq H$

5) Show that every cyclic group is abelian. Is the converse true ? Justify .

6) Show that If G is a cyclic group generated by a , then a^{-1} also generated by G.

7) Show that every subgroup of a cyclic group is cyclic.

8) Let H be a subgroup of a group G. prove that

i) $a \in H \Leftrightarrow Ha = H$ ii) $a \in H \Leftrightarrow aH = H$

9) Let H be a subgroup of a group G. Prove that

$Ha = Hb \Leftrightarrow ab^{-1} \in H$

$aH = bH \Leftrightarrow b^{-1}a \in H , \quad \forall a,b \in G$.

10) Let H be a subgroup of a group G . Prove that

i) Any two right cosets of H are either disjoint or identical.

ii) Any two left cosets of H are either disjoint or identical.

11) If H is a subgroup of a finite group G. Then prove that 0(H) / 0(G)

12) Prove that every group of prime order is cyclic and hence abelian

13) Order of every element 'a' of a finite group G is a divisor of order of a group i.e. 0(a) / 0(G)

14) If a is an element of a finite group G then $a^{o(G)} = e$

15) If an integer a is relatively prime to a natural number n then prove that

$a^{\phi(n)} \equiv 1 \pmod{n}$, ϕ being the Euler's function .

16) Prove that If P is a prime number and a is an integer such that $P \nmid a$,
then $a^{p-1} \equiv 1 \pmod{p}$

Unit - 05

De-moiver's Theorem, Elementary Functions.

Marks – 02

1) State De-Moiver's Theorem for integral indices.

2) List n – nth roots of unity.

3) Write 3- distinct cube roots of unity.

4) Find the sum of all n- nth roots of unity.

5) Simplify $(\cos 3\theta + i \sin 3\theta)^8 \cdot (\cos 4\theta - i \sin 4\theta)^{-2}$

6) Simplify $\frac{(1+i)(1+\sqrt{3}\,i)}{i(1-\sqrt{3}\,i)}$, using De-Moiver's Theorem.

7) Find 4- fourth roots of unity.

8) Solve the equation $x^2 - i = 0$, using De-Moiver's Theorem.

9) Separate into real and imaginary parts of $e^{5+\frac{\pi}{2}i}$

10) Separate into real and imaginary parts of $e^{(5+3i)^2}$

11) Define sin z and cos z, $z \in C$.

12) Define sinh z and cosh z, $z \in C$.

13) Prove that $\cos^2 z + \sin^2 z = 1$, using definitions of cos z and sin z.

14) Prove that $\tan z = \frac{2 \tan z}{1 - \tan^2 z}$

15) Prove that sin iz = i sinh z

16) Prove that sinh (iz) = i sin z

17) Prove that cos (iz) = cosh z

18) Prove that cosh (iz) = cos z

19) Prove that tanh (iz) = i tan z

20) Prove that tan (iz) = i tanh z

21) The four fourth roots of unity are ----, ----,---- and ----

22) If $z = \sqrt{3} - i$, then z^{12} = ----

23) $e^{-\pi i}$ = ---- , and $e^{4\pi i}$ = ----

24) Period of sin z is ----

Period of cos z is ----

25) Period of sinh z is ----

Period of cosh z is ----

26) Express $\frac{(\sqrt{3}-i)^2}{(1+i)^{10}}$ in the form p + iq where p, q are reals.

27) $(\cos\theta + i\sin\theta)^7$ has seven distinct values.

T ☐ F ☐

28) $(\cos\theta + i\sin\theta)^{3/4}$ has 4 distinct values.

T ☐ F ☐

29) $\mathrm{Re}(e^z) = e^{\mathrm{Re}(z)}$

T ☐ F ☐

30) $|e^z| = e^{|z|}$

T ☐ F ☐

31) Match

a)	$\sinh^2 z + \cosh^2 z$	i)	1
b)	$\sinh^2 z - \cosh^2 z$	ii)	-1
c)	i sin (iz)	iii)	e^z
d)	sec z .cos z	iv)	- sinh z
		v)	$\frac{2}{e^{iz} - e^{-iz}}$

Choose the correct answer (32 to 40)

32) Consider

a) The sum of the n, nth roots of unity is always 1

b) The product of any two roots of unity is a root of unity.

A) Both a) & b) are true B) Only a) is true

C) Only b) is true D) Both are false

33) A value of log i is

A) πi B) $\pi i / 2$ C) 0 D) $- \pi i / 2$

34) The real part of sin (x + iy) is

A) sin x . cosh y B) cos x . sinh y

C) sinh x . cos y D) cosh x . sin y

35) 2π is period of

A) cos z B) tan z C) e^z D) cot z

36) a) cos (iz) = cosh z b) sin (iz) = i sinh z

A) Both are true B) Both are false

C) Only a) is true D) Only b) is true

37) $\sinh^2 z - \cosh^2 z$ is equal to

A) cosh 2z B) 1 C) -1 D) sinh 2z

38) If w is an imaginary 9^{th} root of unity, then $w + w^2 + \text{-----} + w^8$ is equal to

A) 9 B) 0 C) 1 D) -1

39) A square root of 2i is

A) 1 - i B) 1 + i C) $\sqrt{2}$ D) $\sqrt{2}\, i$

40) $(\cos \pi/4 + i \sin \pi/4)^{-2}$ is

A) i B) - i C) 1 D) -1

Marks - 04

1. Simplify using De-Moiver's Theorem, the expression

$$\frac{(\cos 2\theta - i\sin 2\theta)^7 (\cos 3\theta + i\sin 3\theta)^{-5}}{(\cos 4\theta + i\sin 4\theta)^{12} (\cos 5\theta - i\sin 5\theta)^{-6}}$$

2. Simplify

$$\frac{(\cos\theta + i\sin\theta)^{8/7} (\cos\theta - i\sin\theta)^{12/7}}{(\cos\theta + i\sin\theta)^{12/7} (\cos 4\theta - i\sin 4\theta)^{5/4}}$$

3. Prove that $\left[\frac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right]^n = \cos[(\pi/2 - \theta)n] + i\sin[(\pi/2 - \theta)n]$

4. If α and β are roots of $x^2 - 2x + 2 = 0$ and n is a positive integer, then prove that

$$\alpha^n + \beta^n = 2^{\frac{n+2}{2}} \cos(n\pi/4)$$

5. Evaluate $(1 + i\sqrt{3})^{10} + (1 - i\sqrt{3})^{10}$

6. Prove that $(1 + i\sqrt{3})^{-10} = 2^{-11}(-1 + i\sqrt{3})$

7. Prove that $(-1 + i)^7 = -8(1 + i)$

8. Prove that $(1 + i\sqrt{3})^8 + (1 - i\sqrt{3})^8 = -256$

9. If $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\beta$ prove that

$$\frac{x-y}{x+y} = i\tan\left(\frac{\alpha-\beta}{2}\right)$$

10. Find $(3 + 4i)^{½} + (3 - 4i)^{1/2}$

11. Find all values of $(1 - i\sqrt{3})^{1/4}$

12. Find all values of $(1 + i)^{1/5}$

    Show that their continued product is 1 + i.

13. Find the continued product of the four values of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{3/4}$

14. If w is a complex cube root of unity, prove that $(1 - w)^6 = -27$

Using De-Moiver's Theorem, solve the following equations (15 to 25)

15. $x^4 - x^3 + x^2 - x + 1 = 0$

16. $x^4 + x^3 + x^2 + x + 1 = 0$

17. $x^8 - x^4 + 1 = 0$

18. $x^9 - x^5 + x^4 - 1 = 0$

19. $x^{10} + 11x^5 + 10 = 0$

20. $16x^4 - 8x^3 + 4x^2 - 2x + 1 = 0$

21. $x^3 + x^2 + x + 1 = 0$

22. $x^6 - 1 = 0$

23. $x^4 + 1 = 0$

24. $z^7 - z^4 + z^3 - 1 = 0$

25. $z^{12} - z^6 + 1 = 0$

26. Express $\cos^5\theta$ in terms of cosines of multiple of angle θ.

27. Express $\cos^6\theta$ in terms of cosines of multiple of angle θ..

28. Express $\sin^5\theta$ in terms of sines of multiple of angle θ.

29. Prove that $\cos^8 \theta = 1/128 [\cos 8\theta + 8 \cos 6\theta + 28 \cos 4\theta + 56 \cos 2\theta + 35]$

30. Prove that $\cos^7 \theta = 1/64 [\cos 7\theta + 7 \cos 5\theta + 21 \cos 3\theta + 35 \cos \theta]$

31. Prove that $\sin 7\theta = 7\cos^6\theta \sin\theta - 35 \cos^4\theta \sin^3\theta + 21 \cos^2\theta \sin^5\theta - \sin^7\theta$

32. Prove that $\cos 5\theta = \cos^5\theta - 10 \cos^3\theta \sin^2\theta + 5 \cos\theta \sin^4\theta$

33. Prove that $\sin 5\theta = 5 \cos^4\theta \sin\theta - 10 \cos^2\theta \sin^3\theta + \sin^5\theta$

34. If $\sin \alpha + \sin \beta + \sin \gamma = \cos \alpha + \cos \beta + \cos \gamma$ prove that

    a) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3 \sin (\alpha + \beta + \gamma)$

    b) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos (\alpha + \beta + \gamma)$

35. Express $\dfrac{\sin 7\theta}{\sin \theta}$ in powers of $\sin \theta$ only.

36. Prove that $\dfrac{\sin 6\theta}{\sin \theta} = 32 \cos^5\theta - 24\cos^3\theta + 6 \cos\theta$

37. Prove that $\dfrac{\sin 6\theta}{\cos \theta} = 32 \sin^5\theta - 32 \sin^3\theta + 6 \sin\theta$

38. Using definitions of cos z and sin z, prove that $\sin^2 z + \cos^2 z = 1$

39. If z_1 and z_2 are complex numbers, show that

$\cos (z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2$

40. Prove that $\cosh (z_1 + z_2) = \cosh z_1 \cosh z_2 + \sinh z_1 \sinh z_2$

41. Prove that a) $2\cosh^2 z - 1 = \cosh 2z$

b) $2\sinh^2 z + 1 = \cosh 2z$

42. Find the general values of a) Log (-i) b) Log (-5)

Separate into real and imaginary parts of (43 to 55)

43. log (4 + 3i)

44. log (3 + 4i)

45. sin (x + iy)

46. cos (x + iy)

47. tan (x + iy)

48. sec (x + iy)

49. cosec (x + iy)

50. cosh (x + iy)

51. coth (x + iy)

52. sech (x + iy)

53. cosech (x + iy)

54. tanh (x + iy)

55. cot (x + iy)

Prove the following (56 to 60)

56. $\sinh 2z = 2 \sinh z \cosh z$

57. $\sinh 2z = \dfrac{2 \tanh z}{1 - \tanh^2 z}$

58. $\cosh 2z = \dfrac{1 + \tanh^2 z}{1 - \tanh^2 z}$

59. $\tanh 2z = \dfrac{2 \tanh z}{1 + \tanh^2 z}$

60. $\cosh 3z = 4 \cosh^2 z - 3 \cosh z$

61. If $\cos(x + iy) = \cos\alpha + i \sin\alpha$ show that $\cos 2x + i \cosh 2y = 2$

62. If $\sin(x + iy) = \tan\alpha + i \sec\alpha$ show that $\cos 2x \cosh 2y = 3$

63. If $\sin(\alpha + i\beta) = x + iy$, prove that

$$\frac{x^2}{\cosh^2\beta} + \frac{y^2}{\sinh^2\beta} = 1 \text{ and } \frac{x^2}{\sin^2\alpha} - \frac{y^2}{\cos^2\alpha} = 1$$

64. If $x + iy = \cosh(u + iv)$, show that

$$\frac{x^2}{\cosh^2 u} + \frac{y^2}{\sinh^2 u} = 1 \text{ and } \frac{x^2}{\cos^2 v} - \frac{y^2}{\sin^2 v} = 1$$

65. If $x + iy = \cosh(u + iv)$, show that $x^2 \operatorname{sech}^2 u + y^2 \operatorname{cosech}^2 u = 1$

66. If $x + iy = \cosh(u + iv)$, show that $(1 + x)^2 + y^2 = (\cosh v + \cos u)^2$

67. If $x + iy = \cos(u + iv)$, show that $(1 - x)^2 + y^2 = (\cosh v - \cos u)^2$

68. If $\cos(x + iy) = r(\cos\alpha + i \sin\alpha)$, show that $2y = \log\left[\frac{\sin(x - \alpha)}{\sin(x + \alpha)}\right]$

69. If $u = \log[\tan(\frac{\pi}{4} + \frac{x}{2})]$. prove that $\tanh\frac{u}{2} = \tan\frac{x}{2}$

70. If $\tan(x + iy) = A + iB$ then show that $\frac{A}{B} = \left[\frac{\sin 2x}{\sinh 2y}\right]$

71. Prove that $\sin[\log(i^i)] = -1$

72. Show that $\sin\left[i \log\left(\frac{1 + ie^{-i\theta}}{1 - ie^{-i\theta}}\right)\right]$ is purely real.

73. Find the 5-5th roots of -1.

74. Find the modulus and principal value of the argument of $\frac{(1 + i\sqrt{3})^7}{(\sqrt{3} - i)^{11}}$

75. Express $\frac{(\sqrt{3} - i)^7}{(1 + i)^{10}}$ in the form $a + ib$, where a and b are reals.

76. If $z = -(\sqrt{3} + i)$, find z^{10}

77. If $x_i^2 + 1 = 2 x_i \cos\theta$ (i = 1, 2,3), then prove that one of the value of $x_1 x_2 x_3 + \frac{1}{x_1 x_2 x_3}$ is $2\cos(\theta_1 + \theta_2 + \theta_3)$

78. If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, prove that one of the values of

$$x^m y^n + \frac{1}{x^m y^n} \text{ is } 2\cos(m\alpha + n\beta)$$

79. If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$, prove that

$$\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\theta - n\phi)$$

80. Solve the equation $x^2 - i = 0$, using De-moivre's theorem.

Marks – 04 / 06

1) State and prove De-Moiver's Theorem for integral indices.

2) State and prove De-Moiver's Theorem for rational indices.

3) State De-Moiver's Theorem. Obtain the formula for n-nth roots of unity.

4) Find n-nth roots of unity and represent them geometrically.

5) Show that the product of any two roots of unity is the root of unity.

6) Show that the 7^{th} roots of unity form a series in G.P. and find their sum.

7) Show that the sum of n-nth roots of unity is zero.

8) Find n-nth roots of a complex number z = x + iy.

9) Prove that

$$(x + iy)^{m/n} + (x - iy)^{m/n} = 2(x^2 + y^2)^{m/2n} \cdot \cos[(m/n)\tan^{-1}(y/x)]$$

10) If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$ then show that

$$\frac{x^m}{y^n} + \frac{y^n}{x^m} = 2\cos(m\theta - n\phi)$$

11) Define sin z, cos z and sinh z, cosh z. Prove that sin z and cos z are periodic functions with period 2π.

12) Define tan z. Prove that tan z is a periodic function with period π.

13) Define sinh z, and cosh z. Prove that sinh z and cosh z are periodic functions with period $2\pi i$.

14) Obtain the relation between circular functions sinz, cosz and hyperbolic functions sinhz, coshz.

15) Define Log z , $z \in C$ Separate Log z into real and imaginary parts.

16) Prove that $i \log\left\lfloor \frac{x-i}{x+i} \right\rfloor = \pi - 2\tan^{-1}x$

17) Prove that $\cos\left\{i\log\left(\frac{a+ib}{a-ib}\right)\right\} = \frac{a^2-b^2}{a^2+b^2}$

18) Prove that $\tan\left\{i\log\left(\frac{a-ib}{a+ib}\right)\right\} = \frac{2ab}{a^2-b^2}$

19) Using definition prove that $\cosh^2 z - \sinh^2 z = 1$

20) If $\sin^{-1}(\alpha + i\beta) = u + iv$, prove that $\sin^2 u$ and $\cosh^2 v$ are the roots of the quadratic equation $\lambda^2 - (1 + \alpha^2 + \beta^2)\lambda + \alpha^2 = 0$