SAMPLE QUESTION PAPER

Subject : Mathematics **Class :** Senior Secondary

Time : 3 Hours **Maximum Marks :** 100

1. Find 'a' and 'b' if 2
   ai. (3+bi) = 3 – 7i

2. Find the value of $A^2 + I$
   where $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$, I is a unit matrix. 2

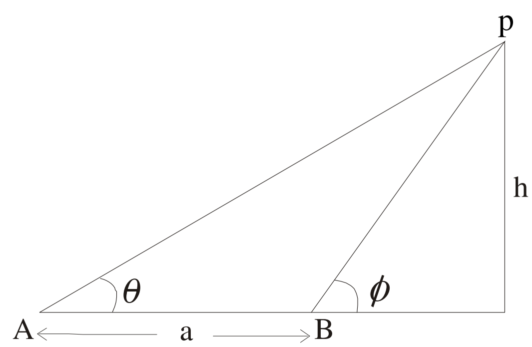
3. Prove that: 2
   $${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$$

4. How many ways can 4 boys and 3 girls be seated in a row of 7 chairs if boys and girls alternate? 2

5. Prove that:
   $$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$$ 2

6. Prove that:
   $$\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$$ 2

7. Find the value of 'h' in terms of θ, ϕ and 'a' as shown in the figure. 2



8. Evaluate : 2
   $$\lim_{x \to 0} \frac{\sin ax}{\tan bx}$$

9. If 1, w, w^2 be the cube roots of unity, then prove that 3
   $$(1 + w - w^2)^7 + (1 - w + w^2)^7 = 128$$

10. Show that : 3

$$\begin{vmatrix} -x^2 & xy & xz \\ xy & -y^2 & yz \\ xy & zy & -z^2 \end{vmatrix} = 4x^2\ y^2\ z^2$$

11. Using geometric progression, express $0.\bar{5}$ as rational number. 3

12. In what ratio does the point (3, -1) divide the join of the points (4, 2) and (5, 5). 3

13. Find the equation of the circle which passes through the origin and cuts off intercepts from the axes equal to 4 and 5. 3

14. Find the derivative from the first principle of the function $\sqrt{ax}$ 3

15. Find the intervals in which function

$f(x) = \dfrac{x^3}{3} - 9x + 27$ is increasing and decreasing. 3

16. Evaluate : $\displaystyle\int \frac{1}{(x+3)\ (2x+3)}\,dx$ 3

17. Find the co-efficient of x^{10} in the expansion of

$\dfrac{1+3x^2}{\left(1-x^2\right)^3}$ mentioning the condition under which the result holds. 4

18. Find the general solution of the equation 4

sin x +sin 2x + sin 3x = 0

19. Find the vertex, focus, directrix and length of latus rectum of the parabola 4

$5x^2 + 24y = 0$

20. Solve the equation 4

$\dfrac{dy}{dx} + \dfrac{y}{x} = \cos\ x$

21. Of all the rectangles inscribed in a given circle, prove that square has the maximum area. 4

22. Find the square root of – 15 – 8i. Hence find the square root of – 15 + 8i 5

23. Solve the system of equations using matrices 5

$x + y + z = 6$

$2x - y + z = 3$

$x - 2y + 3z = 6$

24. Prove that $\frac{1}{2.3} + \frac{1}{4.5} + \frac{1}{6.7} + ... \infty = 1 - \log 2$ 5

25. If $e^{\sin^{-1} x} + x^y + y^x = C$, find $\frac{dy}{dx}$ 5

26. Find the area of the region enclosed by the parabolas
$y^2 = 4ax$ and $x^2 = 4ay$ for $a > 0$. 5

27. Evaluate : 5

$$\int \frac{2x-3}{\sqrt{4x-x^2-3}}\, dx$$

OPTION – I

(Statistics and Probability)

28. In a study to test the effectiveness of a new variety of wheat, an experiment was performed with 50 experimental fields and the following results were obtained: 3

Yield per hectare (in quintals)	No. of fields
31-35	2
36-40	3
41-45	8
46-50	12
51-55	16
56-60	5
61-65	2
66-70	2

If the mean yield per hectare is 50 quintals, find variance and standard deviation.

29. If A and B are two events, such that 3
$P(A) = 0.8$, $P(B) = 0.6$, $P(A \cap B) = 0.5$

then find the value of
(i) $P(A \cup B)$ (ii) $P(B/A)$ (iii) $P(A/B)$

30. A pair of dice is thrown 10 times. If getting a doublet (same number on both) is considered a success, find the probability of (i) 4 successes (ii) No success 4

OPTION – II
(Linear Programming)

28. Solve the following, by simplex method

Minimize $z = x_1 + x_2$ 3

Subject to

$2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1 \geq 0, x_2 \geq 0$

29. Four person A, B, C and D are to be assigned four jobs I, II, III and IV. The cost matrix is given as under: 3

Job \ Man	A	B	C	D
I	8	10	17	9
II	3	8	5	6
III	10	12	11	9
IV	6	13	9	7

Find the proper assignment.

30. Solve the following by using graphical method: 4

Minimize $z = 60x_1 + 40x_2$

Subject to the conditions

$3x_1 + x_2 \geq 24$

$x_1 + x_2 \geq 16$

$x_1 + 3x_2 \geq 24$

$x_1 \geq 0, x_2 \geq 0$

OPTION – III
(Vectors and Analytical Solid Geometry)

28. In a regular hexagon ABCDEF, if $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{BC} = \vec{b}$, then express each of the following in terms of $\vec{a}$ and $\vec{b}$

(i) $\overrightarrow{AC}$ (ii) $\overrightarrow{AD}$ (iii) $\overrightarrow{EA}$ 3

29. Find the equation of the plane through the points (–1,1,1) and (1, –1,1) and perpendicular to the plane x + 2y + 2z – 5 = 0 3

30. Reduce the equations of the line given by

$3x + 2y - z - 4 = 0$

and $4x + y - 2z + 3 = 0$ in symmetric form. 4

MARKING SCHEME
(For Sample Question Paper)

Subject: Mathematics **Class : Sr. Secondary**

1. $ai\ (3 + bi) = 3 - 7i$

$\Rightarrow 3\ ai + ab\ (i)^2 = 3 - 7i$

$\Rightarrow 3\ ai - ab = 3 - 7i$ ½

$\Rightarrow 3\ a = -7$ and $-ab = 3$

$\Rightarrow a = -\frac{7}{3}$ *and* $-\left(\frac{-7}{3}\right) b = 3$ ½

$\Rightarrow \frac{7}{3}b = 3$

$\Rightarrow b = \frac{9}{7}$

$a = \frac{-7}{3}, \quad b = \frac{9}{7}$ ½ + ½

2. Given matrix, $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$

$A^2 + I = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ½

$= \begin{pmatrix} 4+5 & 10+15 \\ 2+3 & 5+9 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ½

$= \begin{pmatrix} 9 & 25 \\ 5 & 14 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ½

$= \begin{pmatrix} 9+1 & 25+0 \\ 5+0 & 14+1 \end{pmatrix} = \begin{pmatrix} 10 & 25 \\ 5 & 15 \end{pmatrix}$ ½

3. To prove: ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$L.H.S. = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$ ½

$= \frac{n!}{(r-1)!(n-r)!} \left[\frac{1}{r} + \frac{1}{n-r+1}\right]$ ½

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$ ½

$$= \frac{(n+1)!}{r!(n-r+1)!}$$

$= {}^{n+1}C_r$ (By definition) ½

4. Starting with boys to take the first seat 4 boys can be accommodated in 4! ways and 3 girls can be accommodated in 3! ways.

Total no. of such arrangement is

$= 4! \times 3!$ 1

$= 4.3.2.1 \times 3.2.1$ ½

$= 144$ ways ½

5. L.H.S $= \sin^6\theta + \cos^6\theta$

$= \left(\sin^2\theta\right)^3 + \left(\cos^2\theta\right)^3$ ½

$= \left(\sin^2\theta + \cos^2\theta\right)\left(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta\right)$ ½

$= \left(\cos^2\theta + \sin^2\theta\right)^2 - 3\sin^2\theta\cos^2\theta$ ½

$= 1 - 3\sin^2\theta\cos^2\theta$ ½

= R.H.S

6. We can write, $\tan 56^\circ$

$= \tan\left(45^\circ + 11^\circ\right)$ ½

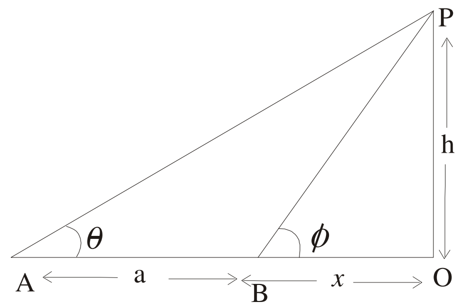
$$= \frac{\tan 45 + \tan 11^\circ}{1 - \tan 45^\circ \tan 11^\circ}$$ ½

$$= \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ}$$

$$= \frac{1 + \frac{\sin 11^\circ}{\cos 11^\circ}}{1 - \frac{\sin 11^\circ}{\cos 11^\circ}}$$ ½

$$= \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$$ (Proved) ½

7. Let BO = x



In $\Delta\ PBO$

$\frac{x}{h} = \cot\ \phi$ ½

Similarly, in $\Delta\ PAO$

$\frac{a+x}{h} = \cot\theta$ ½

$\Rightarrow\ \frac{a + h\cot\ \phi}{h} = \cot\theta$ ½

$\Rightarrow\ a + h\ \cot\phi = \text{h}\cot\theta$

$\Rightarrow\ a = h\ (\cot\theta - \cot\phi)$

$\Rightarrow\ \frac{\text{a}}{\cot\theta - \cot\phi} = h.$ ½

8. $\lim_{x\to 0} \frac{\sin\ ax}{\tan\ \text{b}x}$

$= \lim_{x\to 0} \left(\frac{\text{sin ax}}{\text{ax}} . \frac{\text{ax}}{\text{bx}} . \frac{\text{bx}}{\text{tan bx}} \right)$ ½ + ½

$= \frac{\text{a}}{\text{b}} \lim_{x\to 0} \left(\frac{\text{sin ax}}{\text{ax}} \right) \lim_{x\to 0} \left(\frac{\text{bx}}{\text{tan bx}} \right)$ ½

$= \frac{\text{a}}{\text{b}}.1.1$

$= \frac{a}{b}$ ½

9. To prove $\left(1 - w + w^2\right)^7 + \left(1 + w - w^2\right) = 128$

we know that $1+\text{w}+\text{w}^2=0$ and $\text{w}^3 = 1$.

$\therefore$ L.H.S. = $\left(-w - w\right)^7 + \left(-w^2 - w^2\right)^7$ ½ + ½

$= \left(-2w\right)^7 + \left(-2w^2\right)^7$

$= -128\ \left(w^7 + w^{14}\right)$

$= -128\ \left[\left(\text{w}^3\right)^2 .\text{w} + \left(\text{w}^3\right)^4 .\text{w}^2\right]$ ½ + ½

$= -128\ \left(\text{w} + \text{w}^2\right)$ ½

$= -128\left(-1\right) = 128.$ ½

= R.H.S.

10. $$\begin{vmatrix} -x^2 & xy & xz \\ xy & -y^2 & yz \\ xz & zy & -z^2 \end{vmatrix}$$

$$= xyz \begin{vmatrix} -x & x & x \\ y & -y & y \\ z & z & -z \end{vmatrix}$$ 1

$$= x^2\ y^2\ z^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$ 1

$$= x^2\ y^2\ z^2 \begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} \qquad \left(c_1^{'} = c_1 + c_2\right)$$ ½

Expanding the determinant, we have ½

$= x^2 y^2\ z^2\left[0(1-1)-1(0-2)+1(0+2)\right]$

$= 4\ x^2 y^2 z^2$

11. We know that

$0.\overline{5}$ = 0.5555 .... ½

= 0.5 + 0.05 + 0.005 + ... ½

(this being an infinite G.P. having first term as 0.5 and common ratio 0.1)

$= \dfrac{0.5}{1-(0.1)}$ ½ + ½

$= \dfrac{0.5}{0.9} = \dfrac{5}{9}$ ½ + ½

12. Let (3, –1) divide the join of (4, 2) and (5, 5) in the ratio k : 1

$\therefore\ \dfrac{5k+4}{k+1} = 3$ 1

or 5k + 4 = 3k +3 1

or 2k = -1

or k $= \dfrac{-1}{2}$

$\therefore$ The required ratio is 1:2 (externally) 1

13. Let the general equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ….(i) ½

Since (i) passes through (0,0), (4, 0) and (0, 5), we get

c = 0 ….(ii)

16 + 8g +c = 0 ….(iii)

25 + 10f + c = 0 ….(iv) ½x3=1½

From (ii), (iii) and (iv), we get

$8g = -16 \Rightarrow g = -2$

and $f = \dfrac{-5}{2}$ ½

Substituting these values in (i), we have

$x^2 + y^2 - 4x - 5y = 0$ ½

14. Let $f(x) = \sqrt{ax}$

$f(x+\delta x) = \sqrt{a(x+\delta x)}$ ½

$f(x+\delta x) - f(x) = \sqrt{a(x+\delta x)} - \sqrt{ax}$

$\dfrac{f(x+\delta x) - f(x)}{\delta x} = \dfrac{\sqrt{a(x+\delta x)} - \sqrt{ax}}{\delta x}$ ½

$= \left(\dfrac{\sqrt{a(x+\delta x)} - \sqrt{ax}}{\delta x}\right)\left(\dfrac{\sqrt{a(x+\delta x)} + \sqrt{ax}}{\sqrt{a(x+\delta x)} + \sqrt{ax}}\right)$ ½

$= \dfrac{a(x+\delta x) - ax}{\delta x\left[\sqrt{a(x+\delta x)} + \sqrt{ax}\right]}$

$= \dfrac{a\delta x}{\delta x\left[\sqrt{a(x+\delta x)} + \sqrt{ax}\right]}$ ½

Taking limit $\delta x \to 0$

$\lim_{\delta x \to 0} \dfrac{f(x+\delta x) - f(x)}{\delta x} = \lim_{\delta x \to 0} \dfrac{a}{\sqrt{a(x+\delta x)} + \sqrt{ax}}$ ½

$f'(x) = \dfrac{a}{\sqrt{ax} + \sqrt{ax}}$

$= \dfrac{a}{2\sqrt{ax}}$ ½

15. $f(x)=\frac{x^3}{3}-9x+27$

$f'(x)=\frac{3x^2}{3}-9 \qquad =x^2-9$ ½

$=(x+3)(x-3)$ ½

For increasing function

$f'(x)>0$ i.e. $x^2-9>0$

(i) $(x-3)(x+3)>0$

$x>3, x>-3 \qquad \Rightarrow x>3$ ½

(ii) $x+3<0$ and $(x-3)<0$

$x<-3$ and $x<3 \qquad \Rightarrow x<-3$ ½

For function to be decreasing

i.e., $x^2-9<0$

(i) $(x+3)(x-3)<0$

$x+3<0$ and $x-3>0$

$x<-3$ and $x>3$

No solution. ½

(ii) $x+3>0$ and $x-3<0$

$x>-3$ and $x<3$

$-3<x<3$

½

16. $\frac{1}{(x+3)(2x+3)}$ *in* $I=\int\frac{1}{(x+3)(2x+3)}dx$ being proper rational function,

it can be put as follows :

$\frac{1}{(x+3)(2x+3)}=\frac{A}{x+3}+\frac{B}{2x+3}$ …. (i) ½

Where A and B are to be determined

1=A(2x +3) + B (x+3)

Solving for A and B, we get $A=-\frac{1}{3}$ and $B=\frac{2}{3}$ ½ + ½

Substituting these values in (i), we have

$I=-\int\frac{1}{3(x+3)}dx+\frac{2}{3}\int\frac{1}{2x+3}dx$

$=-\frac{1}{3}\log|x+3|+\frac{1}{3}\log|2x+3|+C$ ½ + ½

$=\frac{1}{3}\log\left|\frac{2x+3}{x+3}\right|+C$ ½

17. $\dfrac{(1+3x^2)}{(1-x^2)^3} = (1+3x^2)(1-x^2)^{-3}$

$= (1+3x^2)\left[1+3x^2+\dfrac{3.4}{2!}x^4+\dfrac{3.4.5}{3!}x^6+\dfrac{3.4.5.6}{4!}x^8+\dfrac{3.4.5.6.7}{5!}x^{10}+........\right]$ 1

Coefficient of x^{10} is 1

$\dfrac{3.4.5.6.7}{5.4.3.2.1}+3.\dfrac{3.4.5.6}{4.3.2.1}$ 1

= 21 + 45 = 66

Condition $|x|<1$. 1

18. $\sin x+\sin 2x+\sin 3x=0$

or $\sin x+\sin 3x+\sin 2x=0$ ½

$\Rightarrow 2.\sin\left(\dfrac{x+3x}{2}\right).\cos\left(\dfrac{x-3x}{2}\right)+\sin 2x=0$ 1

$\Rightarrow 2\sin 2x\cos x+\sin 2x=0$ ½

$\Rightarrow \sin 2x[2\cos x+1]=0$

either $\sin 2x=0$ or $2\cos x+1=0$ 1 + 1

$\Rightarrow 2x=n\pi$ $\Rightarrow \cos x=-\dfrac{1}{2}$

$\Rightarrow x=\dfrac{n\pi}{2}$ $\Rightarrow \cos x=\cos\dfrac{2\pi}{3}$

$\Rightarrow x=2n\pi\pm\dfrac{2\pi}{3}$

19. $5x^2+24y=0$

$\Rightarrow 5x^2=-24y$

$\Rightarrow x^2 = \dfrac{-24}{5}y$

$\Rightarrow x^2 = 4\left(\dfrac{-6}{5}\right)y$ ½

Vertex is (0,0). ½

Focus is (0,a), here $a=\dfrac{-6}{5}$ 1

$\therefore$ Focus is $\left(0,\dfrac{-6}{5}\right)$

Directrix is y = −a

$\Rightarrow y+a=0$

$\Rightarrow y+\left(\frac{-6}{5}\right)=0$

$\Rightarrow 5y-6=0$ 1

Length of latus rectum $= 4a$

$=4\left(-\frac{6}{5}\right)$

$=\left|-\frac{24}{5}\right|=\frac{24}{5}$ 1

20. $\frac{dy}{dx}+\frac{y}{x}=\cos\ x$

The coefficient of $\frac{dy}{dx}$ is unity. ½

Therefore, integrating factor will be

$e^{\int\frac{1}{x}dx}=e^{\log|x|}=x$ 1

Multiplying both sides by the integrating factor (x), and integrating

1

$xy\ =\int x\cos xdx$

$=x(\sin\ x)-\int\sin\ x\ dx$ ½

1

$\therefore\quad xy=x\sin x+\cos x+c$

21. Let the radius of the circle be a.

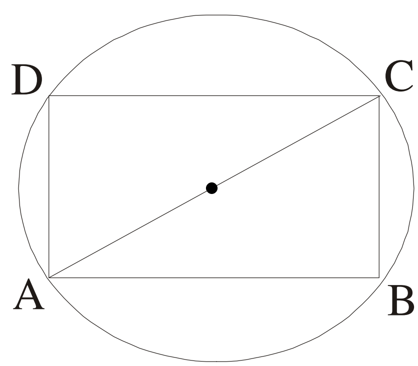
ABCD being a rectangle, $\angle B=90^\circ$

$\therefore AC$ is a diagonal.

Let AB and BC be x and y respectively

$\therefore x^2+y^2=4a^2$ 1

Differentiating w.r.t x, we have



$2x+2y\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=\frac{-x}{y}$ 1

Let A(x) = xy

Differentiating with respect to x,

$A'(x)=y+x\frac{dy}{dx}=0$

or $\quad y+x\left(\frac{-x}{y}\right)=0$

or $\quad -x^2+y^2=0$

or $\quad$ x = y 1

$\Rightarrow ABCD$ is a square.

Again differentiating w.r.t x,

$$A''(x) = -2x + 2y\frac{dy}{dx}$$

$$= -2x + 2y\left(\frac{-x}{y}\right)$$

$$= -2x - 2x = -4x < 0$$ 1

Hence area is maximum when rectangle ABCD is a square.

22. Let $\sqrt{-15-8i} = x + iy$

$\Rightarrow \quad -15-8i = x^2 - y^2 + 2ixy$ 1

$\Rightarrow \quad x^2 - y^2 = -15$ …(i)

and 2xy = – 8 …(ii) ½ + ½

$$\left(x^2+y^2\right)^2 = \left(x^2-y^2\right)^2 + 4x^2y^2$$

$$= (-15)^2 + 64$$

$$= 225 + 64 = 289$$

$\therefore \quad x^2 + y^2 = 17$ …(iii) ½

Adding (i) and (iii), we have

$$2x^2 = 2$$

$\Rightarrow \quad x^2 = 1$ ½

$\Rightarrow \quad x = \pm 1$ ½

$\Rightarrow \quad y = \pm 4$

From (ii), we conclude that x and y are of opposite signs.

There, the required square root is $\pm(1-4i)$ ½

Now the corresponding 2nd equation for the expression –15 + 8i is

2xy = 8

Which implies x and y have the same sign. 1

$\therefore$ The required square root is $\pm(1+4i)$

23. $x + y + z = 6$

$2x - y + z = 3$

$x - 2y + 3z = 6$

The given equations can be written as

AX = B, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & -2 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$$ ½

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & -2 & 3 \end{vmatrix} = 1(-3+2)-1(6-1)+1(-4+1)$$

$$= -1-5-3 = -9 \neq 0$$

It is non singular. 1

$\therefore A^{-1}$ exists.

$a_{11} = -1, \quad a_{12} = -5, \quad a_{13} = -3$

$a_{21} = -5, \quad a_{22} = 2, \quad a_{23} = 3$

$a_{31} = 2, \quad a_{32} = 1, \quad a_{33} = -3$

$$\text{Adj A} = \begin{bmatrix} -1 & -5 & 2 \\ -5 & 2 & 1 \\ -3 & 3 & -3 \end{bmatrix}$$ 1

$$A^{-1} = \frac{Adj\ A}{|A|} = \frac{1}{-9}\begin{bmatrix} -1 & -5 & 2 \\ -5 & 2 & 1 \\ -3 & 3 & -3 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{-9}\begin{bmatrix} -1 & -5 & 2 \\ -5 & 2 & 1 \\ -3 & 3 & -3 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$$ 1

$$= -\frac{1}{9}\begin{bmatrix} -6-15+12 \\ -30+6+6 \\ -18+9-18 \end{bmatrix} = -\frac{1}{9}\begin{bmatrix} -9 \\ -18 \\ -27 \end{bmatrix}$$ 1

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ ½

$\therefore x = 1, y = 2, z = 3.$

24. $$\frac{1}{2.3} + \frac{1}{4.5} + \frac{1}{6.7} + \ldots\ldots + \frac{1}{2n(2n+1)} + \ldots\ldots\infty$$

$$T_n = \frac{1}{2n(2n+1)}$$ 1

$$T_n = \frac{1}{2n} - \frac{1}{2n+1}$$ ½

Replacing n by 1,2,3,……, we have

$$\left.\begin{aligned} T_1 &= \frac{1}{2}-\frac{1}{3} \\ T_2 &= \frac{1}{4}-\frac{1}{5} \\ T_3 &= \frac{1}{6}-\frac{1}{7} \\ &\ldots\ldots \quad \ldots\ldots \\ T_n &= \frac{1}{2n}-\frac{1}{2n+1} \end{aligned}\right]$$ 1

Adding, we get

$$T_1+T_2+\ldots..+T_n+\ldots.=\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\ldots..$$ 1

$$1-\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\ldots..\right)$$ ½

$= 1 - \log(1+1)$ ½

$= 1 - \log 2$ ½

25. $e^{\sin^{-1}x}+x^y+y^x=C$

Differentiating w.r. t x, we get

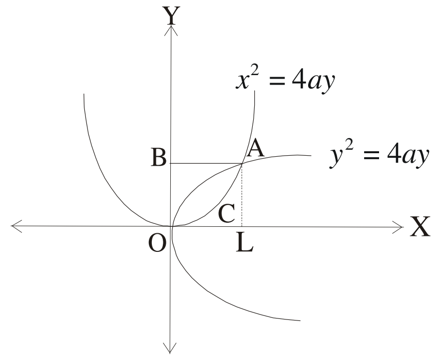
$$\frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}+x^y\left(\frac{y}{x}+\log x.\frac{dy}{dx}\right)+y^x\left(\log y+\frac{x}{y}\frac{dy}{dx}\right)=0$$ 1+1+1

$$\Rightarrow \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}+x^{y-1}.y+x^y\log x\frac{dy}{dx}+y^{x-1}.x.\frac{dy}{dx}+y^x\log y=0$$

$$\Rightarrow \left(x^y\log x+y^{x-1}x\right)\frac{dy}{dx}=-\left(\frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}+yx^{y-1}+y^x\log y\right)$$ 1

$$\frac{dy}{dx}=-\left(\frac{\frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}+yx^{y-1}+y^x\log y}{x^y\log x+y^{x-1}x}\right)$$ 1

26.



$x^2 = 4ay$ (½ mark for correct figure)

$y^2 = 4ax$ ½

The points of intersections are O (0,0), A (4a, 4a)

The area common to both

= Area (OBAL) – Area (OCAL) ½

$$= \int_0^{4a} y\,dx - \int_0^{4a} y\,dx$$ ½ + ½

$$= \int_0^{4a} \sqrt{4ax}\,dx - \int_0^{4a} \frac{x^2}{4a}\,dx$$ ½ + ½

$$= 2\sqrt{a}\left|\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^{4a} - \frac{1}{4a}\left|\frac{x^3}{3}\right|_0^{4a}$$ ½

$$= 2\sqrt{a}.\frac{2}{3}(4a)^{\frac{3}{2}} - \frac{1}{12a}.64a^3$$ ½

$$= \frac{32}{3}a^2 - \frac{16}{3}a^2 \qquad \left(\because (4)^{\frac{3}{2}} = \left(2^2\right)^{\frac{3}{2}} = 2^3 = 8\right)$$

$$= \frac{16}{3}a^2 \, square\ \ units.$$ ½

27. $$\int \frac{2x-3}{\sqrt{4x-x^2-3}}dx$$

$$\left.\begin{array}{l} 2x-3 = \lambda(4-2x)+\mu \\ 2 = -2\lambda \Rightarrow \lambda = -1 \\ -3 = 4\lambda+\mu = -4+\mu \Rightarrow \mu = 1 \end{array}\right]$$ 1

$$\int \frac{2x-3}{\sqrt{4x-x^2-3}}dx = \int\left[\frac{1}{\sqrt{1-(x-2)^2}} - \frac{4-2x}{\sqrt{1-(x-2)^2}}\right]dx$$ 1

$$= \int \frac{1}{\sqrt{1-(x-2)^2}}\,dx - \int \frac{4-2x}{\sqrt{1-(x-2)^2}}\,dx$$

Put $4x - x^2 - 3 = t$

$\Rightarrow (4-2x)\,dx = dt$

$= \sin^{-1}\left(\frac{x-2}{1}\right) - \int \frac{dt}{t^{\frac{1}{2}}} + C$ 1+1

$= \sin^{-1}(x-2) - \int t^{-\frac{1}{2}}\,dt + C$

$= \sin^{-1}(x-2) - 2\sqrt{t} + C$ ½

$= \sin^{-1}(x-2) - 2\sqrt{4x - x^2 - 3} + C$ ½

OPTION – I
(Statistics and Probability)

28.

Yield per hectare (in quintals)	No. of fields	Class mark x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
31 – 35	2	33	− 17	289	578
36 – 40	3	38	− 12	144	432
41 – 45	8	43	− 7	49	392
46 – 50	12	48	− 2	4	48
51 – 55	16	53	+3	9	144
56 – 60	5	58	+8	64	320
61 – 65	2	63	+13	169	338
66 – 70	2	68	+18	324	648
Total	50				2900

For calculating $(x_i - \bar{x})$, $(x_i - \bar{x})^2$ and $\sum f_i (x_i - \bar{x})^2$ correctly, $\frac{1}{2}$ mark each 1½

Thus $\sigma_g^2 = \frac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^2}{N}$ ½

½

$= \frac{2900}{50} = 58$ ½

and $\sigma_g = +\sqrt{58} = 7.61$ (approx)

29. (i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.8 + 0.6 - 0.5$ 1

$= 0.9$

(ii) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.5}{0.8} = \frac{5}{8}$ 1

(iii) $P(A/B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{0.5}{0.6}=\dfrac{5}{6}$ 1

30. Here n = 10

A doublet can be obtained when a pair of dice is thrown and shows (1,1), (2,2), (3,3), (4,4), (5,5) and (6,6) i.e. 6 ways.

$p=\dfrac{6}{36}=\dfrac{1}{6}$ ½

$q=\dfrac{5}{6}$ ½

$(p+q)^n = {}^nC_0 p^n + {}^nC_1 p^{n-1} q + ... + {}^nC_n q^n$ 1

(i) P (4 successes) $= {}^{10}C_4 p^4 q^6$

$=\dfrac{10.9.8.7}{4.3.2.1}\left(\dfrac{1}{6}\right)^4\left(\dfrac{5}{6}\right)^6$

$=210\times\dfrac{5^6}{6^{10}}=\dfrac{7\times5^7}{6^9}$ 1

(ii) p (no success) $= {}^{10}C_0 p^0 q^{10}$

$=\left(\dfrac{5}{6}\right)^{10}$ 1

OPTION – II
(Linear Programming)

28. Changing the given problem to a maximization problem, we have

$$z_1=-z=-x_1-x_2$$
$$-2x_1-x_2\le-4$$
$$-x_1-7x_2\le-7$$
$$x_1\ge0, x_2\ge0$$

Introducing non-negative variables to form equation, we have

$$-2x_1-x_2+s_1=-4$$
$$-x_1-7x_2+s_2=-7$$
$$x_1\ge0, x_2\ge0, s_1\ge0, s_2\ge0$$

The initial simplex table is 1

$$\rightarrow\left[\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & z_1 & \\ -2 & -1 & 1 & 0 & 0 & -4 \\ -1 & \boxed{-7} & 0 & 1 & 0 & -7 \\ \hline 1 & 1 & 0 & 0 & 1 & 0 \end{array}\right]\begin{array}{c} \\ s_1 \\ s_2 \\ z_1 \end{array}$$

Dividing R_2 by –7 and applying the operation $R_1 + R_2, R_3 - R_2$ we get the following table

$$\rightarrow\left[\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & z_1 & \\ \boxed{-\frac{13}{7}} & 0 & 1 & -\frac{1}{7} & 0 & -3 \\ \frac{1}{7} & 1 & 0 & -\frac{1}{7} & 0 & 1 \\ \hline \frac{6}{7} & 0 & 0 & \frac{1}{7} & 1 & -1 \end{array}\right]\begin{array}{c} \\ s_1 \\ x_2 \\ z_1 \end{array}$$ 1

$\uparrow$

Dividing R_1 by $-\frac{13}{7}$ and applying the operation $R_2 - \frac{1}{7}R_1, R_3 - \frac{6}{7}R_1$, we get the following table

½

$$\left[\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & z_1 & \\ 1 & 0 & -\frac{7}{13} & \frac{1}{13} & 0 & \frac{21}{13} \\ 0 & 1 & \frac{1}{13} & -\frac{2}{13} & 0 & \frac{10}{13} \\ \hline 0 & 0 & \frac{6}{13} & \frac{1}{13} & 1 & -\frac{31}{13} \end{array}\right]\begin{array}{c} \\ x_1 \\ x_2 \\ z_1 \end{array}$$

So the optimal solution is

max $z_1 = -\frac{31}{13}$, so min $z = \frac{31}{13}$

This occurs at $x_1 = \frac{21}{13}, x_2 = \frac{10}{13}, s_1 = 0, s_2 = 0$ ½

29. Row reduction ½

Men / Jobs	A	B	C	D
I	0	2	9	1
II	0	5	2	3
III	1	3	2	0
IV	0	7	3	1

Column reduction

Men / Jobs	A	B	C	D
I	0	0	7	1
II	0	3	0	3
III	1	1	0	0
IV	0	5	1	1

½

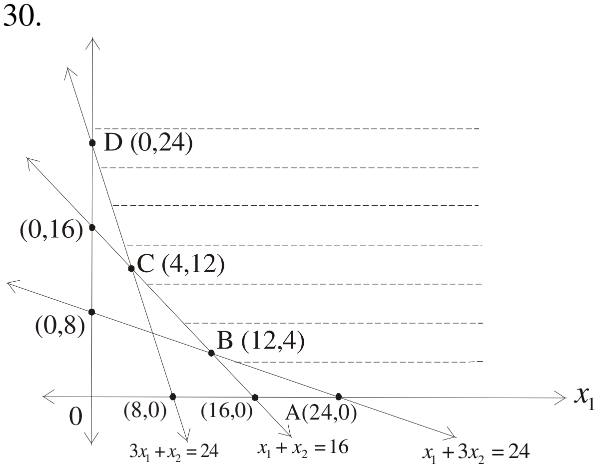
Zero assignment

Men / Jobs	A	B	C	D
I	~~0~~	[0]	7	1
II	~~0~~	3	[0]	3
III	1	1	~~0~~	[0]
IV	[0]	5	1	1

1

Here total assigned zero = 4 (i.e. number of rows or columns)
Thus, the assignment is optional.
From the table, we get I ⟶ B, II ⟶ C, III ⟶ D and IV ⟶ A 1

30.



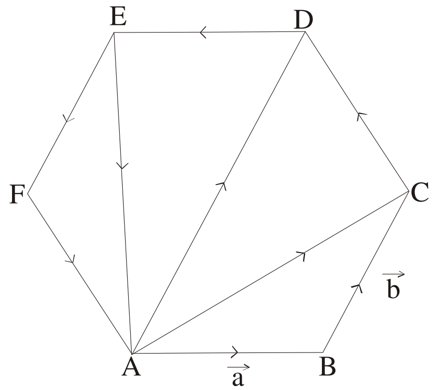
Plotting of inequalities 1
For indicating feasible region and vertices 1

Values of $Z=60x_1 + 40x_2$ at the four vertices
B (12,4) = 880
C (4,12) = 720
D (0,24) = 960 1

Minimum Value is 720 at $x_1 = 4, x_2 = 12$ 1

OPTION – III
(Vectors and Analytical Solid Geometry)

28.



(i) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= \vec{a} + \vec{b}$ ½

(ii) $\overrightarrow{AD} = 2\overrightarrow{BC}$ ½

$= 2\vec{b}$

(iii) Now $\overrightarrow{CD} = \overrightarrow{AD} - \overrightarrow{AC}$

$= 2\vec{b} - \left(\vec{a} + \vec{b}\right) = \vec{b} - \vec{a}$ 1

$\therefore$ $\overrightarrow{EA} = \overrightarrow{EF} + \overrightarrow{FA}$

$= -\vec{b} - \overrightarrow{CD}$ ½

$= -\vec{b} - \left(\vec{b} - \vec{a}\right)$ ½

$= \vec{a} - 2\vec{b}$

29. The equation of any plane passing through the point (–1,1,1) is

a (x + 1) + b (y – 1) + c (z – 1) = 0 …(i) ½

Since the point (1, –1,1) lies on the plane

$\therefore$ 2a – 2b + 0.c = 0 …(ii) ½

Again the plan (i) is perpendicular to the plane x + 2y + 2z –5 =0

$\therefore$ a + 2b + 2c = 0 …(iii) ½

From (ii) and (iii) ,by cross multiplication method we get

$$\frac{a}{-4-0} = \frac{b}{0-4} = \frac{c}{4+2}$$

$$\frac{a}{2} = \frac{b}{2} = \frac{c}{-3}$$ ½

Hence the required equation of the plane is

2 (x + 1) + 2 (y – 1) –3 (z – 1) = 0 1

2x + 2y – 2z + 3 = 0

30. 3x + 2y –z – 4 = 0 ...(i)

4x + y – 2z + 3 = 0 ... (ii)

Let z = 0 be the z – coordinate of a point on each of the planes given by (i) and (ii)

The equation of the planes reduce to

3x + 2y = 4

4x + y = – 3

which on solving gives x = – 2, y = 5 1

The point common to two planes is (– 2, 5, 0)

Let l, m, n be the direction ratios of the line.

As the line is perpendicular to normal to be plane

$$\therefore 3l + 2m - n = 0$$ 1

$$and \quad 4l + m - 2n = 0$$

$$\frac{1}{-4+1} = \frac{m}{-4+6} = \frac{n}{3-8}$$

$$\frac{l}{3} = \frac{m}{-2} = \frac{n}{5}$$ 1

The equations of the lines are

$$\frac{x+2}{3} = \frac{y-5}{-2} = \frac{z}{5}$$ 1