North Maharashtra University , Jalgaon

Question Bank

(New Syllabus w.e.f. June 2008)

Class- S. Y.B.Sc.

Subject : Mathematics

Paper MTH-211
(Calculus of Several Variables)

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Unit -I
(Functions of Two & Three Variables)

<u>I) Objective Type Questions (2 Marks each)</u>

1) Define neighbourhood of a point in a plane .
2) Define simultaneous limit of a function f(x,y) as (x,y)$\rightarrow$ (a,b)
3) Define Continuity of a function f(x,y) at a point (a,b)
4) Find two repeated limits of f(x,y) = $\dfrac{x^2 - y^2}{x^2 + y^2}$, (x,y) $\neq$ (0.0).

   As x $\rightarrow 0$, y$\rightarrow 0$
5) Evaluate the limit,

   $\lim_{(x,y)\rightarrow(0,0)} \dfrac{xy^3}{x^2 + y^6}$ if exists
6) Define f_x(a,b) and f_y(a,b)
7) Define f_{xx}(a,b) and f_{yy}(a,b)
8) Define f_{xy}(a,b) and f_{yx}(a,b)
9) If u=$tan^{-1}\dfrac{y}{x}$, find $\dfrac{\partial u}{\partial x}$ and $\dfrac{\partial u}{\partial y}$
10) Find f_x(x,y) and f_y(x,y) if f(x,y)=e^x sinxy
11) If u(x,y) = $\dfrac{x}{y^2} - \dfrac{y}{x^2}$ find $u_x(1,1)$ and $u_y(1,1)$.
12) Define differentiability of a function f(x,y) at a point (a,b) of its domain.
13) State the necessary condition for differentiability of a function f(x,y) at a point (a,b).
14) State sufficient condition for differentiability of a function f(x,y) at a point (a,b)
15) State Young's theorem for the equality of f_{xy} and f_{yx}
16) State Schwarz's theorem for the equality of f_{xy} and f_{yx}

<u>II) Multipal Choice Questions (1 Marks each)</u>

Choose the correct option .

1) If a function f(x,y) is discontinuous at a point (a,b) then

   a) $\lim_{(x,y)\rightarrow(a,b)}$ f(x,y) exist and equal to f(a,b)

   b) f(x,y) is not differentiable at (a,b)

   c) f(x,y) is differentiable at (a,b)

   d) None of these.

2) If the simultaneous limit

   exists and has the same value along any three different paths then

   a) $\lim_{(x,y)\rightarrow(a,b)}$ f(x,y) exists.

   b) $\lim_{(x,y)\rightarrow(a,b)}$ f(x,y) may or may not exist

   c) $\lim_{(x,y)\rightarrow(a,b)}$ f(x,y) does not exist

d) None of these.

3) If a function f(x,y) is differentiable at a point (a,b) then,

a) f_x(a,b)& f_y(a,b) may or maynot exist.

b) f_x(a,b) & f_y(a,b) both exist .

c) only one of f_x(a,b) and f_y(a,b) exists.

d) f_x(a,b) and f_y(a,b) both does not exist.

4) If f(x,y) = $x^3 + y^3 - 2x^2y^2$ then

f(x,x) (1,1) = --------

a) 1 b) -1

c) 0 d) None of these.

III) Theory and Examples (4- Marks each)

1) If a function f(x,y) is differentiable at a point (a,b) of its domain then show that

i) f(x,y) is continuous at (a,b)

ii) f_x and f_y exist at (a,b)

2) State and prove sufficient condition for differentiability of the function f(x,y).

3) State and prove Schwarz's theorem for equality of f_{xy} and f_{yx} at a point (a,b)

4) State and prove Young's theorem for equality of f_{xy} and f_{yx} at a point (a,b)

5) Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{x^2(x+y)}{x^2+y^2}$$

6) Show that the limit

$$\lim_{(x,y)\to(0,0)} \frac{\tan(x^2+y)}{x+y} \text{ does not exist.}$$

7) Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^4}{(x^2+y^4)^2}$$

8) Discuss the continuity of the function f defined by

$$f(x,y) = \frac{x^3+y^3}{x-y} \quad \text{when } x \neq y \text{ and}$$

$$= 0 \qquad \text{for } x = y$$

at (0,0)

9) Show that the function f(x,y)is continuous at (0,0)

$$\text{where } f(x,y) = \frac{xy(x^2-y^2)}{(x^2+y^2)} \quad \text{for } (x,y) \neq (0,0)$$

$$= 0 \qquad \text{for } (x,y)=(0,0)$$

10) Examine the continuity of the function f(x,y) at (0,0)
Where

$$f(x,y) = \frac{xy \sin\sqrt{x^2+y^2}}{x^2+y^2} \quad \text{for } (x,y) \neq (0,0)$$

$$= 0 \quad \text{for } x = y$$

at (0,0)

11) Show that the function f(x,y) is continuous at (0,0)
Where

$$f(x,y) = y + x\sin\frac{1}{y} \quad \text{when } y \neq 0$$

x may or may not be 0.

$= 0$ when $y = 0$

12) Discuss the continuity of the function f(x,y) at (0,0)
Where

$$f(x,y) = \frac{x^2y^2}{x^2y^2 + (x-y)^2} \quad \text{for } (x,y) \neq (0,0)$$

$$= 0 \quad \text{for } (x,y) = (0,0).$$

13) If u = log (tanx +tany+ tanz)
Prove that

$$\text{Sin}2x\frac{\partial u}{\partial x} + \sin 2y\frac{\partial u}{\partial y} + \sin 2z\frac{\partial u}{\partial z} = 2$$

14) If $u = x^2 \tan^{-1}\frac{y}{x} - y\tan^{-1}\frac{x}{y}$

Find u_{xy}(x,y)

15) If $u = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$, show that $\nabla^2 u = 0$ where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$

16) Let f be a function defined by

$$f(x,y) = \frac{x^2y^2}{x^2+y^2} \quad \text{for } (x,y) \neq (0,0)$$

and f(0,0) = 0

show that f_{xy}(x,y) and f_{yx}(x,y) are equal for $(x,y) \neq (0,0)$
and also show that they are not continuous at (0,0).

17)For the example 16 prove that f_{xy} (0,0)= f_{yx}(0,0)

18) If $f(x,y) = \dfrac{x^2y^2(x^2-y^2)}{x^2+y^2}$ for $x^2+y^2 \neq 0$ and f(0,0)= 0

Show thtat $f_{xy}(0,0) = f_{yx}(0,0)$

19) Let a function f be difined as,

f(x,y) = $x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$ when xy $\neq$ 0

and f(x,y) =0 when xy = 0

show that , $f_{xy}(0,0)$ =1 and $f_{yx}(0,0)$ = -1

20)Let f be function defined by ,

f(x,y) = $\frac{xy}{\sqrt{x^2 + y^2}}$ if (x,y) $\neq (0,0)$

f(0,0) = 0

show that f(x,y) is not differentiable at (0,0).

21) Show that the function f defined by,

f(x,y) = $\frac{x^4 + y^4}{x^2 + y^2}$ when $x^2 + y^2 \neq 0$

= 0 when $x^2 + y^2 = 0$

is differentiable at (0,0).

22) Show that the function f(x,y) = $|x| + |y|$ $\forall$ x,y $\in$ R

Is continuous at the origin but not differentiable there.

23) Discuss the differentiability of a function f , where

f(x,y) = $\frac{xy}{x^2 + y^2}$ when (x,y) $\neq$ (0,0)

f(0,0) = 0 at the origin.

24) Let a function f be defined by ,

f(x,y) = $\frac{xy(x^2 - y^2)}{x^2 + y^2}$ when (x,y) $\neq$ (0,0)

= 0 when x= 0 =y

Show that , $f_x(0,0) = 0 = f_y(0,0)$

Also show that ,

$f_x(x,y) = \frac{x^4 y + 4x^2 y^3 - y^5}{(x^2 + y^2)^2}$ for (x,y) $\neq$ (0,0)

25) Prove that the function defined in example 24 is differentiable at (0,0).

26) Using differentials find an approximate value of (2.01) $(3.02)^2$

27) Estimate f(1.02,1.97), where ,f(x,y)= $\sqrt{x^2 + y^2}$

28) Find approximate value of $(3.9)^2(2.05) + (2.05)^3$

29) Find approximately the value of $(5.12)^2(6.85) - 3(6.85)$

30) Find approximate value of, $\sqrt{(3.012)^2 + (3.977)^2}$

UNIT –II

(Composite Function and Mean Value Theorem)

I) Objective questions (2 marks each)

1) If u =f(x,y) x= Φ (t) , y = $\Psi(t)$

Then state the formula for $\frac{du}{dt}$

2) If w =f(u,v), u = $\Phi(x,y)$, v = $\Psi(x,y)$

Then state the formula for $\frac{\partial w}{\partial x}$.

3) If w =f(u,v) , u = $\Phi(x,y)$, v = $\Psi(x,y)$

Then state the formula for $\frac{\partial w}{\partial y}$

4) Define homogenous function of x and y with degree n.

5) State Euler's theorem for the homogenous function f(x,y) with degree n.

6) If u = $G^{-1}\left[x^n f\left(\frac{y}{x}\right)\right]$ and $G^1(u) \neq 0$

Then what is the value of x $\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

7)If u = f(x,y) is a homogenous function of degree n then what is the value of $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$

8) Using Mean value theorem complete the equality
f(a+h,b+k) =- - - - - - - - - + h f_x (a+θh, b+θk) + kf_y (a+θh, b+θk)

9) If f(x,y) = a $x^2 + 2hxy + by^2$ then find the degree of the homogeneous function using definition.

10) If f(x,y) = $x^4 + y^4$ then find the value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$

11) If f(x,y) = $\left(x^2 + y^2\right)^{\frac{1}{2}}$ then find the value of x $\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$

12)If f(x,y) = $x^2 + y^2$ then find the value of
$x^2 f_{xx} + 2xyf_{xy} + y^2 f_{yy}$

13) If z = $x^2 + y^2$, x = t^2 + 2y = 2t
find $\frac{dz}{dt}$

14) If z =sin (x+y), x = t^2 +1,y = t^2
find $\frac{dz}{dt}$

15) If z = $u^2 + v^2$, u = x+y , v = x-y
Find $\frac{\partial z}{\partial x}$

16)If z = $u^2 + v^2$, u = x+y , v = x-y
Find $\frac{\partial z}{\partial y}$

II) Multiple choice questions (1 marks each)

Choose the correct option from the given four options.

1) If u = $tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ .x ≠ y

Then $xu_x + yu_y$ = -------

a) cos2u b)sinu
c)sin2u d)None of these.

2) If f(x,y) = $\left(x^2+y^2\right)^{\frac{-1}{2}}$ then $xf_x + yf_y$ is

a) 0 b)-f(x,y)

c)f(x,y) d) None of these.

3) If z = x+y, x = 2t, y = 3t.

then $\frac{dz}{dt}$ is

a) 0 b) 3

c)1 d)5

4)If z=xy $f\left(\frac{x}{y}\right)$ then

$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ = --------

a) z **b) 0**

c) $\frac{1}{z}$ d)2z.

II) Theory and Examples (3 – marks each)

1) If u =f(x,y) is a differentiable function of x and y ,
x= $\Phi(t)$,y = $\Psi(t)$ are differentiable functions of t then prove that

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{du}{dt} + \frac{\partial u}{\partial y}\frac{du}{dt}$$

2) If w =f(u,v) is a differentiable function and u= $\Phi(x, y)$, v= $\Psi(x, y)$
are differentiable functions then prove that,

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial x}$$

3) If w =f(u,v) is a differentiable function and u= $\Phi(x, y)$, v= $\Psi(x, y)$
are differentiable functions then prove that,

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial y}$$

4) State and prove Euler's theorem for homogenous function in x and y.

5) If, u = $G^{-1}\{x^n f\left(\frac{y}{x}\right)\}$ and $G^1(u) \neq 0$ Then prove that

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{G(u)}{G^1(u)}$ Hence find , $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ if u= $sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$

6) If u = f(x,y) is homogenous function of degree n in x and y then prove that,

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

7) State and prove mean value theorem for the function f(x,y).

8) Find $\frac{du}{dt}$ if u = $x^3 + y^3$, where x = a cost, y = b sint

9) Find $\frac{dz}{dt} if z = xy^2 + x^2 y$

Where , x = at^2 , y =2at

10) If u = f$\left(e^{y-z}, e^{z-x}, e^{x-y}\right)$

Prove that, $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

11) If u = $f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$ show that, $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$

12) If u = f (y-z, z-x, x-y) , prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

13) If z is a function f x and y where x = $e^{u}+e^{-v}$ and y = $e^{-u}-e^{v}$

Show that, $\frac{\partial z}{\partial u}-\frac{\partial u}{\partial v}=x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}$

14) Let z = f (u,v) where u = 2x-3y and v = x +2y, prove that,

$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=3\frac{\partial z}{\partial v}-\frac{\partial z}{\partial u}$

15) If u = $tan^{-1}\frac{y}{x}$ where x = $e^{t}-e^{-t}$, y = $e^{t}+e^{-t}$, $find\ \frac{du}{dt}$

16) If z = f (x,y) = $tan^{-1}\frac{x}{y}$, x = u+v , y = u-v , show that

$\frac{\partial z}{\partial u}+\frac{\partial z}{\partial v}=\frac{u-v}{u^{2}+v^{2}}$

17) If z =f(x,y), x = uv, y = $\frac{u+v}{u-v}$ then prove that,

$2x\frac{\partial z}{\partial x}=u\frac{\partial z}{\partial u}+v\frac{\partial z}{\partial v}$

18) If z = f(u,v) ,u = $x^{2}-y^{2}-2xy, v=y$ prove that the equation

$(x+y)\frac{\partial z}{\partial x}+(x-y)\frac{\partial z}{\partial y}=0$ is equivalent to $\frac{\partial z}{\partial v}=0$

19) Verify Euler's Theorem for function, f(x,y) = $x^{3}+y^{3}-3x^{2}y$

20) If u = $cosec^{-1}\left[\frac{x^{\frac{1}{2}}+y^{\frac{1}{2}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}\right]^{\frac{1}{2}}$ show $x^{2}\frac{\partial^{2}u}{\partial x^{2}}+2xy\frac{\partial^{2}u}{\partial x\partial y}+y^{2}\frac{\partial^{2}u}{\partial y^{2}}=\frac{\tan u}{12}\left[\frac{13}{12}+\frac{\tan^{2}u}{12}\right]$

21) If u = $sin^{-1}\left[\frac{x^{2}+2xy}{\sqrt{x-y}}\right]^{\frac{1}{5}}$ find the values of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ and $x^{2}\frac{\partial^{2}u}{\partial x^{2}}+2xy\frac{\partial^{2}u}{\partial x\partial y}+y^{2}\frac{\partial^{2}u}{\partial y^{2}}$

22) If u = log ($x^{3}+y^{3}-x^{2}y-xy^{2}$) prove that,

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} \text{ and } x^{2}\frac{\partial^{2}u}{\partial x^{2}}+2xy\frac{\partial^{2}u}{\partial x\partial y}+y^{2}\frac{\partial^{2}u}{\partial y^{2}}=-3$$

23) If u = $sin^{-1}\sqrt{x+y^{2}}$ prove that $x^{2}\frac{\partial^{2}u}{\partial x^{2}}+2xy\frac{\partial^{2}u}{\partial x\partial y}+y^{2}\frac{\partial^{2}u}{\partial y^{2}}=\tan^{3}u$

24) If u = $\Phi\left(\frac{x}{y}\right)+f\left(\frac{y}{x}\right)$ then prove that, $x^{2}\frac{\partial^{2}u}{\partial x^{2}}+2xy\frac{\partial^{2}u}{\partial x\partial y}+y^{2}\frac{\partial^{2}u}{\partial y^{2}}=0$

25) If u = $sin^{-1}\left(x^{2}+y^{2}\right)^{\frac{1}{5}}$, prove that, ,

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{2}{25}\tan u\left(2\tan^2 u-3\right)$$

26) If u = $\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ show that

i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$

ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$ = sin2u$\left(1-4\sin^2 u\right)$

27) Verify Euler's theorem for the function f(x,y) = $x^4\log\frac{y}{x}$

28) If f(x,y) = x^3-xy^2 show that θ used in the mean value theorem applied to the points (2,1) and (4,1) satisfies the quadratic equation $3\theta^2+6\theta-4=0$

29) If f(x,y) $=x^2y+2xy^2$ show that the value of θ in the expression of the mean value theorem applied to the line segment joining the point (1,2) to (3,3) satisfies the equation

$12\theta^2+30\theta-19=0$

30) If u = $\tan^{-1}\frac{y}{x}$ show that $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}=0$

IV) Compulsory Examples (2 marks each)

1) If z = f(x,y) where, $x=r\cos\theta, y=r\sin\theta$ then prove that,

$$\frac{\partial z}{\partial r}=\cos\theta\frac{\partial z}{\partial x}+\sin\theta\frac{\partial z}{\partial y}$$

2) If z = f(x,y) where x = rcosθ, y = rsinθ then prove that

$$\frac{\partial z}{\partial \theta}=-r\sin\theta\frac{\partial z}{\partial x}+r\cos\theta\frac{\partial z}{\partial y}$$

3) Verify Euler's theorem for the function f(x,y) = $\tan^{-1}\frac{y}{x}$

4) Verify Eulers theorem for the function f(x,y) = $\left(x^2+y^2\right)^{\frac{1}{2}}$

5) If u = $\tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{4}\sin 2u$

6) If u $=\sin^{-1}\left(x^2+y^2\right)^{\frac{1}{5}}$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{2}{5}\tan u$

7) If u = x+y and x $=e^{t}, y=t^3$ then find $\frac{du}{dt}$ at t =1

8) If u = $\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$

Unit – III (Taylor's Theorem and Extreme values)

I) Objective questions (2 marks each)

1) Write the simplified mathematical Taylor's expansion for f(x,y)
2) Write the simplified mathematical Maclaurin's expansion for f(x,y).
3) Define absolute maximum of the function f(x,y) at (a,b)
4) Define absolute minimum of the function f(x,y) at (a,b)

5) State the necessary condition for the extreme value.
6) Write the condition for critical point (a,b) to become a function f(x,y) maximum.
7)Write the condition for critical point (a,b) to become function f(x,y) minimum.
8) Under what condition the critical point (a,b) will be saddle point ?
9) prove that, $e^{x+y} = 1 + (x+y) + \frac{(x+y^2)}{2!} + \text{- - - - - - - - -}$
10)State the working rule to determine extreme value of the function f(x,y)
11) prove that

sin (x+y) = (x+y) - $\frac{(x+y)^3}{3!} -$ ------------.

12) prove that cos (x + y) = 1 - $\frac{(x+y)^2}{2!} + \frac{(x+y)^3}{3!} -$ ----------------
13) Find the Critical point for f(x,y) = xy + $\frac{50}{x} + \frac{20}{y}$
14) Find the Stationary points for f(x,y) = $x^3y^2(1-x-y)$
15) Find the Stationary points for f(x,y) = $x^3 + y^3 - 3axy$
16) Find the minimum value of f(x,y) = 1+ $x^2 + y^2$

<u>II) Multiple Choice questions (1 mark each)</u>

Choose the correct option from the given four options :

1) If f(x,y) = $x^2 - y^2 + 4$ then f has extreme value at - - - - -
   a) (1,1) b) (0,0)
   b) (1,0) c) None of these.
2) To expand $x^2 + 2xy + y^3$ in powers of x -4 and y+2 by Taylor's theorem the values of h & k are - - - - - .respectively.
   a) 4 , 2 b) -4 , 2
   c) 4 , -2 d) None of these.
3) If (a,b) is the Stationary point of the function f(x,y) and $f_{xx}(a,b)\rangle 0, f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2 \langle 0$ then f(a,b) is,
   a) Minimum b) Saddle point
   c) Maximum d) None of these
4) If (a,b) is the critical point of the function f(x,y) and $f_{xx}(a,b)\rangle 0, f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2 > 0$ then f(a,b) is,
   a) Maximum b) Saddle point
   c) Minimum d) None of these

<u>III) Theory and Examples.(4 marks each)</u>

1) State and prove Taylor's theorem for f(x,y)
2) State Taylor's theorem and hence obtain Macluarin's expansion in simplified form
3) State and prove necessary condition for extreme values of the function f (x,y)
4) State and prove the sufficient condition for the maximum value of the function f (x,y)
5) State and prove the sufficient condition for the minimum value of the function f (x,y)
6) Explain the Lagrange's method of undetermined multipliers to find extreme values of the function f (x,y,z).
7) Prove that $e^{ax}\sin by = by + abxy + \frac{e^u}{6}\left[\left(a^3x^3 - 3ab^2xy^2\right)\sin u + \left(3a^2bx^2y - b^3y^3\right)\cos v\right]$

Where u $= a\theta x, v = b\theta y$

8) Expand sinxy in powers of (x-1) and (y - $\frac{\Pi}{2}$) upto and including second degree term.

9) Prove that $e^{x+y} = 1 + (x+y) + \frac{1}{2}(x+y)^2 + \frac{1}{6}(x+y)^3 e^{\theta x + \theta y}$

10) Show that for $0 \langle \theta \langle 1$

$$\sin x \sin y = xy - \frac{1}{6}\left[\left(x^3 + 3xy^2\right)\cos\theta x \sin\theta y + \left(y^3 + 3xy^2\right)\sin\theta x \cos\theta y\right]$$

11) Expand $x^3 + y^3 + xy^2$ in powers of (x-1) and (y-2)

12) Expand $x^2 y$ as polynomial in (x-1) and (y+2) by using Taylor's theorem

13) Expand the function f(x,y) $= x^2 + xy - y^2$ by Taylor's theorem in powers of (x-1) and (y+2) .

14) Expand $x^2 y + 3y - 2$ in powers of (x-1) and (y-2)

15) Expand $e^{2x} \cos y$ as a Taylor's series about (0,0) up to first three terms.

16) Write down the Taylor's expansion of $e^{ax} \cos by$ about (0,0) up to and including terms of the second degree.

17) Expand $e^{ax} \log(1+y)$ in powers of x and y up to terms of third degree.

18) Expand $e^x \sin y$ in powers of x and y as for as terms of third degree.

19) Find extreme values of $f(x,y) = x^3 y^2 (12 - 3x - 4y)$

20) Find extreme values of f(x,y) $= xy + \frac{a^3}{x} + \frac{a^3}{y}$

21) Find the Stationary points and determine the nature of the function f(x,y) = $x^3 + y^3 - 3x - 12y + 20$

22)Find the least value of the function, f(x,y) = xy + $\frac{50}{x} + \frac{50}{y}$

23) Investigate the maximum and minimum values of f(x,y) = $3x^2 y - 3x^2 - 3y^2 + y^3 + 2$

24) Discuss the maximum and minimum of the function u $= x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$

25) Find the rectangle of perimeter 12cm which has maximum area.

26) Find the points on the surface $z^2 = xy + 1$ nearest to the origin.

27) Determine the minimum distance from origin to the plane $3x + 2y + z - 12 = 0$

28) Find the minimum and maximum distance from the origin to the curve $5x^2 + 6xy + 5y^2 = 8$

29) If a,b,c,are +ve numbers find the extreme value of f(x,y,z) $= x^a y^b z^c$ subject to the condition x+y+z =1

30) Find extreme values of f(x,y) = xy (a-x-y)

Unit IV (Double and Triple Integrals)

I) Objective questions (2 marks each)

1) Define double integral $\iint_R f(x,y)dR$

2) Evaluate : $\int_1^2 \int_0^x (x + 2y)dydx$

3) Evaluate : $\int_1^2\int_0^1 (x^2+y^2)dxdy$

4) Evaluate : $\int_0^1\int_0^{x^2} e^{\frac{y}{x}} dxdy$

5) Evaluate : $\int_1^2\int_0^x \frac{dxdy}{x^2+y^2}$

6) Evaluate : : $\int_0^a\int_0^b (x^2+y^2)dxdy$

7)Evaluate : $\int_0^4 \int_0^{\sqrt{y}} xydxdy$

8) Evaluate : $\iint xydxdy$ over the rectangle bounded by x=2, x=5, y=1, y=2.

9) Evaluate : $\int_0^1 dx\int_0^1 (x+y)dy$

10)Evaluate : : $\int_0^1\int_0^{1-x} xydydx$

11) Define triple integral $\iiint_v f(x,y,z)dv$

12) Evalute : ; $\int_0^1\int_0^1\int_0^1 (x+y+z)dxdydz$

13) Evaluate ; $\int_0^1\int_0^2\int_1^2 xyzdxdydz$

14) Evaluate : $\int_3^4\left[\int_1^2 \frac{dy}{(x+y)^2}\right]dx$

15)Evaluate : $\int_1^2\int_1^2 (x-y)dxdy$

II) Multiple Choice questions (1 marks each)
Choose the correct option from the given options.

1) The area of region bounded by the circle $x^2+y^2=a^2$ is

a) $\Pi^2 a$ unit b) Πa^2 unit^2
c) Πa^3 unit^3 d) Πa unit

2) The area of the ellipse $\frac{x^2}{16}+\frac{x^2}{9}=1$ is

a) 12Π unit^2 b) 12 unit
c) 12Π unit^3 d) 12Π

3) The volume of the sphere $x^2+y^2+z^2=a^2$ is

a) Πa^2 unit^2 b) Πa^3 unit
c) $\frac{4}{3}\Pi a^3$ unit^3 d) Πa unit

4) The Volume of ellipsoid $\frac{x^2}{1}+\frac{y^2}{4}+\frac{z^2}{9}=1$ is

a) $\frac{4}{3}\Pi$ unit　　　b) $\frac{4}{3}\Pi^2$ unit ^{2}

c) $\frac{24}{3}\Pi$ unit^3　　　d) $\frac{12}{3}\Pi$ unit^3

III) Theory and Examples (6 marks each)

1) a) Draw a sketch of the region of integration

$$\int_{-1}^{2}\int_{x^2}^{x+2} f(x,y)dxdy$$

b) Evaluate : $\int_{y=0}^{3}\int_{x=0}^{2}\int_{z=0}^{1}(x+y+z)dzdxdy$

2) a) Draw a sketch of the region of integration , $\int_0^4 dx \int_0^{\sqrt{25=x^2}} f(x,y)dy$

c) Evaluate : $\int_{x=0}^{1}\int_{y=0}^{1}\int_{z=1}^{2} x^2 yzdzdydx$

3) a) Draw a sketch of the region of integration $\int_0^2 dy \int_y^{4-y} f(x,y)dx$

b) Evaluate : $\int_0^1\int_0^{1-x}\int_0^{1-x-y} xyzdxdydz$

4) a) Draw a sketch of the region of integration $\int_1^2 dx \int_{\frac{x}{2}}^{2x} f(x,y)dy$

b) Evaluate $\iiint (x+y+z)dxdydz$ over the tetrahedron x=0 , y =0, z =0 and x+y+z=1

5) a) Change the order of integration and hence evaluate

$$\int_0^a\int_y^a \frac{xdxdy}{x^2+y^2}$$

b) Evaluate : $\int_0^a\int_0^{\sqrt{1-x^2}}\int^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$

6) a) Evaluate : $\iint xydxdy$ over the region in the positive quadrant for which x+y$\le$1

b) Evaluate : $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}dxdydz$

7)a) Using double integral find the area of the circle $x^2+y^2=a^2$

b) Evaluate : $\int_0^1\int_0^{1-x}\int_0^{x+y} e^z dxdydz$

8) a) Change the order of integration $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}dxdy$

b) Evaluate $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ over the region x $\geq 0, y \geq 0, z \geq 0, x+y+z \geq 1$

9) a) Change the order of integration $\int_1^2 \int_x^{2x} f(x,y)dxdy$

b)Evaluate ; $\int_0^a \int_0^{a-x} \int_0^{a-x-y} x^2 dxdydz$

10)a) Find the area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, using double integration.

b) Evaluate : $\int_0^1 \int_0^{1-x} \int_0^{(x+y)^2} xdxdydz$

11) a) Using double integration find the area of the region bounded by the parabola's $y^2 = 4x$ and $x^2 = 4y$

b) Find the volume of the sphere of radius 5.

12) a) Evaluate $\iint e^{2x+3y} dxdy$ over the triangle bounded by x =0, y = 0, and x+y = 1

b) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

13)a) Evaluate $\iint ydxdy$ over the area bounded by y =x ^{2} and x+y=2

b) Find the volume of the region bounded by x =0 ,y = 0, z = 0 and x+y+z=1

14) a) Evaluate : $\iint x^2 y^2 dxdy$ over the region $x^2 + y^2 \leq 1$

b) Using triple integration find the volume of the sphere of radius a.

15) a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{dxdy}{(1+e^y)\sqrt{1-x^2-y^2}}$

b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes y+z =3, z = 0

III) Compulsory Examples (2 marks each)

1) Find the area of square bounded by lines x =0, x =1, y = 0, y = 1 using double integration.
2) Evaluate $\int_{x=0}^{2} \int_{y=0}^{x} \int_{z=0}^{y} e^p dxdydz$,where p is a real.
3) Find the area bounded by the lines x=0 ,x=a,y =0, y = b.
4) Find the volume of region bounded by the planes
   ,x=0, x = 1, y = 0 , y = 1, z = 0 ,z =1.
5) Find the area of the circle of radius 1 using double integration.
6) Evaluate $\int_2^1 \int_0^2 x^3 y^3 dxdy$
7) Evaluate : $\int_0^1 \int_0^1 (x+y)dxdy$
8) Evaluate: $\int_a^b \int_a^b xydxdy$