

SIGNALS AND SYSTEMS (IT1201)

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(IT 1201)

UNIT I

CLASSIFICATION OF SIGNALS AND SYSTEMS

1. Define Signal.

A signal is a function of one or more independent variables which contain some information.

Eg: Radio signal, TV signal, Telephone signal etc.

2. Define System.

A system is a set of elements or functional block that are connected together and produces an output in response to an input signal.

Eg: An audio amplifier, attenuator, TV set etc.

3. Define CT signals.

Continuous time signals are defined for all values of time. It is also called as an analog signal and is represented by $x(t)$.

Eg: AC waveform, ECG etc.

4. Define DT signal.

Discrete time signals are defined at discrete instances of time. It is represented by $x(n)$.

Eg: Amount deposited in a bank per month.

5. Give few examples for CT signals.

AC waveform, ECG, Temperature recorded over an interval of time etc.

6. Give few examples of DT signals.

Amount deposited in a bank per month,

7. Define unit step, ramp and delta functions for CT.

Unit step function is defined as

$$U(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Unit ramp function is defined as

$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Unit delta function is defined as

$$\delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{otherwise} \end{cases}$$

8. State the relation between step, ramp and delta functions(CT).

The relation ship between unit step and unit delta function is

$$\delta(t) = \frac{d}{dt} u(t)$$

The relationship between delta and unit ramp function is

$$\int \delta(t) dt = r(t)$$

9. State the classification of CT signals.

The CT signals are classified as follows

- (i) Periodic and non periodic signals
- (ii) Even and odd signals
- (iii) Energy and power signals
- (iv) Deterministic and random signals.

10. Define deterministic and random signals.

A deterministic signal is one which can be completely represented by Mathematical equation at any time. In a deterministic signal there is no uncertainty with respect to its value at any time.

Eg: $x(t) = \cos \omega t$

$$x(n) = 2\pi f t$$

A random signal is one which cannot be represented by any mathematical equation.

Eg: Noise generated in electronic components, transmission channels, cables etc.

11. Define power and energy signals.

The signal $x(t)$ is said to be power signal, if and only if the normalized average power p is finite and non-zero.

$$\text{I.e. } 0 < p < \infty$$

A signal $x(t)$ is said to be energy signal if and only if the total normalized energy is finite and non-zero.

Ie. $0 < E < \infty$

12. Compare power and energy signals.

Sl.No	POWER SIGNAL	ENERGY SIGNALS
1	The normalized average power is finite and non-zero	Total normalized energy is finite and non-zero.
2	Practical periodic signals are power signals	Non-periodic signals are energy signals

13. Define odd and even signal.

A DT signal $x(n)$ is said to be an even signal if $x(-n)=x(n)$ and an odd signal if $x(-n)=-x(n)$.

A CT signal $x(t)$ is said to be an even signal if $x(t)=x(-t)$ and an odd signal if $x(-t)=-x(t)$.

14. Define periodic and Aperiodic signals.

A signal is said to be periodic signal if it repeats at equal intervals. Aperiodic signals do not repeat at regular intervals.

A CT signal which satisfies the equation $x(t)=x(t+T_0)$ is said to be periodic and a DT signal which satisfies the equation $x(n)=x(n+N)$ is said to be periodic.

15. State the classification or characteristics of CT and DT systems.

The DT and CT systems are according to their characteristics as follows

- (i). Linear and Non-Linear systems
- (ii). Time invariant and Time varying systems.
- (iii). Causal and Non causal systems.
- (iv). Stable and unstable systems.
- (v). Static and dynamic systems.
- (vi). Inverse systems.

16. Define linear and non-linear systems.

A system is said to be linear if superposition theorem applies to that system. If it does not satisfy the superposition theorem, then it is said to be a non-linear system.

17. Define Causal and non-Causal systems.

A system is said to be a causal if its output at anytime depends upon present and past inputs only.

A system is said to be non-causal system if its output depends upon future inputs also.

18. Define time invariant and time varying systems.

A system is time invariant if the time shift in the input signal results in corresponding time shift in the output.

A system which does not satisfy the above condition is time variant system.

19. Define stable and unstable systems.

When the system produces bounded output for bounded input, then the system is called bounded input, bounded output stable.

A system which doesnot satisfy the above condition is called a unstable system.

20. Define Static and Dynamic system.

A system is said to be static or memoryless if its output depends upon the present input only.

The system is said to be dynamic with memory if its output depends upon the present and past input values.

PART-B

1. Discuss the classification of DT and CT signals with examples.

- Deterministic and random signals
- Periodic and Aperiodic signals
- Energy and power signals
- Noise signals
- Physically Realisable & non-realisable signals.

2. Discuss the classification of DT and CT systems with examples.

- Linear and Non-Linear systems
- Time invariant and Time varying systems
- Causal and Non-causal systems
- Stable and unstable systems
- Static and dynamic systems
- Inverse systems

3. Problems on the properties & classifications of signals & systems

Find whether the following signals are periodic or not

a. $x(t)=2\cos(10t+1)-\sin(4t-1)$

Ans:Periodic signal.

b. $x(t)=3\cos4t+2\sin\pi t$

Ans:Non periodic signal

Check whether the following system is

1. Static or dynamic
2. Linear or non-linear
3. Causal or non-causal
4. Time invariant or variant

$$y(n) = \text{sgn}[x(n)]$$

Ans: static, non linear, Causal, Time invariant.

UNIT II ANALYSIS OF CT SIGNALS

1. Define CT signal

Continuous time signals are defined for all values of time. It is also called as an analog signal and is represented by $x(t)$.

Eg: AC waveform, ECG etc.

2. Compare double sided and single sided spectrums.

The method of representing spectrums of positive as well as negative frequencies are called double sided spectrums.

The method of representing spectrums only in the positive frequencies is known as single sided spectrums.

3. Define Quadrature Fourier Series.

Consider $x(t)$ be a periodic signal. The fourier series can be written for this signal as follows

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos w_0 n t + \sum_{n=1}^{\infty} b_n \sin w_0 n t$$

This is known as Quadrature Fourier Series.

4. Define polar Fourier Series.

$$x(t) = D_0 + \sum_{n=1}^{\infty} D_n \cos((2\pi n t / T_0) +$$

The above form of representing a signal is known as Polar Fourier series.

5. Define exponential fourier series.

$$x(t) = \sum_{n=1}^{\infty} C_n e^{j2\pi n t / T_0}$$

The method of representing a signal by the above form is known as exponential fourier series.

6. State Dirichlets conditions.

- (i). The function $x(t)$ should be single valued within the interval T_0

- (ii). The function $x(t)$ should have atmost a finite number of discontinuities in the interval T_0
- (iii). The function $x(t)$ should have finite number of maxima and minima in the interval T_0
- (iv). The function should have absolutely integrable.

7. State Parsevals power theorem.

Parsevals power theorem states that the total average power of a periodic signal $x(t)$ is equal to the sum of the average powers of its phasor components.

8. Define Fourier Transform.

Let $x(t)$ be the signal which is the function of time t . The fourierr transform of $x(t)$ is given by

$$X(w) = \int x(t)e^{-j\omega t}.dt$$

9. State the conditions for the existence of fourier series.

- (i). The function $x(t)$ should be single valued in any finite time interval T
- (ii). The function $x(t)$ should have atmost finite number of discontinuities in any finite time interval T .
- (iii). The function $x(t)$ should have finite number of maxima and minima in any time interval T .
- (iv) The function $x(t)$ should be absolutely integrable.

10. Find the Fourier transform of function $x(t)=\delta(t)$

Ans: 1

11. State Rayleigh's energy theorem.

Rayleigh's energy theorem states that the energy of the signal may be written in frequency domain as superposition of energies due to individual spectral frequencies of the signal.

12. Define laplace transform.

Laplace transform is the another mathematical tool used for analysis of continuous time signals and systems. It is defined as

$$F(s) = \int f(t) e^{-st} dt$$

13. Obtain the laplace transform of ramp function.

Ans: $1/s^2$

14. What are the methods for evaluating inverse Laplace transform.

The two methods for evaluating inverse laplace transform are

- (i). By Partial fraction expansion method.

(ii). By convolution integral.

15. State initial value theorem.

If $x(t) \xrightarrow{L} X(s)$, then value of $x(t)$ is given as,

$$x(0+) = \lim_{s \rightarrow \infty} [sX(s)]$$

provided that the first derivative of $x(t)$ should be laplace transformable.

16. State final value theorem.

If $x(t)$ and $X(s)$ are laplace transform pairs, then the final value of $x(t)$ is given as ,

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} [sX(s)]$$

17. State the convolution property of fourier transform.

If $x_1(t)$ and $X_1(f)$ are fourier transform pairs and $x_2(t)$ and $X_2(f)$ are fourier transform pairs, then

$$\int x_1(\tau)x_2(f-\tau)d\tau \text{ is fourier transform pair with } X_1(f)X_2(f)$$

18. What is the relationship between Fourier transform and Laplace transform.

$$X(s) = X(j\omega) \text{ when } s = j\omega$$

This states that laplace transform is same as fourier transform when $s = j\omega$.

19. Find the fourier transform of sgn function.

$$\text{Ans: } 2/j\Omega$$

20. Find out the laplace transform of $f(t) = e^{at}$

$$\text{Ans: } 1/(s-a)$$

PART- B

1. State and prove properties of fourier transform.

- Linearity property
- Shifting property
- Frequency shifting
- Differentiation in time domain
- Integration in time domain
- Convolution in time domain

2. State the properties of Fourier Series.

- Linearity property

- Shifting property
- Convolution in time domain
- Multiplication in time domain
- Duality property
- Parsevals theorem

3. State the properties of Laplace transform.

- Linearity property
- Shifting property
- Complex translation
- Differentiation in time domain
- Integration in time domain
- Initial value theorem
- Final value theorem
- Convolution in time domain

4. Problems on fourier series, Fourier transform and laplace transform.

a. Find the fourier series of of the periodic signal $x(t)=t$ $0 \leq t \leq 1$

Ans: $C_0=1/2$, $C_n=j/2\pi n$

b. Find the fourier transform of $x(t)=e^{-at}u(t)$

Ans: $1/a+j\Omega$

c. Find the laplace transform of the signal $x(t)= e^{-at}u(t)+ e^{-bt}u(-t)$

Ans: $1/(s+b)+1/(s+a)$

5. State and prove parsevals power theorem and Rayleigh's energy theorem.

Ans:

Statements

Proofs.

UNIT III

LTI- CT SYSTEMS

1. Define LTI-CT systems.

In a continuous time system if the time shift in the input signal results in the corresponding time shift in the output, then it is called the LTI-CT system

2. What are the tools used for analysis of LTI-CT systems?

The tools used for the analysis of the LTI-CT system are

Fourier transform
Laplace transform

3. Define convolution integral.

The convolution of two signals is given by

$$y(t) = x(t) * h(t)$$

where

$$x(t) * h(t) = \int x(\tau) h(t - \tau) d\tau$$

This is known as convolution integral.

4. List the properties of convolution integral.

- a. commutative property
- b. distributive property
- c. associative property
- d. shift property
- e. convolution with an impulse
- f. width property

5. State commutative property of convolution.

The commutative property of convolution states that

$$x_1(t) * x_2(t) = x_2(t) * x_1(t)$$

6. State the associative property of convolution.

Associative property of convolution states that

$$x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$$

7. State distributive property of convolution.

The distributive property states that

$$x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$$

8. When the LTI-CT system is said to be dynamic?

In LTI CT system, the system is said to be dynamic if the present output depends only on the present input.

9. When the LTI-CT system is said to be causal?

An LTI continuous time system is causal if and only if its impulse response is zero for negative values of t .

10. When the LTI-CT system is said to be stable?

A LTI-CT system is said to be stable if the impulse response of the system is absolutely integrable. That is

$$\int |h(t)| dt < \infty$$

11. Define natural response.

Natural response is the response of the system with zero input. It depends on the initial state of the system. It is denoted by $y_n(t)$

12. Define forced response.

Forced response is the response of the system due to input alone when the initial state of the system is zero. It is denoted by $y_f(t)$.

13. Define complete response.

The complete response of a LTI-CT system is obtained by adding the natural response and forced response.

$$y(t) = y_n(t) + y_f(t)$$

14. Draw the direct form I implementation of CT systems.

15. Draw the direct form II implementation of CT systems.

16. Mention the advantages of direct form II structure over direct form I structure.

No. of integrators are reduced to half

17. Define Eigen function and Eigen value.

In the equation given below,

$$y(t) = H(s)e^{st}$$

$H(s)$ is called Eigen value and e^{st} is called Eigen function.

18. Define Causality and stability using poles.

For a system to be stable and causal, all the poles must be located in the left half of the s plane

19. Find the impulse response of the system $y(t) = x(t-t_0)$ using laplace transform.

Ans:

$$h(s) = \delta(t-t_0)$$

20. The impulse response of the LTI CT system is given as $h(t) = e^{-t} u(t)$. Determine transfer function and check whether the system is causal and stable.

Ans:

$$H(s) = 1/(s+1)$$

The system is causal, stable.

1. Derive convolution integral and also state and prove the properties of the same.

Ans:

Convolution integral derivation
Properties
Proofs.

2. Explain the properties of LTICT system interms of impulse response.

Ans:

Dynamicity
Causality
Stability
Step response

3. Problems on properties of LTI CT systems.

Check whether the following system are stable and causal

1. $h(t) = e^{-2t}u(t-1)$

Ans: Causal, Stable.

2. $h(t) = e^{-4t}u(t+10)$

Ans: Non,causal and stable.

4. Problems on differential equation.

Determine the forced response of the system

5. $d/dt[y(t)] + 10y(t) = 2x(t)$

Ans: $y^p(t) = 0.4$, $y^f(t) = 0.4(1 - e^{-2t})$

5. Realization of LTI CT system using direct form I and II structures.

6. Finding frequency response using Fourier methods.

Steps:

1. Take fourier transform for the given Differential equation
2. Find system transfer function $H(\omega)$
3. The frequency response can be obtained from the transfer function by separating the real and imaginary parts.

7. Solving differential equations using Fourier methods

Steps:

1. Take Fourier transform for the given Differential Equation
2. Then find $Y(s)$ using the given initial conditions.
3. Then find $y(t)$ by taking inverse Fourier transform

8. Solving Differential Equations using Laplace transforms.

Steps:

1. Take laplace transform for the given Differential Equation
2. Then find $Y(s)$ using the given initial conditions.
3. Then find $y(t)$ by taking inverse Laplace transform.

9. Obtaining state variable description.

Steps:

1. The state variable description consists of differential equations that describe state of the system.
2. The output of the system is related to current state and input.
3. The state is the minimal set of signals that represent system's entire pass memory
4. The state equations for LTI CT system can be written as
$$\frac{d}{dt}[q(t)] = A q(t) + b x(t)$$
$$y(t) = c q(t) + D x(t)$$
hence A, b, c, D are the matrices representing internal structure of the system.

10. Obtaining frequency response and transfer functions using state variable.

Steps for obtaining frequency response:

UNIT-4

ANALYSIS OF DISCRETE TIME SIGNALS

PART-A

1. Define DTFT.

Let us consider the discrete time signal $x(n)$. Its DTFT is denoted as $X(w)$. It is given as

$$X(w) = \sum_{n=-\infty}^{\infty} x(n)e^{-jwn}$$

2. State the condition for existence of DTFT?

The conditions are

- If $x(n)$ is absolutely summable then
$$\sum_{n=-\infty}^{\infty} |x(n)| < \infty$$
- If $x(n)$ is not absolutely summable then it should have finite energy for DTFT to exist.

3. List the properties of DTFT.

- Periodicity
- Linearity
- Time shift
- Frequency shift
- Scaling
- Differentiation in frequency domain
- Time reversal

- Convolution
- Multiplication in time domain
- Parseval's theorem

4. What is the DTFT of unit sample?

The DTFT of unit sample is 1 for all values of w .

5. Define DFT.

DFT is defined as $X(w) = \sum_{n=0}^{N-1} x(n)e^{-jwn}$.

Here $x(n)$ is the discrete time sequence

$X(w)$ is the fourier transform of $x(n)$.

6. Define Twiddle factor.

The Twiddle factor is defined as $W_N = e^{-j2\pi/N}$

7. Define Zero padding.

The method of appending zero in the given sequence is called as Zero padding.

8. Define circularly even sequence.

A Sequence is said to be circularly even if it is symmetric about the point zero on the circle.

$$x(N-n) = x(n), 1 \leq n \leq N-1.$$

9. Define circularly odd sequence.

A Sequence is said to be circularly odd if it is anti symmetric about point $x(0)$ on the circle

10. Define circularly folded sequences.

A circularly folded sequence is represented as $x((-n))_N$. It is obtained by plotting $x(n)$ in clockwise direction along the circle.

11. State circular convolution.

This property states that multiplication of two DFT is equal to circular convolution of their sequence in time domain.

12. State parseval's theorem.

Consider the complex valued sequences $x(n)$ and $y(n)$. If

$$x(n) \xleftrightarrow{\text{DFT}} X(k)$$

$$y(n) \xleftrightarrow{\text{DFT}} Y(k)$$

then $\sum_{n=0}^{N-1} x(n)y^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)Y^*(k)$

13. Define Z transform.

The Z transform of a discrete time signal $x(n)$ is denoted by $X(z)$ and is given by $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$.

14. Define ROC.

The value of Z for which the Z transform converged is called region of convergence.

15. Find Z transform of $x(n) = \{1, 2, 3, 4\}$

$$x(n) = \{1, 2, 3, 4\}$$

$$\begin{aligned} X(z) &= \sum_{n=0}^3 x(n)z^{-n} \\ &= 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} \\ &= 1 + \frac{2}{z} + \frac{3}{z^2} + \frac{4}{z^3} \end{aligned}$$

16. State the convolution property of Z transform.

The convolution property states that the convolution of two sequences in time domain is equivalent to multiplication of their Z transforms.

17. What z transform of $(n-m)$?

By time shifting property

$$Z[A(n-m)] = AZ^{-m} \sin Z[n] = 1$$

18. State initial value theorem.

If $x(n]$ is causal sequence then its initial value is given by

$$x(0) = \lim_{z \rightarrow \infty} X(z)$$

19. List the methods of obtaining inverse Z transform.

Inverse z transform can be obtained by using

- Partial fraction expansion.
- Contour integration
- Power series expansion
- Convolution.

20. Obtain the inverse z transform of $X(z) = 1/z - a, |z| > |a|$

$$\text{Given } X(z) = z^{-1}/1 - az^{-1}$$

By time shifting property

$$X(n) = a^n \cdot u(n-1)$$

PART – B

1. State and prove properties of DTFT

- ❖ Periodicity
- ❖ Linearity
- ❖ Time shift
- ❖ Frequency shift
- ❖ Scaling
- ❖ Differentiation in frequency domain
- ❖ Time reversal
- ❖ Convolution
- ❖ Multiplication in time domain
- ❖ Parseval's theorem.

2. State and prove the properties of DFT.

- Periodicity
- Linearity
- Circular symmetries of a sequence
- Symmetry properties
- Circular convolution
- Time reversal of a sequence
- Circular time shift of a sequence
- Circular frequency shift
- Complex conjugate properties
- Circular correlation
- Multiplication of two sequences
- Parsevals theorem

3. State and prove the properties of z transform.

- ✓ Linearity
- ✓ Time shifting
- ✓ Scaling in z domain
- ✓ Time reversal
- ✓ Differentiation in z domain
- ✓ Convolution in time domain
- ✓ Correlation of two sequences
- ✓ Multiplication of two sequences
- ✓ Conjugation of a complex sequence
- ✓ Z transform of real part of the sequence
- ✓ Z transform of imaginary part of the sequence
- ✓ Parsevals relation
- ✓ Initial value theorem

4. Find the DFT of $x(n) = \{1, 1, 1, 1, 1, 1, 0, 0\}$
5. Find the circular convolution of $x_1(n) = \{1, 2, 0, 1\}$
 $X_2(n) = \{2, 2, 1, 1\}$
6. Problems on z transform and inverse z transform.

UNIT-5
LINEAR TIME INVARIANT DISCRETE TIME SYSTEMS
PART-A

1. Define convolution sum?

If $x(n)$ and $h(n)$ are discrete variable functions, then its convolution sum $y(n)$ is given by,

$$y(n) = \sum x(k) h(n-k)$$

2. List the steps involved in finding convolution sum?

- folding
- Shifting
- Multiplication
- Summation

3. List the properties of convolution?

- Commutative property of convolution
 $x(n) * h(n) = h(n) * x(n) = y(n)$
- Associative property of convolution
 $[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$
- Distributive property of convolution
 $x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$

4. Define LTI causal system?

A LTI system is causal if and only if $h(n) = 0$ for $n < 0$. This is the sufficient and necessary condition for causality of the system.

5. Define LTI stable system?

The bounded input $x(n)$ produces bounded output $y(n)$ in the LTI system only if, $\sum |h(k)| < \infty$. When this condition is satisfied, the system will be stable.

6. Define FIR system?

The systems for which unit step response $h(n)$ has finite number of terms, they are called Finite Impulse Response (FIR) systems.

7. Define IIR system?

The systems for which unit step response $h(n)$ has infinite number of terms, they are called Infinite Impulse Response (IIR) systems.

8. Define non recursive and recursive systems?

When the output $y(n)$ of the system depends upon present and past inputs then it is called non-recursive system.

When the output $y(n)$ of the system depends upon present and past inputs as well as past outputs, then it is called recursive system.

9. State the relation between fourier transform and z transform?

The fourier transform is basically the z-transform of the sequence evaluated on unit circle.

$$\text{i.e., } X(z)|_{z=e^{j\omega}} = X(\omega) \text{ at } |z|=1 \text{ i.e., unit circle.}$$

10. Define system function?

$H(z) = \frac{Y(z)}{X(z)}$ is called system function. It is the z transform of the unit sample

$$X(z)$$

response $h(n)$ of the system.

11. What is the advantage of direct form 2 over direct form 1 structure?

The direct form 2 structure has reduced memory requirement compared to direct form 1 structure.

12. Define butterfly computation?

In the figure the two values 'a' and 'b' are available as input. From these two values

'A' and 'B' are computed at the output. This operation is called Butterfly computation.

13. What is an advantage of FFT over DFT?

FFT algorithm reduces number of computations.

14. List the applications of FFT?

- Filtering
- Spectrum analysis
- Calculation of energy spectral density

15. How unit sample response of discrete time system is defined?

The unit step response of the discrete time system is output of the system to unit

sample sequence. i.e., $T[\delta(n)] = h(n)$. Also $h(n) = \mathcal{Z}^{-1}\{H(z)\}$.

16. A causal DT system is BIBO stable only if its transfer function has _____.

Ans: A causal DT system is stable if poles of its transfer function lie within the unit circle.

17. If $u(n)$ is the impulse response response of the system, What is its step response?
 Here $h(n) = u(n)$ and the input is $x(n) = u(n)$.
 Hence the output $y(n) = h(n) * x(n)$
 $= u(n) * u(n)$
18. Convolve the two sequences $x(n) = \{1, 2, 3\}$ and $h(n) = \{5, 4, 6, 2\}$
 Ans: $y(n) = \{5, 14, 29, 26, 22, 6\}$
19. State the maximum memory requirement of N point DFT including twiddle factors?
 Ans: $[2N + N/2]$
20. Determine the range of values of the parameter 'a' for which the linear time invariant system with impulse response $h(n) = a^n u(n)$ is stable?
 Ans: $H(z) = \frac{z}{z-a}$, There is one pole at $z=a$. The system is stable, if all its poles.
 $z-a$
 i.e., within the unit circle. Hence $|a| < 1$ for stability.

PART-B

1. State and prove the properties of convolution sum?
- Commutative property of convolution
 $x(n) * h(n) = h(n) * x(n) = y(n)$
 - Associative property of convolution
 $[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$
 - Distributive property of convolution
 $X(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$
- PROOF:
2. Determine the convolution of $x(n) = \{1, 1, 2\}$ $h(n) = u(n)$ graphically?
 Ans: $y(n) = \{1, 2, 4, 4, \dots\}$
3. Determine the forced response for the following system

$$y(n) - \frac{1}{4} y(n-1) - \frac{1}{8} y(n-2) = x(n) + x(n-1)$$
 for $x(n) = (1/8)^n u(n)$. Assume zero initial conditions?
 Ans: $f^{(f)}(n) = \frac{8}{3} (1/2)^n - \frac{2}{3} (1/4)^n - (1/8)^n$
4. Compute the response of the system

$$y(n) = 0.7 y(n-1) - 0.12 y(n-2) + x(n-1) - x(n-2)$$
 to the input $x(n) = n u(n)$. Is the system is stable?
 Ans: Stable. $Y(n) = [38.89 (0.1)^n - 26.53 (0.3)^n - 12.53 + 4.76] u(n)$
5. Derive the 8 point DIT and DIF algorithms