

MCA I Semester
MCA T111: Discrete Structures & Graph Theory
Question bank

Section I

Unit I

- Q1 Relation " is perpendicular to" is
(a) Equivalence (b) Partial order (c) both (a) and (b) (d) None
- Q2. A Homomorphism f is said to be Isomorphism if
(a) f is one-to-one and into (b) f is one-to-many and into
(c) f is one-to-one and onto (d) f is one-to-many and onto
- Q3 If $A = \{a,b,c,d\}$ and $B = \{1,2,3\}$ then cardinality of $(A \times B)$ is
(a) 7 (b) 6 (c) 12 (d) 14
- Q4 What is the probability that a none leap has 53 Mondays :
(a) $2/7$ (b) $1/7$ (c) $3/7$ (d) None
- Q5 What is the probability of getting a sum as 18 on rolling two dice at a time:
(a) $1/6$ (b) $3/36$ (c) 0 (d) $3/10$
- Q6 If A and B are two subsets of a Universal set then
(a) $A-B=A1B'$ (b) $A-B=\phi$ (c) $A-B =A$ (d) None
- Q7 For mutually exclusive events:
(a) $A \cup B = \phi$ (b) $A1B = \phi$ (c) $A1B = \{\phi\}$ (d) None
- Q8 For any event A which of the following is false :
(a) $P(A) + P(A') = 1$ (b) $0 \leq P(A) \leq 1$ (c) $P(A) = P(A')$ (d) None
- Q9 Relation " is parallel" is
(a) Equivalence (b) Partial order (c) both (a) and (b) (d) None
- Q10 Which of the following is not a partition set of the set $A \{1,2,3\}$.
(a) $\{1,2,3\}$ (b) $\{\{1\},\{2,3\}\}$ (c) $\{\{2\},\{1,3\}\}$ (d) None
- Q11 Value of $A \oplus B$ is
(a) $A'B + B'A$ (b) $A'B' + AB$ (c) $A'B - B'A$ (d) none
- Q12 Value of $A \cup (B1C)$ is
(a) $(A \cup B) \cup (A1C)$ (b) $(A \cup B) 1(A \cup C)$ (c) $(A1B) \cup (A1C)$ (d) None
- Q13 Let set $A = \{1,2,3\}$ and $B = \{2,3,4\}$ then
(a) $(A-B) 1 (B-A) = \phi$ (b) $(A-B) \cup (B-A) = \phi$ (c) $(A-B) 1 (B-A) = \{2\}$ (d) None
- Q14 If S is the poset and A is subset of S then $m(m \neq S)$ is said to be lower bound of A if
(a) $m \exists b, \square b \in A$ (b) $m \leq b, \square b \in A$ (c) $m = b, \square b \in A$ (d) None
- Q15 Relation " \leq " is :
(a) Reflexive (b) Transitive (c) Antisymmetric (d) All
- Q16 Let set $A = \{1,2,3\}$ and $B = \{2,3,4\}$ then $n(A \cup B)$ is
(a) 5 (b) 4 (c) 2 (d) 6
- Q17 Relation " is congruent modulo m " is :
(a) Equivalence (b) Partial order (c) both (a) and (b) (d) None

Q18 Let set $A = \{1,2,3\}$ and $B = \{2,3,4\}$ then $n(A \cap B)$ is

- (a) 2 (b) 3 (c) 1 (d) 0

Q19 Relation " \exists " is : (a) Reflexive (b) Transitive (c) Antisymmetric (d) All

Q20 Value of $A \cap (B \cup C)$ is

- (a) $(A \cup B) \cap (A \cap C)$ (b) $(A \cup B) \cap (A \cup C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) None

Unit II

Q1 consider the following

p: this computer is good

q: this computer is cheap

so the symbolic form of following statement "this computer is good or cheap"

- (a) $(\sim p) \wedge q$ (b) $(\sim q) \wedge p$
(c) $p \wedge q$ (d) $p \vee q$

Q2 which of the following logical form not represent tautologies.

- (a) $p \Rightarrow p$ (b) $p \Rightarrow (q \Rightarrow p)$
(c) $(q \Rightarrow p) \Rightarrow p$ (d) $(\sim p \Rightarrow \sim q) \Rightarrow (q \Rightarrow p)$

Q3 which of following is involution law.

- (a) $p \wedge q = q \wedge p$ (b) $\sim \sim p = p$
(c) $\sim p(\vee q) = \sim p \wedge \sim q$ (d) $\sim(\sim p = p)$

Q4 which of following sentence are proposition.

- (a) is this true? (b) four is even
(c) may god bless you (d) what a hit !

Q5 which of the following pair of proposition is not logically equivalent.

- (a) $p \vee (p \wedge q)$ and q (b) $p \wedge q$ and $q \wedge p$
(c) $p \wedge (\sim q \vee q)$ and p (d) $\sim p \wedge q$ and $p \vee \sim q$

Q6 which of the following proposition is tautologies.

- (a) $(p \vee q) \Rightarrow q$ (b) $p \vee (q \Rightarrow p)$
(c) $p \vee (p \Rightarrow q)$ (d) $p \rightarrow (p \Rightarrow q)$

Q7 the proposition $\sim(p \wedge q)$ is equivalent to.

- (a) $\sim p \wedge \sim q$ (b) $\sim p \vee \sim q$
(c) $p \vee q$ (d) none of the above

Q8 the proposition $p \wedge q$ is equivalent to.

- (a) 1 (b) p (c) $\sim p$ (d) none of these

Q9 consider the following

p: today is tuesday

q: it is raining

r: it is cold

so the meaning of following $\neg p \Rightarrow (q \vee r)$

(a) if it is tuesday, then it is raining

(b) if today is not tuesday, then it is raining and it is cold

(c) it is cold and raining day

(d) none of these

Q10 The proposition $\neg (p \vee q)$ is equivalent to

(a) $p \vee q$

(b) $\neg p \wedge \neg q$

(c) $\neg q \vee \neg p$

(d) None of above

Q11 p: anil is rich

q: kanchan is poor

write following statement in symbolic form "neither anil nor kanchan is poor"

(a) $(\neg p) \wedge q$

(b) $p \vee q$

(c) $p \wedge q$

(d) $\neg p \wedge q$

Q12 which of the following proposition is contradiction.

(a) $p \wedge \neg q$ and $(p \vee q) \wedge (\neg p) \wedge (\neg q)$

(b) $p \vee q$ and $\neg (p \wedge q)$

(c) $p \vee \neg q$ and $\neg p \wedge \neg q$

(d) none of these

Q13 which of the following is demorgan's law

(a) $p \vee p = p$

(b) $p \wedge T = p$

(c) $p \vee q = q \vee p$

(d) $\neg (p \vee q) = \neg p \wedge \neg q$

Q14 Which of following is not logically equivalent

(a) $p \Leftrightarrow q$ and $(p \Rightarrow q) \wedge (q \Rightarrow p)$

(b) $p \wedge q$ and $q \wedge p$

(c) $\neg (p \Leftrightarrow q)$ and $(p \wedge \neg q) \vee (\neg p \wedge q)$

(d) none of these

Q15 Consider the following

p: It is cold

q: The temperature is 5°C

So what is write proposition is following sentence

"It is false that it is cold day or temp. is 5°C"

(a) $\neg (p \vee q)$

(b) $\neg p \wedge \neg q$

(c) $\neg (p \wedge q)$

(d) $\neg (\neg p \vee \neg q)$

Q16 which is not true:

(a) $p \Leftrightarrow q = (p \Rightarrow q) \wedge (q \Rightarrow p)$

(b) $p \Leftrightarrow q = (p \vee q) \Rightarrow (p \wedge q)$

- (c) $\sim q \Rightarrow \sim p = p \Rightarrow q$
 (d) none of these

Q17 which of the following is complement law

- (a) $p \vee q = q \vee p$
 (b) $p \wedge q = q \wedge p$
 (c) $p \wedge F = F$
 (d) $p \wedge \sim p = F$

Q18 p: manju is intelligent

q: manju is hardworking

"manju is intelligent and manju is hard working."

find the symbolic form of given statement.

- (a) $\sim p \wedge \sim q$ (b) $p \wedge q$
 (c) $p \rightarrow q$ (d) none of these

Q19 which sentence are not proposition.

- (a) it is raining today. (b) this statement is true.
 (c) close the door. (d) sheela is inside her house.

Q20 let p: he is rich q: he is happy

What is the statement of given symbolic form

$$-p \Leftrightarrow -q$$

- (a) if he is rich, then he is happy.
 (b) to be poor is to be unhappy.
 (c) he is neither nor happy.
 (d) he is rich and happy

Unit III

Q1 Let $G = \{ \dots, 2^{-2}, 2^{-1}, 2^0, 2^1, \dots \}$ with respect to multiplication is an

1. Finite group 2. Finite abelian group 3. Infinite abelian group
 4. None of these

Q2 The set of all +ve rational number form an abelian group with respect to '*' defined by

$$a * b = \frac{ab}{2}$$

The identity element is

1. 0 2. 1 3. 2 4. None of these

Q3 The cube root of unity with respect to multiplication is

1. Not a group 2. Group 3. Abelian group 4. Ring

Q4 If in a group G , $(ab)^2 = a^2b^2$ then G is

1. Abelian 2. Ring 3. Semigroup 4. None of these

Q5 The S_n of all permutation of degree n is a

1. Group of order n 2. Group of order $n!$ 3. Abelian group of order n
4. Abelian group of order $n!$

Q6 Let a, a^k be any two elements in a group. $O(a) = m$ and $O(a^k) = n$ then

1. $m = n$ 2. $m \leq n$ 3. $n \leq m$ 4. None of these

Q7 Let a and x be any two elements in a group G . Then

1. $O(a) > O(x^{-1}ax)$ 2. $O(a) < O(x^{-1}ax)$ 3. $O(a) = O(x^{-1}ax)$ 4. None of these

Q8 In a group G , every element except identity is of order 2 then G is

1. Finite 2. Abelian 3. Ring 4. None of these

Q9 Let a and b are two elements of abelian group and $O(a) = m$, $O(b) = n$, g.c.d

$(m, n) = 1$ then $O(ab) =$

1. $m + n$ 2. $m - n$ 3. mn 4. Can't say

Q10 Let H be any subset of group G such that for all $a, b \in H$, $ab^{-1} \in H$ then H is

1. Subgroup of G 2. Semi group 3. Field 4. Ring

Q11 Let G be a finite group and H is a subset of G such that for all $a, b \in H$, $ab \in H$

then H is

1. Satisfies closure property 2. Subgroup of G 3. Ring 4. Semi group

Q12 Let H and K be any two subgroups of G then

1. HK is a subgroup of G 2. $H \cap K$ is a subgroup of G 3. $H \cup K$ is a subgroup of G 4. All of these

Q13 Let N be a subgroup of G such that every left coset of N in G is a right coset of N in G then N is

1. Cyclic 2. Normal 3. Field 4. None of these

Q14 If N is a normal subgroup of G then product of two right cosets of N in G is a

1. Right coset
2. Left coset
3. Identity
4. None of these

Q 15 Let G be a finite group of order n and $a \in G$ such that $O(a) = n$, then G is

1. Cyclic
2. Normal
3. Ring
4. None of these

Q 16 Let H be a subgroup of G of order 5. Then

1. $H = G$
2. $H = \{e\}$
3. $H = G$ or $H = \{e\}$
4. Can't say

Q 17 Is the set of integers Z with binary operation $a \cdot b = a - b$ for all $a, b \in Z$ a group?

1. Yes
2. No
3. May be
4. Can't say

Q 18 Let Z be a set of integers, then under ordinary multiplication Z is

1. Monoid
2. Semi group
3. Group
4. Abelian group

Q 19 If H_1 and H_2 are two right cosets of H then

1. $H_1 \cap H_2 = \emptyset$ or $H_1 = H_2$
2. $H_1 \cap H_2 \neq \emptyset$
3. $H_1 \cup H_2 = \emptyset$
4. $H_1 \cap H_2 \neq \emptyset, H_1 \neq H_2$

Q 20 A one – one mapping of finite group onto itself is

1. Isomorphism
2. Homomorphism
3. Automorphism
4. None of these

Unit IV

Q1 Find the closed form of generating function for the following function

$\{0,0,1,1,1,1,\dots\}$

- (a) $x^2/(1-x)$
- (b) $x^3/(1+x)$
- (c) $x^2/(1+x)$
- (d) $x^3/(1-x)$

Q2 Find the closed form of generating function for the following function

$\{0,1,0,0,1,0,0,1,\dots\}$

- (a) $x/(1+x^3)$
- (b) $x/(1-x^3)$
- (c) $1/(1-x^3)$
- (d) $1/(1+x^3)$

Q3 Find the closed form of generating function for the following function

$\{1,0,1,0,1,0,\dots\}$

- (a) $1/(1-x^2)$
- (b) $x/(1-x^2)$
- (c) $1/(1-x^3)$
- (d) $x/(1+x^3)$

Q4 the generating function for following sequence.

$c(8,0), c(8,1), c(8,2), \dots, c(8,8), 0, 0, \dots$

- (a) $(1-x)^8$
- (b) $(x-1)^8$

(c) $(1+x)^8$ (d) x^8

Q5 the generating function for following sequence.

3,-3,3,-3,3,-3,.....

(a) $1/3+x$ (b) $3/1+x$
(c) $3/1-x$ (d) $1/3-x$

Q6 find the generating function of following sequence.

27,27,9,1,0,0,0.....

(a) $(3+x)^3$ (b) $(3-x)^3$
(c) $(x-3)^3$ (d) $(1-x)^3$

Q7 Find the sequence of corresponding to the ordinary generating function. $(3x^3+e^{2x})$

(a) $1,2,(2^2)/2!,(2^3)/3!+3,(2^4)/4!.....$
(b) $1,3,(3^2)/2!,(3^3)/3!+3,(3^4)/4!.....$
(c) $1,e,(e^2)/2!,(e^3)/3!+3,(e^4)/4!.....$
(d) none of these

Q8 find particular solution of the recurrence relation.

$a_{n+2} - 5a_{n+1} + 6a_n = 5^n$

(a) $6^n/5$ (b) $5^n/6$
(c) $-n5^{(n+1)}$ (d) $(5^n)a$

Q9 find the solution of $a_n = -4a_{n-1} + 2a_{n-2} + 4a_{n-3}$

(a) $a_n = (C_1 + nC_2)(-2)^n$ (b) $a_n = (nC_1 + C_2)(-2)^n$
(c) $a_n = (C_1 + nC_2)(-4)^n$ (d) $a_n = (nC_1 + C_2)(-4)^n$

Q10 find the solution of $a_n = -4a_{n-1} - 3a_{n-2}$

(a) $(-3)^n C_2 + (-1)^n C_1$ (b) $(-3)^n C_1 + (-1)^n C_2$
(c) $(-1)^n C_2 + (-3)^n C_1$ (d) $(-1)^n C_1 + (-3)^n C_2$

Q11 find the particular soln. of recurrence relation

$a_{n+2} - 5a_{n+1} + 6a_n = 2^n$

(a) $-n*5^{(n+1)}$ (b) $n*2^{(n-1)}$
(c) $-n*2^{(n-1)}$ (d) $n*5^{(n+1)}$

Q12 find the solution of given recurrence relation.

$a_n = a_{n-1} + 2a_{n-2}, n \geq 2$ with the initial condition $a_0=0, a_1=1$

(a) $a_n = (1/3)2^{(n-1)} - (1/3)(-1)^n$
(b) $a_n = (1/3)2^n - (1/3)(-1)^{(n-1)}$
(c) $a_n = (1/3)2^n - (1/3)(-1)^n$
(d) $a_n = (1/3)2^n + (1/3)(-1)^n$

Q13 Find the solution of recurrence relation.

$a_n = 4(a_{n-1} - a_{n-2})$ with initial condition $a_0 = a_1 = 1$

(a) $a_n = (1-n/2)2^n$ (b) $a_n = (1+n/2)2^n$
(c) $a_n = (1-n/4)2^n$ (d) $a_n = (1+n/4)2^n$

Q14 find generating function for the sequence

1,a,(a^2),.....

(a) $1/(1+ax)$ (b) $a/(1+ax)$
(c) $1/(1-ax)$ (d) $a/(1-ax)$

Q15 find the generator function of sequence $\{a_k\}$ if $a_k = 2 + 3k$

(a) $F(x) + G(x) = \{2/(1-x)\} + \{3k/(1-x)\}$
(b) $F(x) + G(x) = \{2/(1+x)\} + \{3k/(1+x)\}$
(c) $F(x) + G(x) = \{2/(1-x)^2\} + \{3k/(1+x)\}$

$$(d) F(x) + G(x) = \left\{ \frac{2}{(1-x)} \right\} + \left\{ \frac{3k}{(1-x)^2} \right\}$$

Q16 if $a_0=0, a_1=1, a_2=4, a_3=12$ satisfy the recurrence relation $a_n + c_1 a_{n-1} + c_2 a_{n-2} = 0$, so find the value of " a_n "

- (a) $a_n = n \cdot 2^n$ (b) $a_n = n \cdot 2^{n+1}$
 (c) $a_n = n+1 \cdot 2^n$ (d) $a_n = n \cdot 2^{n-1}$

Q17 find the sequence $\{a_n\}$ having the generating function $G(x)$ given by $\left\{ \frac{x}{(1-x)^2} + \frac{x}{(1-x)} \right\}$

- (a) $a_n = n-1$ (b) $a_n = n+1$
 (c) $a_n = n/2$ (d) $a_n = n$

Q18 find the sequence of (Y_n) having the generating function G given by

$$G(x) = \left(\frac{3}{1-x} \right) + \left(\frac{1}{1-2x} \right).$$

- (a) $3+2^{n-1}$ (b) $3+2^n$
 (c) $3+2^{n+1}$ (d) none of these

Q19 In how many ways can three boys and two girls sit in a row ?

- (a) 48 (b) 24
 (c) 120 (d) 36

Q20 A student is to answer 10 out of 13 questions on an exam . how many if he must answer the first or second question but not both.

- (a) 110 (b) 286
 (c) 80 (d) 165

Unit V

Q 1 ($a+b$) $a' \cdot b'$ is equivalent to:

- a) 0 b) 1 c) none

Q2 $a \cdot b + a \cdot b' + a' \cdot b + a' \cdot b'$ is equal to:

- a) 0 b) 1 c) a d) none

Q3 $a' + a \cdot b$ is equal to:

- a) $a' + b$ b) $a + b$ c) a d) none

Q4 $x(yz)'$ is equivalent to:

- a) $xy'z + xy'z' + xzy + xzy'$ b) $x'y'z' + xyz + x'y'z + x'yz$ c) none

Q5 The less than relation $<$, on real is

- (a) A partial ordering since it is asymmetric and reflexive .
 (b) A partial ordering since it is anti-symmetric and reflexive .
 (c) Not a partial order because it is not asymmetric and not a reflexive .
 (d) Not a partial order because it is not anti-symmetric and not a reflexive .

Q6 A partial order \leq is defined on the set $S = \{x, a_1, a_2, a_3, \dots, a_n, y\}$ as $x \leq a_i$, for all i and $a_i \leq y$ for all i , where $n \geq 1$. The number of total orders on the set S which contain the partial order \leq is

- (a) 1 (b) n (c) $n+2$ (d) $n!$

Q7 Let $X = \{2, 3, 6, 12, 24\}$, let \leq be the partial order defined by $X \leq Y$ if X divides Y .

Number of edges in the hasse diagram of (X, \leq) is

- (a) 3 (b) 4 (c) 5 (d) None of these

Q8 A self-complemented, distributive lattice is called

- (a) Boolean algebra (b) Modular lattice
(c) Complete lattice (d) Self dual lattice
- Q9 Every finite subset of a lattice has
(a) A LUB and GLB (b) Man LUBs and GLB
(c) Many LUBs and many GLBs (d) Either some LUBs or some GLBs
- Q10 If the lattice (C, \leq) is complemented chain, then
(a) $|C| \leq 1$ (b) $|C| \leq 2$ (c) $|C| > 1$ (d) C doesn't exist
- Q11 The absorption law is defined as
(a) $a * (a * b) = b$ (b) $|C| \leq 2$
(c) $a * (a * b) = b \oplus b$ (d) C doesn't exist
- Q12 How many truth tables can be made from one function table
(a) 1 (b) 2 (c) 3 (d) 8
- Q13 The minimum order of edges in a connected graph with n vertices.
(a) n-1 (b) n (c) n+1 (d) none of these
- Q14 The number of distinct simple graphs with up to three nodes is
(a) 15 (b) 10 (c) 7 (d) 9
- Q15 A graph is planar if and only if does not contain
(a) Subgraphs homomorphic to K_3 and $K_{3,3}$
(b) Subgraphs isomorphic to K_5 or $K_{3,3}$
(c) Subgraphs isomorphic to K_3 and $K_{3,3}$
(d) Subgraphs homomorphic to K_3 or $K_{3,3}$
- Q16 Maximum number of edges in an n-node undirected graph without self loops is
(a) n^2 (b) $n(n-1)/2$ (c) n-1 (d) $n(n+1)/2$
- Q17 Consider a simple connected graph G with n vertices and n edges ($n > 2$). Then
Which of the following statements are True?
(a) G has no cycle
(b) G has at least one cycle
(c) The graph obtained by removing any edge from G is not connected.
(d) None
- Q18 In any undirected graph, the sum of degrees of all the vertices
(a) must be even (b) is twice the number of edges
(c) must be odd (d) both a and b
- Q19 The total number of edges in a complete graph of n vertices is
(a) n (b) $n/2$ (c) $n^2 - 1$ (d) $n(n-1)/2$
- Q20 The number of binary trees with 3 nodes which when traversed in postorder
Gives the sequence A,B,C is-
(a) 3 (b) 9 (c) 7 (d) 5

Section II

Unit I

- Q1 Find all partitions of $\{2,3,4\}$

Q2 Let P and Q be the multisets where $P = \{ 5.a, 7.b, 8.c \}$ and $Q = \{ 3.a, 5.b, 9.c \}$. Find

(1) $P \cup Q$ (2) $P \cap Q$

Q3 Prove that the set $\{ 1, 4, 9, 16, 25, \dots \}$ is a countable set ?

Q4 If a set A has n elements, How many relations are there from A to A.

Q5 Consider the relation R from X to Y. $X = \{1, 2, 3\}$; $Y = \{7, 8\}$ and,

$R = \{(1, 7), (2, 7), (1, 8), (2, 8)\}$, Find (i) R^{-1} (ii) R'

Q6 Show that the relation ' is congruent modulo 4 to ' on the set of integer $\{0, 1, 2, \dots\}$ is an equivalence relation.

Q7 Let $X = \{1, 2, 3, 4\}$, if for $x, y \in X$.

$R = \{(x, y) : x - y \text{ is an integral non-zero multiple of } 2\}$

$S = \{(x, y) : x - y \text{ is an integral non-zero multiple of } 3\}$

Q8 Determine which of the following function $f: R \rightarrow R$ are one to one and which are onto

R. (1) $f(x) = x+1$ (2) $f(x) = x^3$ (3) $f(x) = |x|$

Q9 Show that the functions F and G defined as $F(n) = 3n^4 - 5n^2$ and $G(n) = n^4$, where n is a positive integer, have the same order.

Q10 Prove that $2^{1/2}$ is irrational.

Q11 Prove that for every positive integer n, $n^3 + n$ is even

Q12 Prove that the difference of any rational and any irrational is always rational

Unit II

Q1 Given that the statement $p \rightarrow q$, show that its converse $q \rightarrow p$ and inverse $\neg p \rightarrow \neg q$ are logically equivalent.

Q2 Show that $(\neg p \leftrightarrow q) \Leftrightarrow [(p \wedge \neg q) \vee (q \wedge \neg p)]$

Q3 Show that $[(p \vee q) \wedge (\neg q \vee \neg r)] \wedge (p \vee r) \Leftrightarrow (\neg p) \wedge (\neg q)$

Q4 Show that $\neg(p \wedge (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

Q5 Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not equivalent.

Q3 Show that $\neg p \rightarrow q$ and $p \leftrightarrow \neg q$ are logically equivalent

Q7 Show that $\exists x P(x) \wedge \exists x Q(x)$ and $\exists x (P(x) \wedge Q(x))$ are not logically equivalent.

Q8 Obtain the disjunctive normal form of the following---

(i) $p \wedge (p \Rightarrow q)$ (ii) $p \vee (\neg p \Rightarrow (q \vee (q \Rightarrow \neg r)))$ (iii) $\neg(p \vee q) \Leftrightarrow p \wedge q$ (iv) $\neg(\neg(p \Leftrightarrow q) \wedge r)$

Q9 Obtain the conjunctive normal form of the following---

(i) $p \wedge (p \Rightarrow q)$ (ii) $\neg(p \vee q) \Leftrightarrow p \wedge q$ (iii) $[q \vee (p \wedge q)] \wedge \neg[(p \vee r) \wedge q]$

Q10 Rewrite the following argument using quantifiers, variable and predicate symbols. Prove the validity of the argument.

All healthy people eat an apple a day.

Ram does not eat an apple a day.

Therefore, Ram is not a healthy person.

Q11 Negate the statement

For all real number x , If " $x > 3$ " then " $x^2 > 9$ ".

Q12 Prove the validity of the following argument "If I get the job and work hard, then I will get promoted. If I get promoted. Then I will be happy. I will not be happy. Therefore, either I will not get the job or I will not work hard."

Unit III

Q1 Let G be a group and let $a \in G$ have order k . If p is a prime divisor of k , and if there

is $x \in G_p$ with $x^p = a$, prove that x has order pk

Q2 Prove that if $G = \langle a \rangle$ is a cyclic group with generator a , then G is generated by the

subset $X = \{a\}$.

Q3 If G is a group in which $x^2 = 1$ for every $x \in G$, prove that G must be abelian.

Q4 Let T be the set of even integer, show that the semi group $(\mathbb{Z}, +)$ and $(T, +)$ are isomorphic .

Q5 Determine whether the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$ is even or odd.

Q6 Let \mathbb{R}^+ be the group of non-zero real number under multiplication, and let r be a positive integer. Show that the $f(x) = x^r$ is a homomorphism from \mathbb{R}^+ to \mathbb{R}^+

Q7 Let S be a set of 2×2 matrices of the form $\begin{pmatrix} 0 & 0 \\ a & b \end{pmatrix}$, where a, b are integer is a ring but not a field.

Q8 A Gaussian integer is a number of form $(a + ib)$, where a and b are integer, check whether it is a field or not.

Q9 Prove that the set G of $(p-1)$ integers $1, 2, 3, \dots, p-1$, p being prime, is a finite abelian group of order $p-1$ the composition being multiplication modulo p.

Q10 let a binary operation* on G be defined by $(a,b)*(c,d)=(ac, bc+d)$ for all ordered pairs (a,b) of real numbers, $a \neq 0$. Show that $(G,*)$ is a non abelian group.

Q11 Show that the order of each subgroup of a finite group is a divisor of the order of the group?

Q12 Let $S = \{1, 3, 7, 9\}$ and $G = (S, \text{multiplication mod } 10)$. determine all the left and right cosets of the subgroup $\{1, 9\}$.

Unit IV

Q1 Determine the numeric function corresponding to each of the following generating function: -

1. $A(x) = (1+x)^2 / (1-x)^2$
2. $A(x) = x^4 / (1-2x)$

Q2 Solve the difference equation:-
 $a_r + 6a_{r-1} + 12a_{r-2} + 8a_{r-3} = 0$.

Q3 Solve the differential equation: - $a_n - 5a_{n-1} + 6a_{n-2} = 0$
with the initial conditions, $a_0 = 2, a_1 = 5$.

Q4 A ball is dropped to the floor from a height 40 m. Suppose that the ball always rebounds to reach the half of height from it falls. Find the numeric function a_r , Where a_r denotes the height it reaches in the r^{th} rebound.

Q5 let a_r denotes the altitude of an aircraft, in thousands of feet, at the r^{th} minute
Suppose the aircraft takes off after spending 10 minutes on the ground climbs up a uniform speed to a cruising altitude of 30,000 feet in 10 minutes, starts to Descend uniformly after 110 minutes of flying time and has lands 10 minutes later. Find the numeric function a.

Q6 A person deposits Rs. 200 in a saving account at an interest rate 6% per year

compounded annually. Obtain the amount in this account at the end of r year. Find the numeric function.

Q7 Solve the difference equation $t_r + 4t_{r-1} + 4t_{r-2} = r^2 - 3r + 5$.

Q8 solve the recurrence relation $t_{r+2} + p^2 t_r = \sin pr$.

Q9 solve the difference equation $a_{r+1} - 3a_r = 2^r \cos r \pi/2$.

Q10 solve the following recurrence relation $a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0$ for $n \geq 3$ with $a_0 = 0, a_1 = 1, \text{ and } a_2 = 10$.

Q11 solve the recurrence relation $a_{n+2} - 5a_{n+1} + 6a_n = 2, a_0 = 1, a_1 = 2$, by the method of generating function.

Q12 Solve following recurrence relation by using generating function method.
 $a_r - 9a_{r-1} + 26a_{r-2} - 24a_{r-3} = 2^r + r$ for $n \geq 3$ and $a_0 = 0, a_1 = 1, a_2 = 10$

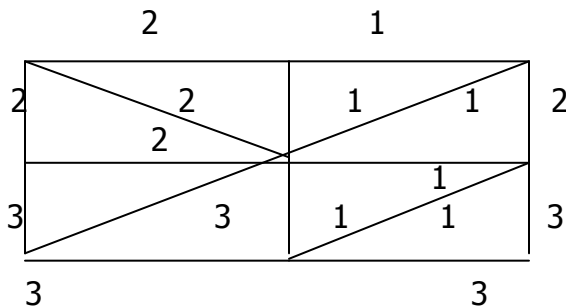
Unit V

Q1 What is Tree? Explain the binary search Tree.

Q2 Give a directed Tree representation of the following formula: $(P \vee (\neg P \wedge Q)) \wedge ((\neg P \vee Q) \wedge \neg R)$
 Form this representation obtain the corresponding prefix formula.

Q3 What is minimum spanning Tree ? Explain.

Q4 Find a minimal spanning tree T for the weighted graph G in figure:



Q5 What is lattice? Give an example of an infinite lattice L with finite length.

Q6 If L is a lattice, then prove that

- (i) $a \wedge b = a$ if and only if $a \vee b = b$.
- (ii) The relation $a \leq b$ (defined by $a \wedge b = a$ or $a \vee b = b$) is a partial order on L .

Q7 If L is a finite distributive lattice, then prove every a in L can be written uniquely (except for order) as the join of irredundant join irreducible elements.

Q8 find the number of nonisomorphic posets with three elements a, b, c , and draw Their diagrams.

Q9 Show that the following "weak" distributive laws hold for any lattice:

(a) $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

(b) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

Q10 Let R be a ring. Let L be the collection of all ideals of R . for any ideals J and K of R , we define: $J \vee K = J + K$ and $J \wedge K = J \cap K$ Prove that L is a bounded lattice.

Q11 Let B be a Boolean algebra

(a) Show that complements of the atoms of B are the maxterms.

(b) Show that any element x in B can be expressed uniquely as a product of maxterms

Section III

Unit I

Q1 If the mapping $f: R \rightarrow R$ be given by $f(x) = 4x - 1$ and the mapping $g: R \rightarrow R$ be given by $g(x) = x^3 + 2$. Find $(g \circ f)x$ and $(f \circ g)x$ where R being the set of real numbers.

Q2 Let $A = \{-2, -1, 0, 1, 2\}$ and let the function $f: A \rightarrow R$ be defined by $f(x) = x^2 + 1$. Find the range of f .

Q3 Let R be a relation over the set $I \times I$ defined as $(a, b) R (c, d)$.If $a = b = c = d$ where $a, b, c, d \in I$. Show that R is an equivalence relation

Q4 Let R be a relation over the set $I \times I$ defined as $(a, b) R (c, d)$.If $a + d = b + c$ where $a, b, c, d \in I$. Show that R is an equivalence relation

Q5 What do you mean by mapping . Explain One-One into and One -One onto mapping with example

Q 6 Show that the set of all positive rational numbers (Q) forms an abelian group under the composition defined by $a * b = (ab)/2$ where $a, b \in Q$.

Unit II

Q1 Given p and q are true and r and s are false. Find the truth value of the following expressions.

(i) $p \vee (q \vee r)$ (ii) $(\neg(p \wedge q) \vee \neg r) \vee (((\neg p \wedge q) \vee \neg r) \wedge s)$

Q2 Find the truth table of the following propositions---

(i) $\neg(p \vee q) \vee (\neg p \wedge \neg q)$ (ii) $(p \wedge q) \vee (\neg p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$

Q3 Show that the following propositions are tautology ,contradiction or contingency---

$$(i) p \wedge \neg q \quad (ii) (p \vee q) \wedge (\neg p) \wedge (\neg q) \quad (iii) p \Rightarrow p \quad (iv) p \wedge (p \vee q) \Leftrightarrow p$$

Q4 Obtain the conjunctive normal form of the following---

$$(i) p \wedge (p \Rightarrow q) \quad (ii) \neg(p \vee q) \Leftrightarrow p \wedge q \quad (iii) [q \vee (p \wedge q)] \wedge \neg[(p \vee r) \wedge q]$$

Q5 What are quantifiers? Explain the Existential and Universal quantifiers?

Q6 Obtain principal conjunctive normal form.

$$(i) p \wedge q \quad (ii) (\neg p \Rightarrow r) \wedge (q \Leftrightarrow p) \quad (iii) (p \wedge q) \vee (\neg p \wedge q) \vee (q \wedge r)$$

Unit III

Q1 Prove that $R = \{0, 1, 2, 3, 4, 5\}$ under composition $+$ and \times is a commutative Ring.

Q2 Show that the set of four permutations $I, (1\ 2)(3\ 4), (1\ 3)(2\ 4),$ and $(1\ 4)(2\ 3)$ on four symbols $1, 2, 3, 4$ is an abelian group with respect to permutation multiplication

Q3 Prove that the set of all real numbers of the form $m+n\sqrt{2}$ where m & n are rational numbers is the ring under the usual addition and multiplication

Q4 Prove that the relation defined by "is perpendicular to" in the set of straight lines in a plane are symmetric but neither reflexive nor transitive

Q5 Prove that the set of integers is an infinite abelian group for the operation multiplication $(*)$ defined by $a*b = a+b+1$.

Q6 If R is a group of real numbers under addition and let R^+ is the group of positive real numbers under multiplication and if $f: R \rightarrow R^+$ be defined as $f(x) = e^x$ then prove that f is an isomorphism

Unit IV

Q1 Determine the generating function of a numeric function a_r where

$$a_r = \begin{cases} 2^r & \text{if } r \text{ is even} \\ -2^r & \text{if } r \text{ is odd} \end{cases}$$

Q2 Obtain partial fraction decomposition and identify the sequence having the expression as a generating function.

$$(a) \frac{1}{5 - 6z + z^2}$$

$$(b) \frac{3 - 5z}{1 - 2z - 3z^2}$$

Q3 Solve the difference equation (Recurrence Relations) $2a_r - 5a_{r-1} + 2a_{r-2} = 0$ and find particular solutions, such that $a_0 = 0$ and $a_1 = 1$.

Q4 Define generating function. What is generating function of sequence $\{a^k\}$,

$$k = 0, 1, 2, \dots ?$$

Q5 Find first four terms of each of the following Recurrence Relations.

$$a_k = 2 a_{k-1} + k, \text{ for all integers } k \geq 2, a_0 = 1.$$

$$a_k = a_{k-1} + 3a_{k-2}, \text{ for all integers } k \geq 2, a_0 = 1, a_1 = 2.$$

Q6 Find the generating function for the sequence $1, a, a^2, a^3, \dots$ where a is a fixed constant.

Unit V

Q1 Consider a POSET $(P(S), \infty)$ where $S = \{1, 2, 3, 4\}$. Find following.

- (i) Draw Hasse diagram
- (ii) Find maximal elements
- (iii) Find minimal elements
- (iv) Find all upper bounds of $\{\{2\}, \{4\}\}$
- (v) Find LUB of $\{\{2\}, \{4\}\}$ if exists
- (vi) Find all lower bound of $\{1, 3, 4\}$
- (vii) Find GLB of $\{1, 3, 4\}$ if exists

Q2 Simplify the following Boolean function using "K' map".

$$F(A, B, C, D) = \sum (0, 1, 2, 4, 5, 7, 11, 15).$$

Q3 Write short notes on:

- (i) Graph Coloring (with example)
- (ii) Chromatic number (with example)
- (iii) Graph Isomorphism (with example)
- (IV) Planar Graph (with example)

Q4 Convert the following Boolean function

$$F(x, y, z) = (x' + y + z')(x' + y + z)(x + y' + z) \text{ in DNF.}$$

Q5 Let $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be ordered by relation "x divides y". Draw the Hasse diagram.

Q6 Show that the maximum number of edges in a simple graph with n vertices is $n(n-1) / 2$