BCA 3rd Semester
Discrete mathematics
C-305
Question bank

Unit-1

1) Explain the principle of inclusion and exclusion with an appropriate example.
2) In a survey of 300 Students, 64 had taken a mathematics course, 94 had taken an English course, 58 had taken a computer course, 28 had taken both mathematics and computer course, 26 had taken both an English and mathematics course, 22 had taken both English and a computer course, 14 had taken all three courses.
    (i) How many students were surveyed who has taken none of the three courses?
    (ii) How many had taken exactly one of the 3 subjects?
3) Among the first 1000 positive integers :
    (i) Determine the integers divisible by 5, nor by 7, nor by 9.
    (ii) Determine the integers divisible by 5, but not by 7, not by 9.
4) Prove the following
    (i) $|A \cup B| = |A| + |B| - |A \cap B|$
    (ii) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$
5) Define the Recurrence Relations. Explain linear Recurrence Relations with constant coefficients with appropriate examples.
6) Sole the difference equation (Recurrence Relations) $2a_r - 5a_{r-1} + 2a_{r-2} = 0$ and find particular solutions, such that $a_0 = 0$ and $a_1 = 1$.
7) State and prove pigeonhole principle.
8) What is inclusion – exclusion principle? How many bit strings of length eight start with one bit or end with the two bits 00?
9) What is the solution of Recurrence Relations :

    $$a_n = a_{n-1} + 2a_{n-2}$$

    With $a_0 = 2$ and $a_1 = 7$?
10) Define generating function. What is generating function of sequence $\{a^k\}, k = 0,1,2 \ldots\ldots$?
11) Find the Boolean product of A and B, where:

    $$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

12) (i) Find the Recurrence Relations of the sequences.
    (a) S = {5, 8, 11, 14, 17} (b) S = {1, 1, 2, 3, 5, 8, 13}

    (ii) Find first four terms of each of the following Recurrence Relations.

(a) $a_k = 2 a_{k-1} + k$, for all integers $k \geq 2$, $a_0 = 1$.
(b) $a_k = a_{k-1} + 3a_{k-2}$, for all integers $k \geq 2$, $a_0 = 1$, $a_1 = 2$.

13) Solve $a_{n+2} – 5 a_{n+1} + 6a_n = 2$ with initial condition $a_0 = 1$ and $a_1 = -1$.

14) (i) Find the generating function for the sequence $1, a, a^2, a^3 \ldots$ where a is fixed constant.
(ii) Find the closed form for the generating function for each of the following sequence.

(a) $1 - z + z^2 - z^3 + z^4 - z^5 + \ldots$
(b) $1 + 2z + 4z^2 + 8z^3 + 16z^4 + 32z^5 + \ldots$
(c) $z + z^2 + z^3 + z^4 + z^5 + \ldots$
(d) $3 - 4z + 4z^2 - 4z^3 + 4z^4 - 4z^5 + \ldots$
(e) $1 - z^2 + z^4 - z^6 + z^8 - z^{10} + \ldots\ldots$

(iii) Obtain partial fraction decomposition and identify the sequence having the expression as a generating function.

(a) $$\frac{1}{5 – 6 z + z2}$$

(b) $$\frac{3 – 5z}{1 – 2z – 3z2}$$

15) Write short notes on the following.
(i) Generating function
(ii) Recurrence Relation
(iii) Closed form
(iv) Polynomials

Unit-2

1) Consider the following—
   P: Anil is rich
   Q: Kanchan is poor

Write each of the following statement in the symbolic forms
   (i) Anil and Kanchan are both rich.
   (ii) Anil is poor and Kanchan is rich.
   (iii) Neither Anil nor Kanchan is poor.
   (iv) It is not true that Anil and Kanchan are both rich.
   (v) Eithrt Anil or Kanchan is poor

2) Given p and q are true and r and s are false.Find the truth value of the following expressions.
   (i) $p \vee (q \vee r)$ (ii) $(-(p \wedge q) \vee -r) \vee (((-p \wedge q) \vee -r) \wedge s)$

3)Find the truth table of the following propositions---
   (i) $-(p \vee q) \vee (-p \wedge -q)$ (ii) $(p \wedge q) \vee (-p \wedge q) \vee (p \wedge -q) \vee (-p \wedge -q)$ (iii) $p \wedge (q \vee r)$ (iv) $-p \vee q \Rightarrow -q$

3) Consider the following—
   p : x is even
   q : x is divisible by 2

write in simple sentences the meaning of the following.
   (i) $-p \wedge -q$ (ii) $p \Leftrightarrow q$ (iii) $-p \Rightarrow -q$

4) Write the negation of the following—
   (i) If she studies,s will pass in exam.
   (ii) If it rains,then they will not go for picnic.
   (iii) If the determinant of a systemof linear equations is zero then either the system has no solution or it has an infinite number of solutions.

5) Prove that the following propositions are tautology---
   (i) $p \vee -p$ (ii) $(p \wedge q) \Rightarrow p$ (iii) $-(p \vee q) \vee [(-p) \wedge q] \vee p$ (iv) $(p \wedge q) \Rightarrow (p \Rightarrow q)$

6) Show that the following propositions are tautology ,contradiction or contingency---
   (i) $p \wedge -q$ (ii) $(p \vee q) \wedge (-p) \wedge (-q)$ (iii) $p \Rightarrow p$ (iv) $p \wedge (p \vee q) \Leftrightarrow p$ (v) $(p \wedge -q) \Rightarrow r$

7) Show that the following pair of propositions are logically equivalent----
   (i) $-(p \wedge q)$ and $-p \vee -q$ (ii) $p \wedge q$ and $q \wedge p$ (iii) $p \vee (p \wedge q)$ and q

8) Establish the equivalences---
   (i) $p \wedge (-q \vee q)$ and p (ii) $-(p \Leftrightarrow q) \equiv (p \wedge q) \vee (-p \wedge q)$ (iii) $p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

9) Using law of algebra of propositions ,show that---
   (i) $(p \wedge q) \vee p \equiv p$ (ii) $(p \wedge q) \vee (p \wedge -q) \equiv p$ (iii) $(p \Rightarrow q) \wedge (r \Rightarrow q) \equiv (p \vee r) \Rightarrow q$

10) What is proposition calculus? How many connectives are aused in compound proposition?

11) Write short notes on the following-----
(i) Conjunction (ii) Biconditional (iii) Negation (iv) disjunction (v) Implication (vi) Converse, Inverse and Contra positive.

12) Write an equivalent formula for p ^ (q ⇔ r) v (r ⇔ p) which does not involve biconditional.

13) State the Converse , Inverse and Contra positive of the following---
   (i) If today is Easter, Then tomorrow is Monday.
   (ii) If P is square, then P is rectangle.
   (iii) If n is prime, Then n is odd or n is 2.

14) What is predicate calculus? Write the predicate variable and propositional function also?

15) Show that p ⇔ q ≡ (p v q) => (p ^ q) , using
   (a) Truth table (b) algebra of proposition

Unit-3

1) Obtain the disjunctive normal form of the following---
   (i) p ^ (p => q) (ii) p v (-p => (q v (q => -r))) (iii) –(p v q) ⇔ p ^ q (iv) –(-(p ⇔ q) ^ r)

2) Obtain the conjunctive normal form of the following---
   (i) p ^ (p => q) (ii) -(p v q) ⇔ p ^ q (iii)[q v (p ^ q)] ^ - [(p v r) ^ q)

3) Write short notes on the following---
   (i) Disjunctive normal form (ii) conjunctive normal form (iii) Principal disjunctive normal form (iv) Principle conjunctive normal form (v) Maxterms (vi) Minterms.

4) Find the principal disjunctive normal form of the following---
   (i) p => q (ii) q v (p v –q) (iii) –p v q (iv) (p ^ -q ^ -r) v (q ^ r)

5) Obtain principal conjunctive normal form.
   (i) p ^ q (ii) (-p => r) ^ (q ⇔ p) (iii) (p ^ q) v (-p ^ q) v (q ^ r)

6) Represent the argument
      If I study hard, then I get A's
      I study hard

         I get A's
   Symbolically and determine whether the argument is valid or not.

7) Represent the given argument is valid or not

If it rains today, then we will not have a party today

If we do not have party today, then we will have a party tomorrow

----

Therefore, if it rains today, then we will have a party tomorrow

8) Show that **t** is a valid conclusion from the premises

p => q , q => r , r => s ,-s .

9) Prove the validity of the following argument "If I get the job and work hard, then I will get promoted. If I get promoted. Then I will be happy. I will not be happy. Therefore, either I will not get the job or I will not work hard."

10) Show that **s** is valid conclusion from the given premises

p => q , q => r , r => s ^ t , p .

11) What are quantifiers? Explain the Existential and Universal quantifiers?

12) Write the different phrases where the Existential and Universal quantifier used?

13) Using rule of inferences, determine whether the following inference pattern are valid or not.

-t => -r

-s

t => w

r v s

Therefore, w

14) Rewrite the following argument using quantifiers, variable and predicate symbols.Prove the validity of the argument.

All healthy people eat an apple a day.

Ram does not eat an apple a day.

Therefore, Ram is not a healthy person.

15) Negate the statement

For all real number x, If "x > 3" then "x2 >9".

Unit-4

1) (i) Write the definition of simple graph, multi graph and pseudo graph with example?

(ii) How many vertices and how many edges do the following graphs have?

a) K_n (b) C_n (c) W_n (d) $K_{m,n}$ (e) Q_n

2) Draw a graph having the properties or explain why no such graph exists.

(i) Graph with four vertices of degree 1, 1, 2 and 3.

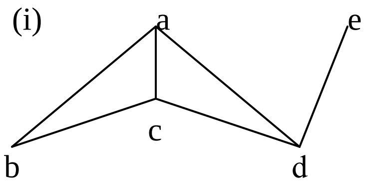
(ii) Graph with four vertices of degree sequence 1, 1, 3, and 3.

(iii) Graph with four vertices of degree sequence 3, 3,3,3,2.
(iv) Graph with four vertices of degree sequence 0, 1,2,2,3.

3) If a graph has exactly two vertices of odd degree, then they must be connected by a path.

4) Show that the following Graphs are isomorphic.

(i)



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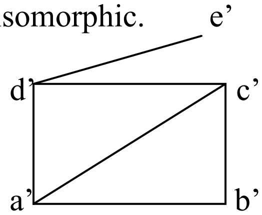

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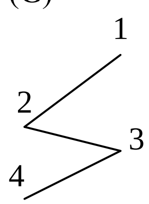
(ii)



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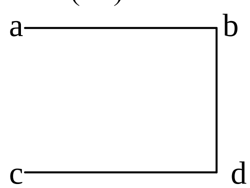


(G')

5) Prove that if G=(V,E) be an undirected graph with e edges. Then

$$\sum_{v \in V} \deg G\,(v) = 2e$$

6) Write definitions of the following with an example.
(i) Diagraph or directed graph
(ii) Complete graph
(iii) Bipartite graph
(iv) Sub graphs
(v) Induced sub graphs
(vi) Planner graph
(vii) Cut edge
(viii) Walk
(ix) Sub graph
(x) Multigraph

7) Consider the ahead graph. Determine the following.

(i) Pendent vertex

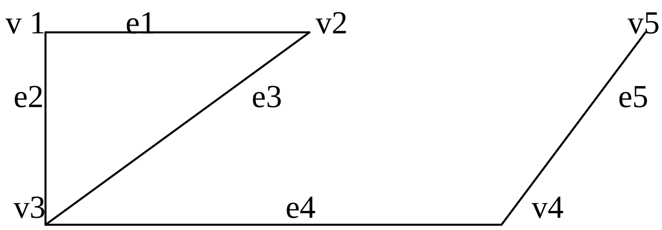
(ii) Odd vertices

(iii) Even vertices

(iv) Incident vertices

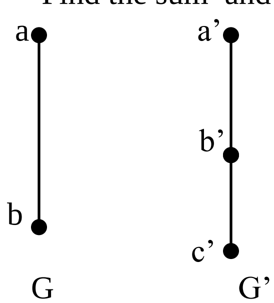
(v) Adjacent edges

(vi) In degree and Out degree

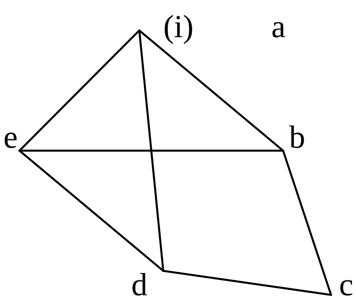


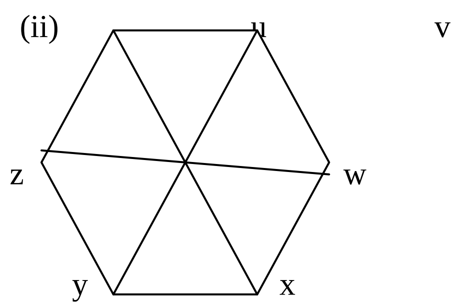
8) A simple graph with **n** vertices and **k** components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges.

9) Find the sum and composition of these two graphs.

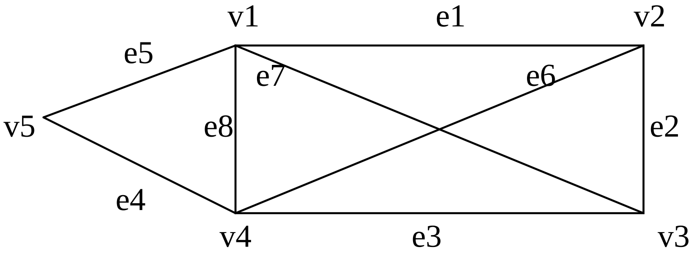


10) Draw the complement of these two graphs.



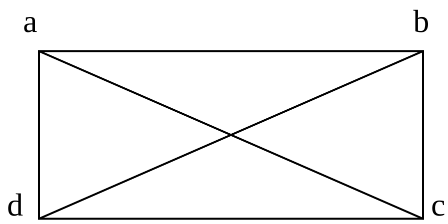


11) Consider the graph given below. Determine which of the following sequences are Path, Simple path, Cycle, Simple cycle

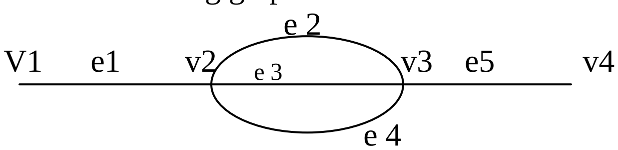


(i) v1 e1 v2 e6 v4 e3 v3 e2 v2
(ii) v1 e1 v2 e2 v3 e3 v4 e4 v5
(iii) v1 e8 v4 e3 v3 e7v1 e8 v4
(iv) v5 e5 v1 e8 v4 e3 v3 e2 v2 e6 v4 e4 v5
(v) v2 e2 v3 e3 v4 e4 v5 e5 v1 e1 v2

12) Consider the following graph draw all spanning graph and sub graphs.



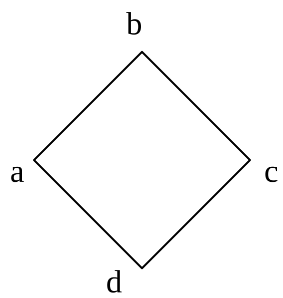
13) Consider the following graph.



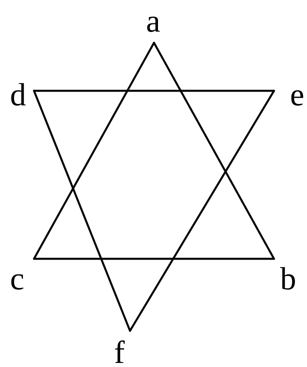
(i) How many simple path there from v1 to v4.
(ii) How many trails are there from v1 to v4.
(iii) How many paths are there from v1 to v4?

14) Find the number of connected components of each of the following graphs.

(i)

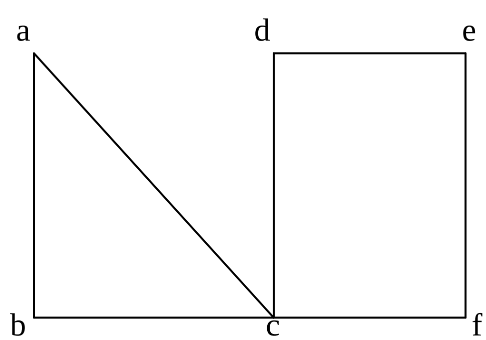


(ii)

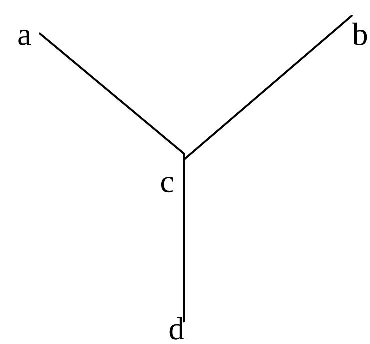


15) Find all the cut vertex of the following graphs.

(i)



(ii)



Unit -5

1) Define the following with a example:
   - (i) Euler line
   - (ii) Decomposition
   - (iii) Components
   - (iv) Ring sum of two graphs
   - (v) Planner graph
   - (vi) Hamiltonian path and circuit
   - (vii) Eulerian path and circuit
   - (viii) Binary tree
   - (ix) Height of tree

2) (i) Prove that a tree with n vertices has n-1 edges.
   (ii)Prove that respect to any of its spanning trees, a connected graph of n vertices and e edges has n-1 tree branches and e-n+1 chords.

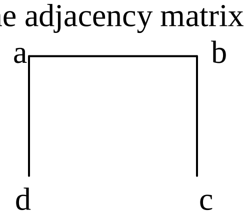
3) (i) Draw a connected graph that becomes disconnected when any edged remove from it.
   (ii) Simple graph and multi graph with two, three, and four vertices.

4) (i) A graph with n vertices and with no loops or parallel edges which has at least ½ (n-1) (n-2) +2 edges is Hamiltonian.
   (ii) Give an example of a graph which is Hamiltonian but not Eulerian and Vice-versa.

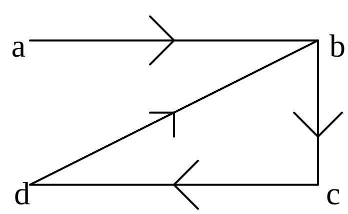
5) (i) If a connected Planner graph **G** has **n** vertices, e edges, and **r** regions then **n – e + r = 2.**
   (ii) If g is connected simple Planner graph with **n (≥ 3)** and **e** edges then **e ≤ 3n -6**.

6) Find the adjacency matrix of the following.

(i)

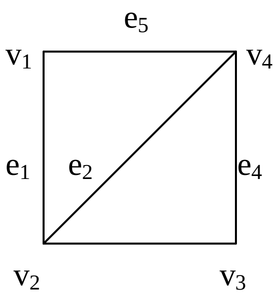


(ii)

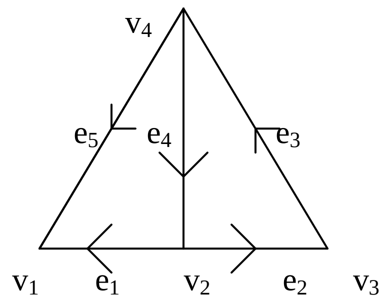


7) Find the incident matrix of the following graphs.

(i)

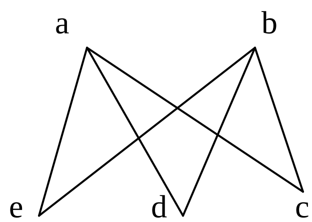


(ii)

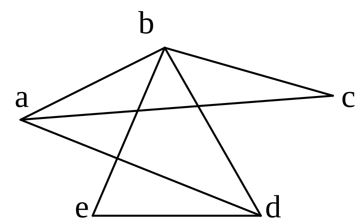


8) Show that each planar by redrawing it so that no edges cross.

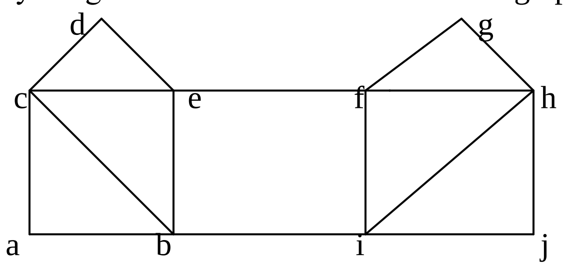
(i)



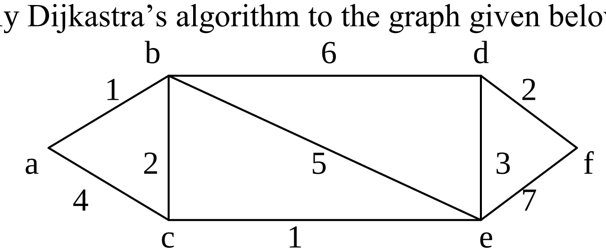
(ii)



9) Using Fleury's algorithm finds Euler circuit in the graph.



10) Apply Dijkastra's algorithm to the graph given below find the shortest path from a to f.

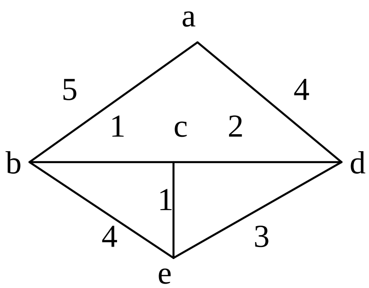


11) (i) Prove that a tree T with n vertices has n – 1 edges.

(ii) For any positive integer n, If G is connected graph with n vertices and n – 1 edges. then G is a tree.

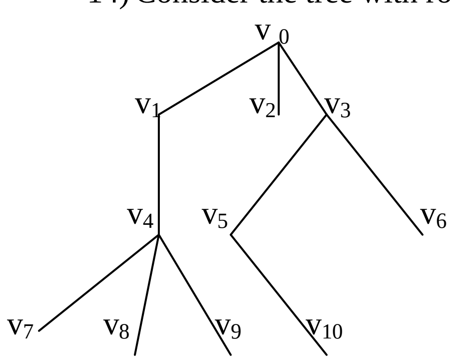
(iii) There are at most m^h leaves in an m-ary tree of height h.

12) Show how Kruskal's algorithm find a minimal spanning tree for the graph given below.



13) (i) Give an example of a graph that has Eulerian circuit which has also a Hamiltonian circuit.
(ii) Give an example of a graph that has Eulerian circuit but not a Hamiltonian circuit.
(iii) Give an example of a graph that has Hamiltonian circuit. but not a Eulerian circuit

14) Consider the tree with root v0 shown in figure.



(i) What are the level of v_0 and $v_{4?}$
(ii) What are the children of v_3?
(iii) What is the height of rooted tree
(iv) What is the parent of v_5
(v) What is sibling of v_7?
(vi) What are the descendants of $v_{3?}$

15) Define a spanning tree. Give an example. Find the spanning tree for the graph given below.