



1. In a nucleus, the attractive central potential which binds the proton and the neutron is called the Yukawa potential. The associated potential energy  $U(r)$  is [4]

$$U(r) = -\alpha \frac{e^{-r/\lambda}}{r}$$

Here  $\lambda = 1.431$  fm (fm =  $10^{-15}$  m),  $r$  is the distance between nucleons, and  $\alpha = 86.55$  MeV·fm is the nuclear force constant ( $1$  MeV =  $1.60 \times 10^{-13}$  J). Assume nuclear force constant  $\alpha$  to be  $A\hbar c$ . Here  $\hbar = h/2\pi$  and  $h$  is Planck's constant. In order to compare the nuclear force to other fundamental forces of nature within the nucleus of Deuterium ( ${}^2\text{H}$ ), let the constants associated with the electrostatic force and the gravitational force to be equal to  $B\hbar c$  and  $C\hbar c$  respectively. Here  $A, B$  and  $C$  are dimensionless. State the expression and numerical values of  $A, B$  and  $C$ .

$A =$	Value of $A =$
$B =$	Value of $B =$
$C =$	Value of $C =$

Detailed answers can be found on page numbers:

2. An opaque sphere of radius  $R$  lies on a horizontal plane. On the perpendicular through the point of contact, there is a point source of light at a distance  $R$  above the top of the sphere (i.e.  $3R$  from the plane).

- (a) Find the area of the shadow of the sphere on the plane. [2]

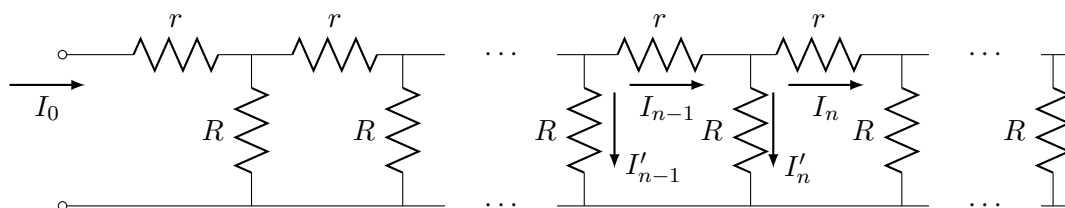
Area =

- (b) A transparent liquid of refractive index  $\sqrt{3}$  is filled above the plane such that the sphere is just covered with liquid. Find the area of the shadow of the sphere on the plane now. [6]

Area =

Detailed answers can be found on page numbers:

3. Consider an infinite ladder of resistors. The input current  $I_0$  is indicated in the figure.



- (a) Find the equivalent resistance of the ladder. [2]

Equivalent resistance =

- (b) Find the recursion relation obeyed by the currents through the horizontal resistors  $r$ . You will get a relationship where  $I_n$  will be related to (may be several)  $I_i$ s,  $i < n$ ,  $n > 0$ . [2]

Relation :

- (c) Solve this for the special case  $R = r$  to obtain  $I_n$  and  $I'_n$  as explicit functions of  $n$ . You may have to make a reasonable assumption about the behaviour of  $I_n$  as  $n$  becomes large. [4]

$I_n =$

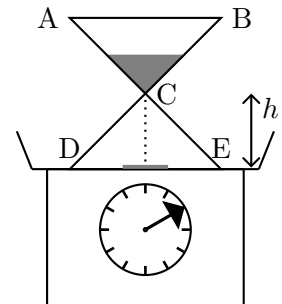
$I'_n =$

- (d) If the ladder is chopped off after the  $N$ -th node (so that  $I_{N+1} = 0$ ) what will the form of  $\frac{I_n}{I_N}$  be for  $n \leq N$ ? [4]

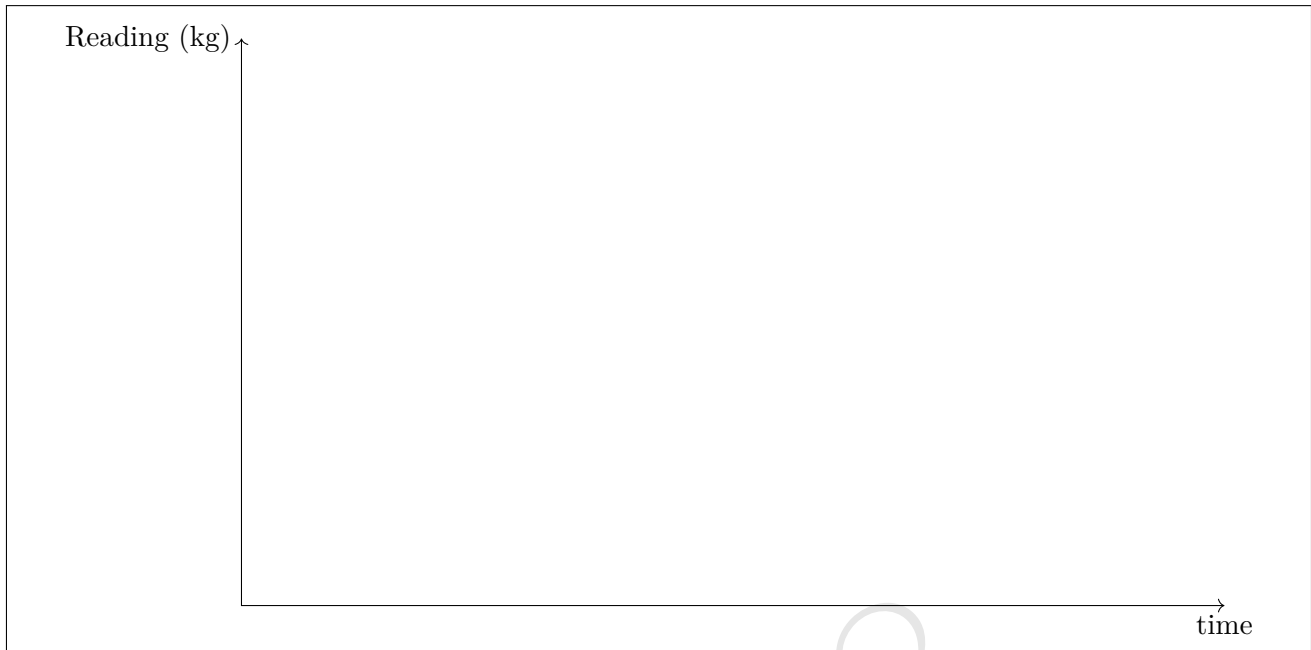
$\frac{I_n}{I_N} =$

Detailed answers can be found on page numbers:

4. An hour glass is placed on a weighing scale. Initially all the sand of mass  $m_0$  kg in the glass is held in the upper reservoir (ABC) and the mass of the glass alone is  $M$  kg. At  $t = 0$ , the sand is released. It exits the upper reservoir at constant rate  $\frac{dm}{dt} = \lambda$  kg/s where  $m$  is the mass of the sand in the upper reservoir at time  $t$  sec. Assume that the speed of the falling sand is zero at the neck of the glass and after it falls through a constant height  $h$  it instantaneously comes to rest on the floor (DE) of the hour glass. Obtain the reading on the scale for all times  $t > 0$ . Make a detailed plot of the reading vs time.



Reading on the scale :



Detailed answers can be found on page numbers:

5. (a) Consider two short identical magnets each of mass  $M$  and each of which may be considered as point dipoles of magnetic moment  $\vec{\mu}$ . One of them is fixed to the floor with its magnetic moment pointing upwards and the other one is free and found to float in equilibrium at a height  $z$  above the fixed dipole. The magnetic field due to a point dipole at a distance  $r$  from it is

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi r^3} (3(\vec{\mu} \cdot \hat{r})\hat{r} - \vec{\mu})$$

Obtain an expression for the magnitude of the dipole moment of the magnet in terms of  $z$  and related quantities.

$\vec{\mu} =$

- (b) i. Consider a ring of mass  $M_r$  rotating with uniform angular speed about its axis. A charge  $q$  is smeared uniformly over it. Relate its angular momentum  $\vec{S}_r$  to its magnetic moment ( $\vec{\mu}_r$ ). [1½]

$\vec{\mu}_r =$

- ii. Assume that the electron is a sphere of uniform charge density rotating about its diameter with constant angular speed. Also assume that the same relation as in the previous part holds between its angular momentum  $\vec{S}$  and its magnetic dipole moment ( $\vec{\mu}_B$ ). Further assume that  $S = h/(2\pi)$  where  $h$  is Planck's constant. Calculate  $\mu_B$ . [1]

$\mu_B =$

- iii. Assume that the sole contribution to the dipole moment of a  $\text{ZnFe}_2\text{O}_4$  molecule comes from an unpaired electron. Also assume that the magnets in the part (5a) are 0.482 kg each of  $\text{ZnFe}_2\text{O}_4$  and the unpaired electrons of the molecules are all aligned. Calculate the height  $z$ . (Note: The molecular weight of  $\text{ZnFe}_2\text{O}_4 = 211$ ) [3]

$z =$

- iv. In an experiment  $\mu_B$  is aligned along a magnetic field of 1 T. It is flipped in a direction anti-parallel to the magnetic field by an incident photon. What should be the wavelength of this photon? [1½]

Wavelength =

Detailed answers can be found on page numbers:

### 6. The Van der Waals Gas:

Consider  $n$  mole of a non-ideal (realistic) gas. Its equation of state may be described by the Van der Waals equation

$$\left(P + \frac{an^2}{V^2}\right) \left(\frac{V}{n} - b\right) = RT$$

where  $a$  and  $b$  are positive constants. We take one mole of the gas ( $n = 1$ ). You must bear in mind that one is often required to make judicious approximations to understand realistic systems.

- (a) For this part *only* take  $a = 0$ . Obtain expressions in terms of  $V$ ,  $T$  and constants for

- i. the coefficient of volume expansion ( $\beta$ ); [1]

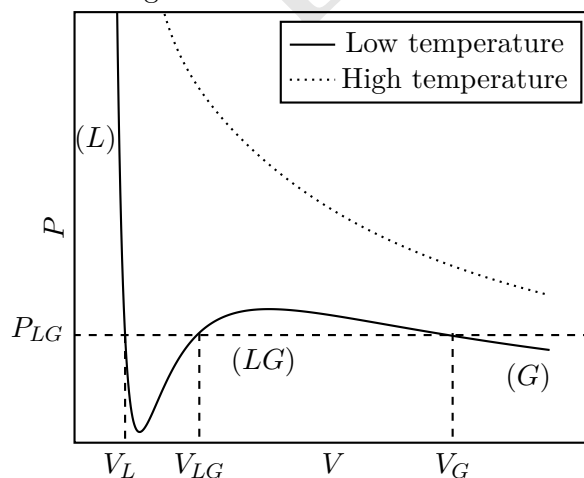
$\beta =$

- ii. the isothermal compressibility ( $\kappa$ ). [1]

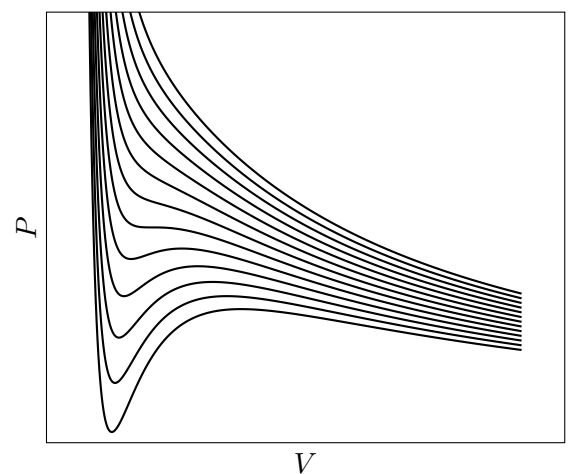
$\kappa =$

- (b) **Criticality:**

The Van der Waals gas exhibits phase transition. A typical isotherm at low temperature is shown in the figure. Here  $L$  ( $G$ ) represents the liquid (gas) phase and at  $P_{LG}$  there are three possible solutions for the volume ( $V_L, V_{LG}, V_G$ ). As the temperature is raised, at a certain temperature  $T_c$ , the three values of the volume merge to a single value,  $V_c$  (corresponding pressure being  $P_c$ ). This is called the point of criticality. As the temperature is raised further there exists only one real solution for the volume and the isotherm resembles that of an ideal gas.



Isotherms of a Van der Waals gas at low and high temperatures



A family of isotherms of a Van der Waals gas

- i. Obtain the critical constants  $P_c$ ,  $V_c$  and  $T_c$  in terms of  $a$ ,  $b$  and  $R$ . [4]

$P_c =$	$V_c =$	$T_c =$
---------	---------	---------

- ii. Obtain the values of  $a$  and  $b$  for  $\text{CO}_2$  given  $T_c = 3.04 \times 10^2 \text{ K}$  and  $P_c = 7.30 \times 10^6 \text{ N}\cdot\text{m}^{-2}$ . [1]

$a =$	$b =$
-------	-------

- iii. The constant  $b$  represents the volume of the gas molecules of the system. Estimate the size  $d$  of a  $\text{CO}_2$  molecule. [1]

$d =$
-------

(c) **The gas phase:**

For the gaseous phase the volume  $V_G \gg b$ . Let the pressure  $P_{LG} = P_0$ , the saturated vapour pressure.

- i. Obtain the expression for  $V_G$  in terms of  $R$ ,  $T$ ,  $P_0$  and  $a$ . [1½]

$V_G =$
---------

- ii. State the corresponding expression for  $V_I$  for an ideal gas. [½]

$V_I =$
---------

- iii. Obtain  $(V_G - V_I)/V_I$  for water given  $T = 1.00 \times 10^2 \text{ }^\circ\text{C}$ ,  $P_0 = 1.00 \times 10^5 \text{ Pa}$ ,  $b = 3.10 \times 10^{-5} \text{ m}^3\cdot\text{mol}^{-1}$  and  $a = 0.56 \text{ m}^6\cdot\text{Pa}\cdot\text{mol}^{-2}$ . Comment on your result. [2]

$\frac{V_G - V_I}{V_I} =$	Comment:
---------------------------	----------

(d) **The liquid phase:**

For the liquid phase  $P \ll a/V_L^2$ .

- i. Obtain the expression for  $V_L$ . [1½]

$V_L =$
---------

- ii. Obtain the density of water ( $\rho_w$ ). You may take the molar mass to be  $1.80 \times 10^{-2} \text{ kg}\cdot\text{mole}^{-1}$ . [1½]

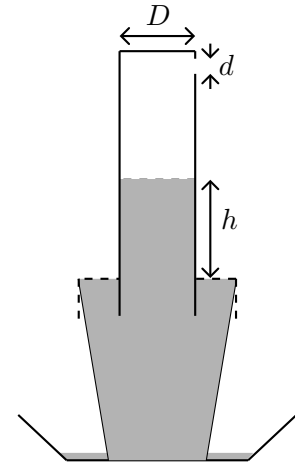
$\rho_w =$
------------

- iii. The heat of vaporization is the energy required to overcome the attractive intermolecular force as the system is taken from the liquid phase ( $V_L$ ) to the gaseous phase ( $V_G$ ). The term  $a/V^2$  represents this. Obtain the expression for the specific heat of vaporization per unit mass ( $L$ ) and obtain its value for water. [2]

$L =$	Value of $L =$
-------	----------------

Detailed answers can be found on page numbers:

7. A small circular hole of diameter  $d$  is punched on the side and the near the bottom of a transparent cylinder of diameter  $D$ . The hole is initially sealed and the cylinder is filled with water of density  $\rho_w$ . It is then inverted onto a bucket filled to the brim with water. The seal is removed, air rushes in and height  $h(t)$  of the water level (as measured from the surface level of the water in the bucket) is recorded at different times ( $t$ ). The figure below and the table in part (c) illustrates this process. Assume that air is an incompressible fluid with density  $\rho_a$  and its motion into the cylinder is a streamline flow. Thus its speed  $v$  is related to the pressure difference  $\Delta P$  by the Bernoulli relation. Take the outside pressure  $P_0$  to be atmospheric pressure  $= 1.00 \times 10^5$  Pa.



- (a) Obtain the dependence of the instantaneous speed  $v_w$  of the water level in the cylinder on  $h$ . [3]

$v_w =$
---------

- (b) Obtain the dependence of  $h$  on time. [3]

$h =$
-------

- (c) The table gives the height  $h$  as function of time  $t$ . Draw a suitable linear graph ( $t$  on  $x$  axis) from this data on the graph paper provided. Two graph papers are provided with this booklet in case you make a mistake.

$t(\text{sec})$	$h(\text{cm})$
0.57	21.54
1.20	20.10
1.81	18.67
2.47	17.23
3.07	15.80
3.86	14.36
4.55	12.92
5.34	11.49

- (d) From the graph and the following data:  $D = 6.66$  cm,  $\rho_a = 1.142$  kg/m<sup>3</sup>,  $\rho_w = 1.000 \times 10^3$  kg/m<sup>3</sup> obtain [5]

- i. The height  $h_0$  at  $t = 0$ .

$h_0 =$
---------

- ii. The value of  $d$ .

$d =$
-------

- iii. The initial speed ( $v_w$ ) of the water level.

$v_w(t = 0) =$
----------------

Detailed answers can be found on page numbers:

HBC18



HBC18

HBC18

HBC18

HBC18

HBC18

HBC18

HBC18

HBC18



HBC18

HBC18

HBC18

HBC18

HBC18

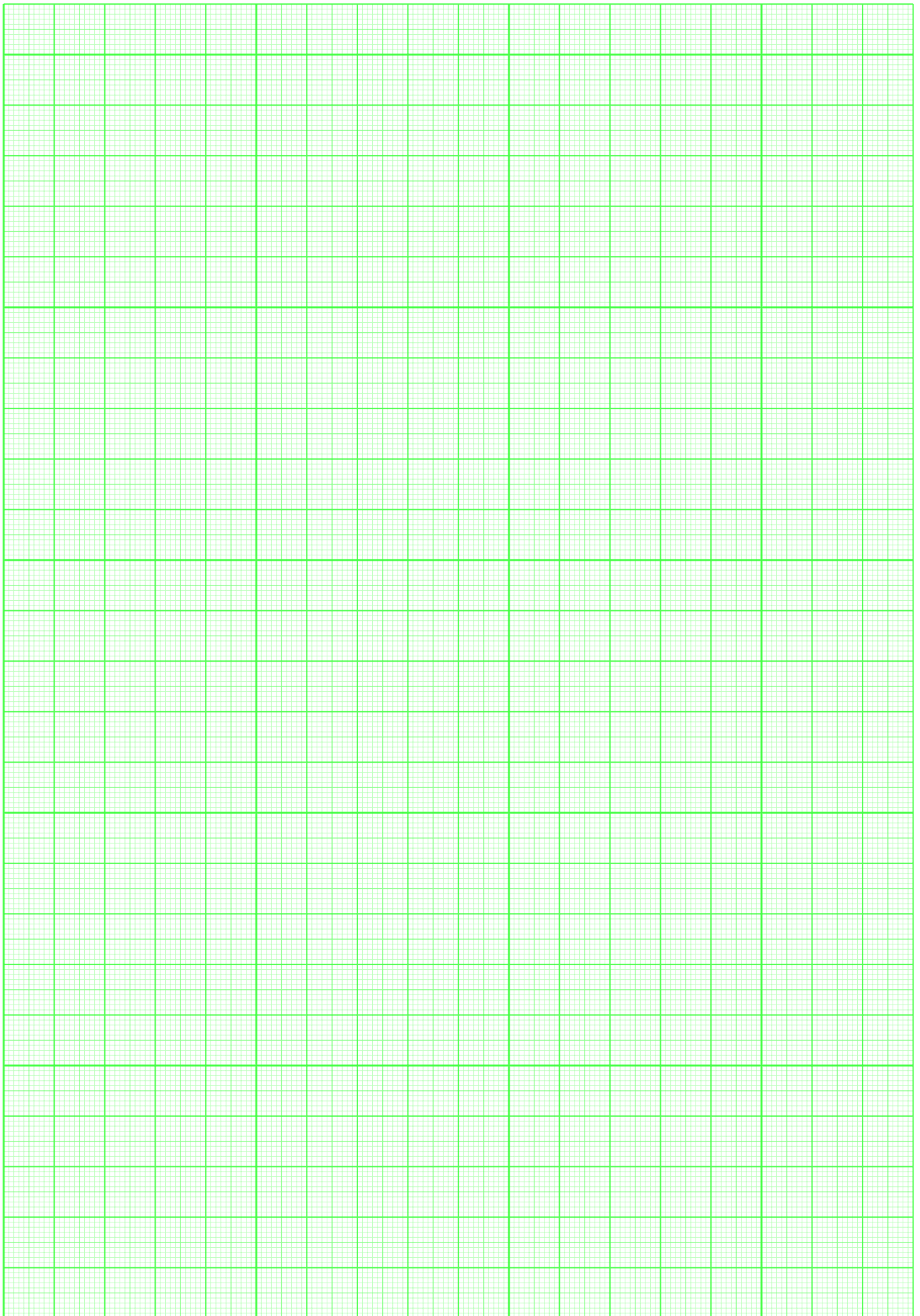
HBC18

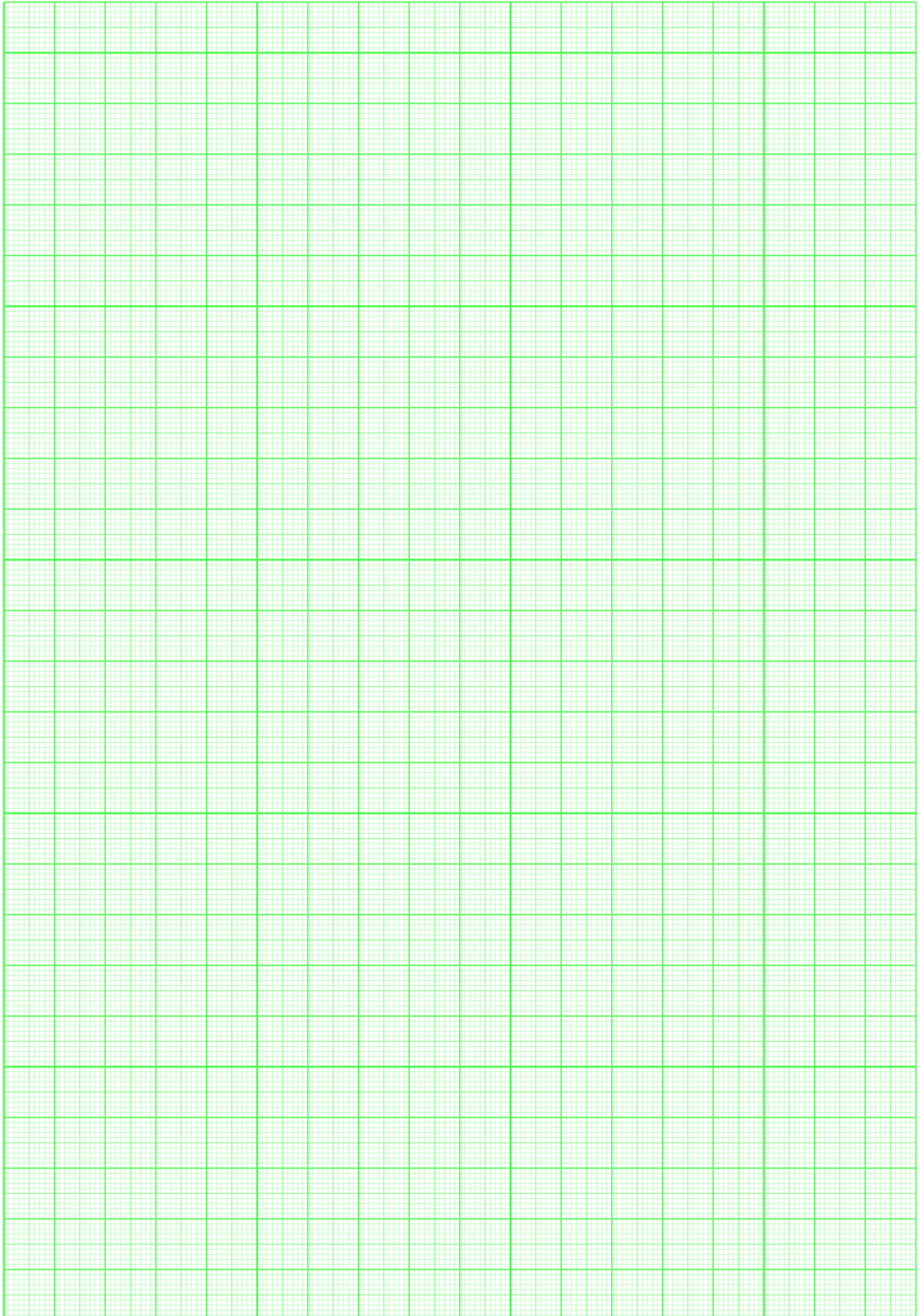
HBC18

HBC18



HBC18





HBC18