"QUESTION BANK: STATISTICAL METHODS – II"

By, SHRIRAM KARGAONKAR (HOD, STATISTICS)

CHAPTER 1) PERMUTATION AND COMBINATION

Q1) State the fundamental principals of counting with one illustrations each.

Q2) How many 3 letter words can be formed using the letters of the word 'COMPUTER'.

Q3) How many three digit numbers divisible by 5 can be formed out of 5,6,7,8,9, if i) no digit is to be repeated?, ii) a digit may be repeated any number of times?

Q4) A committee of 4 persons is to be formed from 10 persons. Find the number of possible ways?

Q5) A cricket team of 11 players is to be formed from 18 players consisting of 7 bowlers, 3 wicket-keepers and 8 batsmen. In how many ways the team can be formed so that it contains exactly 5 bowlers and 2 wicket-keepers?

Q6) In a basket there are 5 mangoes and 6 oranges. If any 3 fruits are to be selected from these, find the number of ways in which: i) exactly 2 mangoes are selected ii) atleast 2 mangoes are selected and iii) no mango is selected.

CHAPTER 2) SAMPLE SPACE AND EVENTS

Q1) Distinguish between deterministic and non-deterministic experiment (i.e Random Experiment) with two examples each.

Q2) Explain the following with suitable examples: i) Sample Space ii) Event
iii) Mutually exclusive events iv) Exhaustive events and v) Certain or Sure event
vi) Relative Complement of A w. r. t. B.

Q3) What do you mean by relative complementation? Explain with an example.

Q4) Three coins are tossed and outcome on the uppermost face is recorded.
Let A: Exactly two coins show tails and B: Atleast two coins show tails. Determine whether the events A and B are mutually exclusive? Are they exhaustive?

Q5) A statistical experiment consists of asking 3 housewives at random, if they wash their dishes with brand X detergent.

a) List elements of the sample space using 'Y' for yes and 'N' for no.

b) List elements of the event: "Second woman interviewed uses brand X".

c) Find the probability of the event in (ii) above if it is assumed that elements of sample space are equally likely to occur

CHAPTER 3) THEORY OF PROBABILITY

Q1) State the classical definition of probability with one illustration. Also state its limitations.

Q2) State the axioms of probability.

Q3) State and prove the addition theorem of probability.

Q4) Define the independence of two events? Does independence of two events imply that the events are mutually exclusive? Justify your answer.

Q5) Explain the concept of conditional probability. State and prove the multiplication theorem for two events defined on sample space.

Q6) Define the partition of a sample space? State the Bayes' theorem.

Q7) If a pair of unbiased coins is tossed, obtain the probability of getting:
i) both heads **ii)** One head and **iii)** Atleast one head.

Q8) From a well shuffled pack of 52 cards, four cards are drawn at random. Find the probability of getting i) two red and two black cards ii) all different cards iii) all cards same and iv) one is king.

Q9) Find the probability of getting 53 Sundays in i) Leap year and ii) Non Leap year.

Q10) i) The letters of the word 'SEMINAR' are arranged at random. Find the probability that the vowels are occupied in the even places.
ii) If a three digit number is to be formed out of 1,2,3,4 and 5: **a)** with repetition and **b)** without repetition, find the probability that it is divisible by 5

Q11) If P(A)=0.6, P(B)= 0.5, P(A∩ B)=0.3. Calculate P(A'), P(AUB), P(A'∩ B), P(A'∩ B') and P(A'UB').

Q12) The probability that a contractor will get a plumbing contract is 0.4 and that he will get an electrical contract is 0.7. If the probability of getting atleast one contract is 0.6, what is the probability that he will get i) both the contracts and ii) exactly one contract?

Q13) The letters of the word 'COMPUTER' are arranged at random. Find probability that vowels occupy even places.

Q14) An unbiased coin is tossed six times. Find probability of getting at least five heads

Q15) Define independence of two events A and B on Ω. If A and B are two independent events defined on Ω, show that : **i)** A and B′ are independent

ii) A′ and B are independent and

iii) A′ and B′ are independent.

Q16) A die is loaded so that probability of an even number is twice the probability of an odd number. Even numbers are equally likely as well as odd numbers are equally likely. Find the probability that:
i) an even number appears uppermost and **ii)** a prime number appears uppermost

Q17) If A is subset of B, then prove that P(A) ≤ P(B).

Q18) In a group of 10 men, 6 are graduates. If 3 men are selected at random, what is the probability that they consists of **(i)** all graduates **(ii)** atleast one graduate **(iii)** at most two graduates **(iv)** no graduates?

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CHAPTER 4) DISCRETE RANDOM VARIABLE

Q1) Define cumulative distribution function of a discrete random variable and state its important properties. Also, give four examples of discrete random variable.

Q2) If a random variable X takes values 1, 2, 3 and 4 such that 2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4), Find : **i)** Probability Distribution of X

ii) Distribution function of X

iii) Median

iv) Mode

Q3) Consider the following frequency distribution of X :

X	1	2	3	4	5
P(X=x)	3k	5k	2k	k	k

Find : **i)** Value of k,

ii) distribution function of X.

Q4) Let X be a discrete random variable with mean 5 and standard deviation 3.

Compute mean and variance of (3-7X).

Q5) Define each of the following :

i) A discrete random variable.

ii) Probability mass function of a discrete random variable.

iii) Mean of a discrete random variable.

iv) Variance of a discrete random variable.

Q6) Suppose three balanced coins are tossed simultaneously. If X denotes the number of heads, find the probability distribution of X.

Q7) Obtain the probability distribution of the following when two dice are thrown:

i) Sum of the numbers on the uppermost faces.

ii) Number of sixes.

iii) Maximum of the two numbers.

Q8) A box of 20 mangoes contain 4 bad mangoes. Two mangoes are drawn at random without replacement from this box. Obtain the probability distribution of the number of bad mangoes.

Q9) Three cards are drawn at random successively, with replacement, from a well shuffled pack of 52 playing cards. Getting ***'a card of diamond'*** is termed as success. Obtain the probability distribution of the number of successes.

Q10) Verify whether the following can be looked upon as p. m. f. for the given values of X.

i) $P(X) = 1/4$; for X= 0, 1, 2, 3, 4.

ii) $P(X) = (X+1)/10$; X = 0, 1, 2, 3.

iii) $P(X) = X^2/30$; X = 0, 1, 2, 3, 4.

iv) $P(X) = (X-2)/5$; X = 1, 2, 3, 4, 5.

Q11) A couple decides to have children until they have a male child. Find the probability distribution of the number of children they would have before first male child. If probability of a male child in their community is 1/3, how many children are they expected to have before the first male child is born?

Q12) From a bag containing 5 white and 5 red balls, three balls are drawn at random.

Find expected number of red balls drawn.

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CHAPTER 5) STANDARD DISCRETE DISTRIBUTIONS

Q1) Suppose X follows discrete uniform distribution on 1, 2, …… n and mean of distribution is 6. What is variance of X?

Q2) Define Poisson distribution. Give any two real life situations, where the distribution is applicable.

Q3) Describe Bernoulli Experiment. Define Bernoulli distribution. Discuss how it leads to Binomial distribution. **(OR: State the relation between Bernoulli and Binomial Distribution.)** Also state approximation between Poisson Distribution and Binomial Distribution.

Q4) Define Discrete Uniform Distribution with parameter 'n'. Find probability distribution of number of heads obtained when three coins are tossed.
Does it follow uniform distribution?

Q5) If a random variable X follows Poisson distribution such that $P(X = 1) = P(X = 2)$. Find Mean and Standard of the distribution. Also, find $P(X = 0)$.

Q6) Define Binomial Distribution. Give one real life situation where binomial distribution can be applied. State its additive property. Also, state recurrence relation for Binomial probabilities.

Q7) Suburban trains on a certain line run every half hour between midnight and 6 in the morning. Find probability that a person entering the station at random time during this period will have to wait for at least 20 minutes.
Also, find the probability that he has to wait between 12 minutes and 27 minutes.

Q8) Obtain the mean and variance of Discrete Uniform variable taking values 1, 2, … ,n.

Q9) Obtain the mean and variance of Binomial Distribution with parameters 'n' and 'p'. Also show that V(X) is less than E(X).

Q10) Define Geometric Distribution. State two real life examples where Geometric Distribution is used. Also state its mean and variance.

Q11) Let $X \rightarrow B(n=8, p=(1/4))$. Find i) $P(X=3)$ ii) $P(X < 3)$ iii) $P(X \leq 6)$.

Q12) If X and Y are independent binomial variates with $X \rightarrow B(5, 0.5)$ and $Y \rightarrow B(8, 0.5)$.

Find **i)** $P[X+Y = 5]$ **ii)** $P[X+Y \leq 1]$ **iii)** $P[(X+Y)/2 \geq 1]$ **iv)** $P[3(X+Y) \leq]$

Q13) If the probability that any person 65 years old will die within a year is 0.05. Find the probability that out of a group of 7 such persons **(i)** exactly one **(ii)** none **(iii)** at least one **(iv)** not more than one **(v)** all of them will die within a year.

Q14) If a boy is throwing stone at a target. The probability of not hitting the target at any throw is 0.6. What is the probability that he will hit the target at 4^{th} attempt for the first time? Also find the expected number of throws required to hit the target for the first time.

Q15) A couple decides to have children until they have a male child. Find the probability distribution of the number of children they would have before first male child. If the probability of male child in their community is 1/3, how many children are they expected to have before the first male child is born?

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CHAPTER 6) CONTINUOUS RANDOM VARIABLE

Q1) Define: **a)** Continuous Random Variable

b) Probability Density Function

c) Expectation / Mean of a Continuous Random Variable.

d) Expectation of a function of a continuous Random Variable.

e) Variance of a continuous random variable.

Q2) A continuous random variable X has distribution function given by

$F(X) = 1 - (4/X^2)$; $X > 2$

$= 0$; otherwise

Find : **a)** Probability Density Function **b)** $P(X > 3)$ **c)** $E(2X + 3)$ **d)** $V(2X + 3)$.

Q3) Let X be a Continuous random variable with p. d. f.

$f(X) = 3X^2$; $0 \leq X \leq 1$

$= 0$; otherwise

Find $E(3X + 2)$ and $V(3X + 2)$.

Q4) Define cumulative distribution function of a continuous random variable. State its important.

Q5) The distribution function of a Continuous Random Variable X is:

$f(X) = 0$; $X < -1$

$= (X^3 + 1)/9$; $-1 \leq X \leq 2$

$= 1$; $X > 2$

Obtain p. d. f. of X. Also find $P(0 < X \leq 1)$.

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CHAPTER 7) STANDARD CONTINUOUS DISTRIBUTION

Q1) The amount of time, in hours that a computer functions before breaking down has Exponential distribution with probability density function (p. d. f.) given by,

$$f(X) = 0.005\, e^{-0.005\, X} \qquad ; X \geq 0$$

$$= 0 \qquad ; \text{otherwise}$$

Find mean of above distribution. Also, write distribution function.

Q2) Define Exponential Distribution with parameter θ. State its mean and variance. Also, State lack of memory property and its interpretation.

Q3) The quantity of oil per tin filled by a machine is uniformly distributed over 240 ml and 260 ml. What is the probability that an oil tin is filled with more than 250 ml of oil. Further, if 25 tins are filled with oil, what is the expected quantity of oil consumed?

Q4) The life time in hours of a certain electrical component follows exponential distribution with distribution function:

$$F(X) = 1 - e^{-0.01X} \qquad ; X \geq 0$$

What is the probability that the component will survive 75 hours ? Also, what is the probability that it will fail during 100 to 150 hours?

Q5) Define uniform distribution over an interval [a,b]. Find its mean, variance and distribution function. Also, state its applications.

Q6) i) If mean and variance of a U[a, b] r.v. are 6 and 4 respectively, determine the values of 'a' and 'b'
ii) Suppose $X \rightarrow U[0, 10]$. Find mean, variance, $P(X > 4)$, $P(X \leq 3)$ and $P(2 \leq X \leq 6)$.

Q7) On a route, the first bus is at 8:00 am and after every 30 minutes, there is a bus. A passenger arrives at the stop at time X, which is uniformly distributed over the interval [8:15 am, 8:45 am]. What is the probability that the passanger will have to wait for more than 15 minutes for a bus?

Q8) Define exponential distribution. State its mean and variance. Obtain the distribution function of exponential distribution.

Q9) State and prove the lack of memory property exponential distribution.

Q10) A random variable X has an exponential distribution with mean 5.
Find: i) $P[(X > 8)/(X > 4)]$ ii) $P[X > 5]$

Q11) Suppose that the life of a electrical component is exponentially distributed with a mean life of 1600 hrs. What is the probability that the electrical component will work : i) upto 2400 hours ii) after 1000 hours iii) between 1500 and 2000 hours. Also find the distribution function.

Q12) a) Define normal distribution. State its properties.

b) State the normal approximation to binomial and poisson distribution.

Q13) What do you mean by fitting of a normal distribution. Explain the steps involved in it. State the area property of normal distribution.

Q14) Suppose a r. v. X follows normal distribution with mean 3 and variance 16. Find:
i) $P(X > 5)$ ii) $P(X < 1)$ iii) $P(X > 0)$ iv) $P(X < 6)$ v) $(2 < X < 6)$ vi) $P(|X-3| < 3.92)$ and vii) $P(|X| > 4)$.

Q15) There are 1000 students in the university of a certain age group and it is known that their weights are normally distributed with mean 55 kg and standard deviation 4.5 kg. Find the number of students having weight:
i) less than 48 kg ii) between 50 kg and 58 kg and iii) above 65

Q16) Define pareto distribution. State its mean and variance. State three real life situations where pareto distribution is used. If mean and variance of pareto distribution is 1.2 and 0.06 respectively, what is the value of α?

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Lesson] STATISTICAL INFERENCE AND TESTS OF HYPOTHESIS

Q1) Define the following terms: i) population ii) Sample iii) parameter iv) statistic v) hypothesis vi) Null and alternative hypothesis vii) critical region vii) types of errors viii) level of significance and xi) p- value x) one sided and two sided tests.

Q2) Explain SRSWR and SRSWOR methods of sampling.
Also define sampling distribution of a statistic with one illustration.

Q3) Describe briefly the steps involved in the testing of a hypothesis.

Lesson] LARGE SAMPLE TESTS

Q1) Describe the following large sample tests briefly:

i) Test for single population mean (population mean is specified)

ii) Test for equality of two population means

iii) Test for single population proportion (population proportion is specified)

iv) Test for equality of two population proportions.

Also state their underlying assumptions.

Q2) a) The mean diastolic blood pressure for a group of 81 adults was found to be 79.2 mm. Test the hypothesis that the mean diastolic blood pressure is 75 mm. Population standard deviation is found to be 9 mm.

b) A sample of 64 bulbs gave average life 1235 hours. Can this sample be regarded from a population with mean life 1250 hours? It is known that population standard deviation is 50 hours. [Use α=1%]

Q3) A random sample of 1000 men from a locality 'A' gave mean wage of Rs. 2500 with a standard deviation Rs. 1500. A sample of 1500 men from locality 'B' gave mean wage Rs. 2680 with standard deviation Rs. 2000. Test the hypothesis that the mean wages of both localities are equal. [Use α=1%]

Q4) **a)** A coin is tossed 400 times and it turned up head 210 times. Can the coin be regarded as fair?

b) Two hundred people were attacked by a disease and 180 survived. Can we say that 85% people survived if they are attacked by this disease? [Use α=5%]

Q5) In a random sample of 800 persons from rural area 200 were found to be smokers. In a sample of 1000 persons from urban area 350 were found to be smokers. Test that proportion of smokers is same for both populations.

Lesson] <u>SMALL SAMPLE TEST (Tests based on t and chi-square)</u>

Q1) i) Explain the test procedure for testing the independence of two attributes in a ($m \times n$) contingency table. Also explain the procedure in case of 2×2 contingency table.
ii) Describe the chi-square test for testing goodness of fit. Also state underlying assumptions.

Q2) i) Describe t-test for single population mean.
Ii) Describe t-test for testing equality of two population means and
iii) Explain paired t-test. State its application.

Q3) A nationalized bank utilizes four teller windows to render fast service to the customers. On a particular day 800 customers were observed. They were given service at the different windows as shown below:

Window number :	1	2	3	4
Number of customers:	150	250	170	230

Test whether the customers are uniformly distributed over the windows.

Q4) In a locality 100 persons were randomly selected and asked for their educational achievements. The results are given below.

Sex	Primary school education	High school education	College education
MALE	10	15	25
FEMALE	25	10	15

Test whether education depends on sex at 1% level of Significance.

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Lesson] SIMULATION

Q1) Explain the meaning of simulation. State its applications in various fields. Also explain pseudo numbers.

Q2) a) Explain the procedure to obtain random numbers from U(a, b) distribution.

b) Explain the procedure to obtain random numbers from exp(θ).

c) Explain the procedure to obtain random numbers from Normal distribution use Box-Muller transformation.

Q3) a) Write a stepwise procedure of run test. State its use.

b) Write a stepwise procedure of sign test. State its use.

Q3) a) Generate 20 random numbers between 5 and 10 from uniform distribution. Also apply sign test and run test on the sample generated above.

b) Generate 20 random numbers from exponential distribution with mean θ=4. Also apply sign test and run test on the sample generated above.

c) Generate 20 random numbers from N(6000, 500) using Box-Muller transformation . Also apply sign test and run test on the sample generated above.