

QUESTION BANK
BCA V SEMESTER
BCA-505
OPTIMAZATION TECHNIQUES

----- UNIT I -----

1. A firm manufactures 3 products A, B and C. The profits are Rs. 3, Rs. 2 and Rs. 4 resp. The firm has 2 machines and below is the required processing time in minutes for each machine on each product.
Machine G and H have 2000 and 2500 machine-minutes resp. The firm must manufacture 100 A's, 200 B's and 50 C's, but no more than 150 A's.
Formulate an LPP to maximize profit.

2. A farmer has 100 acre farm. He can sell all tomatoes, lettuce or radishes he can raise. The price he can obtain is Re. 1.00 per kg for tomatoes, Rs. 0.75 a head for lettuce and Rs. 2.00 per kg for radishes. The average yield per acre is 2000 kg of tomatoes, 3000 heads of lettuce, and 1000 kg of radishes. Fertilizer is available at Rs. 0.50 per kg and the amount required per acre is 100 kg each for tomatoes and lettuce, and 50 kg for radishes. Labour required for sowing, cultivating and harvesting per acre is 5 man-days for tomatoes and radishes, and 6 man-days for lettuce. A total of 400 man-days of labour are available at Rs. 20.00 per man-day.
Formulate this problem as a LPP to maximize the farmer's total profit.

3. A firm plans to purchase at least 200 quintals of scrap containing high quality metal X and low quality metal Y. It decides that the scrap to be purchased must contain at least 100 quintal of X-metal and not more than 35 quintals of Y-metal. The firm can purchase the scrap from two suppliers (A and B) in unlimited quantities. The percentage of X and Y metals in terms of wt. in the scraps supplied by A and B is given below:

Metals	Supplier A	Supplier B
X	25%	75%
Y	10%	20%

The price of A's scrap is Rs. 200 per quintal and that of B's is Rs. 400 per quintal.
Formulate this problem as LPP and solve it to determine the quantities that the firm should buy from the two suppliers so as to minimize total purchase cost.

4. Solve graphically the following LPP:
a) Min. $z=3x_1+5x_2$ subject to:
 $-3x_1+4x_2 \leq 12$
 $2x_1-x_2 \geq -2$
 $2x_1+3x_2 \geq 12$
 $x_1 \leq 4$

$$x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

- b) Maximize $z = 3x_1 + 2x_2$
 subject to the constraints :
 $x_1 + x_2 \leq 4$
 $x_1 - x_2 \leq 2$
 $x_1, x_2 \geq 0$

5. Solve the following LPP using Simplex method:
 Maximize $z = 3x_1 + 2x_2$, subject to the constraints :
 $x_1 + x_2 \leq 4$,
 $x_1 - x_2 \leq 2$, and
 $x_1, x_2 \geq 0$
6. Minimize $z = x_1 - 3x_2 + 2x_3$ using Simplex method, subject to :
 $3x_1 - x_2 + 3x_3 \leq 7$,
 $-2x_1 + 4x_2 \leq 12$,
 $-4x_1 + 3x_2 + 8x_3 \leq 10$, and
 $x_1, x_2, x_3 \geq 0$.
7. Solve using two-phase Simplex method :
 Minimize $z = 15/2x_1 - 3x_2$, subject to the constraints :
 $3x_1 - x_2 - x_3 \geq 3$,
 $x_1 - x_2 + x_3 \geq 2$,
 $x_1, x_2, x_3 \geq 0$.
8. Solve using two-phase Simplex method:
 Maximize $z = 3x_1 - x_2$
 Subjects to the constraints:
 $2x_1 + x_2 \geq 2$,
 $x_1 + 3x_2 \leq 2$,
 $x_2 \leq 4$, and
 $x_1, x_2 \geq 0$.
9. Solve by using Big-M method the following LPP :
 Max. $z = -2x_1 - x_2$, subject to
 $3x_1 + x_2 = 4$,
 $x_1 + 3x_2 \geq 6$,
 $x_1 + 2x_2 \leq 4$, and
 $x_1, x_2 \geq 0$.
10. Solve the following LPP using Charne's Penalty (Big-M) method:
 Min. $z = 2x_1 + 9x_2 + x_3$, subject to

$$\begin{aligned}
 x_1 + 4x_2 + 2x_3 &\geq 5, \\
 3x_1 + x_2 + 2x_3 &\geq 4, \text{ and} \\
 x_1, x_2, x_3 &\geq 0.
 \end{aligned}$$

11. Obtain the dual of the following LPP :

$$\begin{aligned}
 \text{Max. } z &= 2x_1 + 3x_2 + x_3, \text{ subject to} \\
 4x_1 + 3x_2 + x_3 &= 6, \\
 x_1 + 2x_2 + 5x_3 &= 4, \text{ and} \\
 x_1, x_2, x_3 &\geq 0.
 \end{aligned}$$

12. Use dual Simplex method to solve :

$$\begin{aligned}
 \text{Max. } z &= -2x_1 - x_3, \text{ subject to} \\
 x_1 - x_2 + x_3 &\leq -5, \\
 -x_1 + 2x_2 - 4x_3 &\leq -8, \text{ and} \\
 x_1, x_2, x_3 &\geq 0.
 \end{aligned}$$

13. Solve the following LPP by Revised Simplex method :

$$\begin{aligned}
 \text{Max. } z &= 3x_1 + x_2 + 2x_3 + 7x_4, \text{ subject to :} \\
 2x_1 + 3x_2 - x_3 + 4x_4 &\leq 40, \\
 -2x_1 + 2x_2 + 5x_3 - x_4 &\leq 35, \\
 x_1 + x_2 - 2x_3 + 3x_4 &\leq 100, \text{ and} \\
 x_1 &\geq 2, \\
 x_2 &\geq 1, \\
 x_3 &\geq 3, \\
 x_4 &\geq 4.
 \end{aligned}$$

14. Write any four limitations of Linear Programming.

15. What are the advantages of two-phase method over Big-M method?

-----UNIT- 2 -----

1. Determine the initial basic feasible solution of the following transportation problem using North-West Corner method,

	D1	D2	D3	D4	Supply
O1	6	4	1	5	14
O2	8	9	2	7	16
O3	4	3	6	2	5
Demand	6	10	15	4	35

2. Determine the initial basic feasible solution of the following transportation problem using Row-minima method:

	D1	D2	D3	D4	Supply
O1	6	3	5	4	22

O2	5	9	2	7	15
O3	5	7	8	6	8
Demand	7	12	17	9	

3. Obtain the initial basic feasible solution of the following transportation problem using matrix-minima method:

	D1	D2	D3	D4	Supply
O1	1	2	3	4	6
O2	4	3	2	0	8
O3	0	2	2	1	10
Demand	4	6	8	6	

4. Obtain the initial basic feasible solution of the following transportation problem using Vogel's method:

	D1	D2	D3	D4	Supply
O1	21	16	15	13	11
O2	17	18	14	23	13
O3	32	27	18	41	19
Demand	6	10	12	15	

5. Find the optimum solution to the following TP :

	A	B	C	D	E	Supply
1	3	4	6	8	8	20
2	2	10	1	5	30	30
3	7	11	20	40	15	15
4	2	1	9	14	18	13
Demand	40	6	8	18	6	

6. The unit cost of transportation from site I to site J is given below. At site $i=1,2,3$, stocks of 150, 200, 170 units resp., are available. 300 units are to be sent to site 4 and rest to site 5. Find the cheapest way of doing this.

1 2 3 4 5

1	-	3	4	13	7
2	1	-	2	16	6
3	7	4	-	12	13
4	8	3	9	-	5
5	2	1	7	5	-

7. A Deptt. Head has 4 subordinates, and 4 tasks have to be performed. Subordinates differ in efficiency and tasks differ in their intrinsic difficulty. Time each man would take to perform each task is given in the effectiveness matrix. How the tasks should be allocated to each person so as to minimize the total man-hours?

	1	2	3	4
1	8	26	17	11
2	13	28	4	26
3	38	19	18	15
4	19	26	24	10

8. Solve the following assignment problem :

	1	2	3	4
A	10	12	9	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

9. A team of 5 horses and 5 riders has entered a jumping show contest. The no. of penalty points to be expected when each rider rides any horse is shown below :

	R1	R2	R3	R4	R5
H1	5	3	4	7	1
H2	2	3	7	6	5
H3	4	1	5	2	4
H4	6	8	1	2	3
H5	4	2	5	7	1

How should the horses be allotted to the riders so as to minimize the expected loss of the team?

10. Solve the following cost-minimizing problem :

	A	B	C	D	E
1	2	9	2	7	1
2	6	8	7	6	1
3	4	6	5	3	1
4	4	2	7	3	1
5	5	3	9	5	1

11. Solve the following integer prog. Problem :

Max. $z=7x_1+9x_2$ subject to

$-x_1+3x_2 \leq 6,$

$7x_1+x_2 \leq 35,$

$x_1, x_2 \geq 0$ and integers.

12. A manufacturer of baby dolls makes 2 types of dolls : doll x and doll y. processing of these dolls is done on 2 machines A and B. doll X requires 2 hrs. on A and 6 hrs. on B. Doll Y requires 5 hrs. on A and 5 hrs. on B. There are 16 hrs. of time per day available on A and 30 hrs. on B. The profit gained on both the dolls is same, i.e. 1 Re. per doll. What should be daily production of each of the 2 dolls?

Solve the LPP and if the optimal solution is not integer valued, use Gomory's technique to derive the optimal solution.

13. Use Branch and Bound method to solve the following problem :

Max. $z = 3x_1 + 3x_2 + 13x_3$, subject to

$$-3x_1 + 6x_2 + 7x_3 \leq 8$$

$$6x_1 - 3x_2 + 7x_3 \leq 8$$

$$0 \leq x_j \leq 5, \text{ and } x_j \text{ are integers for } j=1,2,3.$$

14. Write the applications of Integer Programming.

15. Use Branch and Bound method to solve the following problem :

Min. $z = -5x_1 + 7x_2 + 10x_3 - 3x_4 + x_5$, subject to

$$x_1 + 3x_2 - 5x_3 + x_4 + 4x_5 \leq 0$$

$$2x_1 + 6x_2 - 3x_3 + 2x_4 + 2x_5 \geq 4$$

$$x_2 - 2x_3 - x_4 + x_5 \leq -2$$

$$x_i = 0, 1, (i=1,2,3,4,5)$$

----- UNIT III-----

1. The total profit of a restaurant was found to depend mostly on the amount of money spent on advertising and the quality of the preparation of the food (measured in terms of the salaries paid to the chefs). In fact the manager of the restaurant found that if he pays his chefs x Rs. Per hr. and spends Rs. Y a week on advertising., the restaurant's weekly profit (in Rs.) will be

$$Z = 412x + 806y - x^2 - y^2 - xy.$$

What hourly wages should be the manager pay his chefs and how much should he spend on advertising so as to maximize the restaurant's profit?

2. Derive Kuhn-Tucker necessary conditions for an optimal solution to a quadratic prog. problem.
3. What is Quadratic Programming? Explain Wolfe's method of solving it.
4. Mention briefly the Wolfe's algorithm for solving a quadratic PP given in the usual notations :

$$\text{Max. } z=f(x)+(1/2)x^TQx, \text{ s.t. } Ax\leq b \text{ and } x\geq 0.$$

5. Apply Wolfe's method for solving QPP :
 Max. $z=4x_1+6x_2-2x_1^2-2x_1x_2-2x_2^2$, subject to :
 $x_1+2x_2\leq 2$, and
 $x_1, x_2\geq 0$.
6. Use Wolfe's method for solving QPP :
 Max. $z=2x_1+3x_2-2x_1^2$ subject to
 $x_1+4x_2\leq 4$,
 $x_1+x_2\leq 2$ and
 $x_1, x_2\geq 0$.
7. Write the Kuhn-Tucker conditions for the following problem :
 Min. $z=x_1^2+x_2^2+x_3^2$ subject to
 $2x_1+x_2-x_3\leq 0$,
 $1-x_1\leq 0$,
 $-x_3\leq 0$.
 And solve this problem.
8. Use Wolfe's method for solving QPP :
 Max. $z=8x_1+10x_2-x_1^2-x_2^2$ subject to
 $3x_1+2x_2\leq 6$ and
 $x_1, x_2\geq 0$.
9. Use Beale's method for solving the QPP
 Max. $z=4x_1+6x_2-2x_1^2-2x_1x_2-2x_2^2$ subject to
 $x_1+2x_2\leq 2$, and
 $x_1, x_2\geq 0$
10. Solve the following QPP by Beale's method :
 Max. $z=10x_1+25x_2-10x_1^2-x_2^2-4x_1x_2$ subject to
 $x_1+2x_2+x_3=10$,
 $x_1+x_2+x_4=9$ and
 $x_1, x_2, x_3, x_4\geq 0$
11. Use Wolfe's method for solving QPP :
 Min. $z=x_1^2+x_2^2+x_3^2$ subject to
 $x_1+x_2+3x_3=2$,
 $5x_1+2x_2+x_3=5$,
 $x_1, x_2, x_3\geq 0$
12. Solve the following QPP by Beale's method :
 Max. $10x_1+25x_2-10x_1^2-x_2^2-4x_1x_2$ subject to
 $x_1+2x_2+x_3=10$,
 $x_1+x_2+x_4=9$ and

$$x_1, x_2, x_3, x_4 \geq 0$$

Also, solve this problem by Wolfe's method and compare the efficiency of both the methods, with respect to easiness.

13. Solve the following QPP by Beale's method :

$$\text{Min. } z = 6 - 6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2 \text{ subject to}$$

$$x_1 + x_2 \leq 2 \text{ and}$$

$$x_1, x_2 \geq 0$$

14. What is meant by Quadratic Programming? How does quadratic programming problem differ from the LPP?

15. Discuss Beale's method for solving a Quadratic Prog. Problem. Hence or otherwise solve :

$$\text{Min. } z = x_1^2 + 3x_2^2 \text{ subject to the constraints :}$$

$$x_1 + 3x_2 \geq 5,$$

$$0.5x_1 + 2x_2 \geq 2 \text{ and}$$

$$x_1, x_2 \geq 0.$$

----- UNIT IV -----

1. There are 6 jobs to be performed each of which should go through 2 machines A and B in the order A, B. The processing times (in hrs.) for the jobs are given :

Jobs	:	1	2	3	4	5	6
Machine A	:	1	3	8	5	6	3
Machine B	:	5	6	3	2	2	10

Determine the sequence for performing the jobs that would minimize the total elapsed time T. What is T?

2. Find the sequence that minimizes the total elapsed time required to complete the following jobs on two machines.

Jobs	:	J1	J2	J3	J4	J5	J6	J7	J8	J9
Machine A	:	2	5	4	9	6	8	7	5	4
Machine B	:	6	8	7	4	3	9	3	8	11

3. Find the sequence that minimizes the total elapsed time required to complete the following jobs on three machines in order of ABC.

Jobs	:	J1	J2	J3	J4	J5
Machine A	:	8	10	6	7	11
Machine B	:	5	6	2	3	4
Machine C	:	4	9	8	4	5

4. Use graphical method to minimize the total elapsed time needed to process the following jobs on machines shown :

Job 1: Sequence	:	A	B	C	D	E
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Time (hrs.) :	3	4	2	6	2
Job 2 : Sequence :	B	C	A	D	E
Time :	5	4	3	2	6

5. A salesman estimates that the following would be the cost on his route visiting the 5 cities as shown in the table below :

	1	2	3	4	5
1	-	2	4	7	1
2	5	-	2	8	2
3	7	6	-	4	6
4	10	3	5	-	4
5	1	2	2	8	-

Determine the optimal route.

6. Solve the following traveling salesman problem :

	1	2	3	4	5
1	-	6	12	6	4
2	6	-	10	5	4
3	8	7	-	11	3
4	5	4	11	-	5
5	5	2	7	8	-

7. A salesman has to visit 5 cities A, B, C, D and E. The distances (in hundred miles) between 5 cities are as follows :

	1	2	3	4	5
1	-	7	6	8	4
2	7	-	8	5	6
3	6	8	-	9	7
4	8	5	9	-	8
5	4	6	7	8	-

If the salesman starts from city A and back to city A, which route should he select so that the total distance traveled is minimum?

8. Solve the following sequencing problem giving an optimal solution when passing is not allowed.

	A	B	C	D	E
M1	11	13	9	16	16
M2	4	3	5	2	6
M3	6	7	5	8	4
	15	8	13	9	11

M4

9. Find the optimal solution of the following sequencing problem :

	M1	M2	M3	M4
A	24	7	7	29
B	16	9	5	15
C	22	8	6	14
D	21	6	8	32

10. A company is producing a batch of 5 parts A, O, E, N and L using 3 machines X, W, M. All these parts are to be produced in the technological order X, M, W machines. Processing times are as follows:

	X	W	M
N	8	5	3
A	4	6	4
O	7	7	3
L	5	8	4
E	6	4	4

Find the optimum sequence of jobs and machine utilization percentages for each machine.

11. Write Johnson's algorithm for n jobs and 2 machines.
 12. A company has 6 jobs on hand A to F. All jobs have to go through 2 machines M1 and M2. The time required for each job on each machine in hrs. is given below :

	A	B	C	D	E	F
M1:	3	12	18	9	15	6
M2:	9	18	24	24	3	15

Draw a sequence table of 6 jobs on 2 machines.

13. 2 jobs are to processed on 4 machines A, B, C, D. The technological order for these machines is as follows :

Job 1 : Sequence :	A	B	C	D
Time:	4	6	7	3
Job 2: Sequence :	D	B	A	C
Time:	6	4	2	3

14. Find the sequence that minimizes the total elapsed time :

	A	B	C	D	E	F	G	H	I
M1:	2	5	4	9	6	8	7	5	4
M2:	6	8	7	4	3	9	3	8	11

15. There are 5 jobs each of which must go through machines A,B,C in order ABC. Processing times are given below :

	A	B	C
1	8	5	4
2	10	6	9
3	6	2	8
4	7	3	6
5	11	4	5

Determine a sequence of given jobs that will minimize the total elapsed time T.

----- UNIT V -----

1. Write short notes on
 - i) CPM
 - ii) PERT
2. Derive economic order quantity model for an inventory problem when shortages are not allowed.
3. An aircraft company uses rivets at an approximate customer rate of 2500 kg. per year. Each unit costs Rs. 30 per kg. and the company personnel estimate that it costs Rs. 130 to place an order, and that the carrying cost of inventory is 10% per year.
 - i) How frequently should orders for rivets be placed? Also determine the optimum size of each order.
 - ii) If the actual costs are Rs. 500 to place an order and 15% for carrying cost, the optimum policy would change. How much is the company losing per year because of imperfect cost information?
4. If the setup cost instead of being fixed is equal to $(C_3 + Bq)$, where B is the setup cost per unit time produced, then show that there is no change in the optimum order quantity produced due to this change in the setup cost.
5. Given the following data for an item of uniform demand, instantaneous delivery time and back order facility : Annual Demand =800 units, cost of an item=Rs. 40, ordering cost=Rs. 800, inventory carrying cost=40%, back order cost=Rs.10. Find
 - i) minimum cost order quantity
 - ii) maximum no. of back orders
 - iii) maximum inventory level
 - iv) time between two consecutive orders
 - v) total annual cost
6. Consider the inventory system with the following data :
 $R=1000$ units/year, $I=0.30$, $P=Rs. 0.50$ per unit, $C_3=Rs. 10$, $L=2$ yrs.
 Determine optimal order quantity , reorder point, minimum average cost.

15. Find the critical path and calculate the slack time for each event for the following PERT diagram.


