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# GLOBAL TALENT SEARCH EXAMINATIONS (GTSE)

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## CLASS -XI

Max Marks: 80

### MATHEMATICS

**General Instructions:** (*Read Instructions carefully*)

1. All questions are compulsory. First 15 minutes for reading instructions.
2. This paper contains **20 objective type questions**. Each question or incomplete sentence is followed by four suggested answers or completions. Select the one that is the most appropriate in each case and darken the correct alternative on the given answer-column, with a pencil or pen.
3. For each correct answer **4 marks** will be awarded and **1 mark** will be deducted for each incorrect answer.
4. No extra sheet will be provided.
5. Use of calculators & mobile is not permitted in examination hall.
6. Use of unfair means shall invite cancellation of the test

Name of the Student : \_\_\_\_\_

Roll No. :

Centre : \_\_\_\_\_

Invigilator's Signature : \_\_\_\_\_

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## AMITY INSTITUTE FOR COMPETITIVE EXAMINATIONS

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**MATHEMATICS**

1. The centre of circle  $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$  is  
(a)  $-i$  (b)  $-2i$  (c)  $2i$  (d)  $1+i$
2. If  $ab < 1$ , then for equation  $(2x-a)(2x-b) - 1 = 0$   
(a) both roots are positive (b) one root is positive, one is negative  
(c) both roots are negative (d) roots are imaginary
3. A line through  $O$  meets the lines  $2x + y = 1$  and  $2x + y = 4$  at the points  $P$  &  $Q$ , then  $OP : PQ$  is  
(a)  $1 : 3$  (b)  $1 : 4$  (c)  $3 : 1$  (d)  $4 : 1$
4.  $PQ$  and  $RS$  are two perpendicular chords of the rectangular hyperbola  $xy = c^2$ . If  $O$  is the centre of the rectangular hyperbola, then the product of the slopes of  $OP$ ,  $OQ$ ,  $OR$  and  $OS$  is equal to  
(a)  $-1$  (b)  $1$  (c)  $2$  (d)  $4$
5. The value of  $\sum_{i=0}^{10} {}^{10}C_i {}^{31}C_{20-i}$  is equal to  
(a)  ${}^{41}C_{20}$  (b)  ${}^{31}C_{20}$  (c)  ${}^{31}C_{11}$  (d) none of these
6. The maximum value of  $t_1 t_2 t_3$ , where  $t_1 t_2 t_3 = (1 - t_1)(1 - t_2)(1 - t_3)$  &  $0 \leq t_i \leq 1$ , is  
(a)  $\frac{1}{8}$  (b)  $\frac{1}{2}$  (c)  $\frac{1}{2\sqrt{2}}$  (d)  $\frac{1}{4}$

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- : Rough Space : -

7. If a circle passes through the point (1, 2) and cuts the circle  $x^2 + y^2 = 4$  orthogonally, then the locus of its centre is
- (a)  $2x + 4y - 9 = 0$  (b)  $2x + 4y - 1 = 0$   
 (c)  $x^2 + y^2 - 3x - 8y + 1 = 0$  (d)  $x^2 + y^2 - 2x - 6y - 7 = 0$
8. If  $\alpha, \beta, \gamma, \in \mathbf{R}$ , satisfy  $2\gamma^2 + (2\alpha-1)\gamma + \alpha^2 - 2\alpha + 2 = 0$ ,  $2\gamma^2 + (2\beta - 1)\gamma + \beta^2 - 2\beta + 2 = 0$ , then
- (a)  $\min (\alpha \cdot \beta) = \frac{5}{8}$  (b)  $\min (\alpha \cdot \beta) = \frac{15}{8}$  (c)  $\max. (\alpha \cdot \beta) = \frac{5}{8}$  (d) none of these
9. The reflection of the curve  $xy = 1$  in the line  $y = 2x$  is the curve  $12x^2 + rxy + sy^2 + t = 0$ , then the value of  $r$  is
- (a)  $-7$  (b)  $25$  (c)  $-175$  (d) none of these
10. If normal to the parabola  $y^2 = 4x$  at  $P(t_1)$  meets the curve again at  $Q(t_2)$  such that area of the triangle  $OPQ$  ( $O$  is the origin)  $\geq \frac{12}{|t_1|}$ ; then
- (a)  $t_1 \geq 1$  (b)  $t_1 \leq -1$   
 (c)  $P$  lies on the right of the latus rectum (d) all of these
11. If  $P_1 = 100^{101}$ ,  $P_2 = 101^{100}$ ,  $P_3 = 99^{100}$ ,  $P_4 = 100^{99}$ , then which of the following is true
- (a)  $P_1 > P_2 > P_3 > P_4$  (b)  $P_1 > P_3 > P_2 > P_4$   
 (c)  $P_1 > P_4 > P_3 > P_2$  (d) none of these
12. The circumcentre of the triangle formed by the tangents and the chord of contact from the point  $(-1, 4)$  to the parabola  $y^2 = 4x$  is
- (a)  $(9, 4)$  (b)  $(4, 9)$  (c)  $(2, 6)$  (d)  $(6, 2)$

- : Rough Space :-

13. From a point  $(h, k)$ , tangents are drawn to  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  such that the chord of contact passes through the point  $(0, 0)$ , then  $(h, k)$  can be  
 (a)  $(5, 3)$  (b)  $(3, 5)$  (c)  $(4, -5)$  (d) no such point exists
14. If  $y, x, z$  are in A.P., then  $2^{x+y}, 2^{y+z}, 2^{x+z}$  are in  
 (a) A.P. (b) G.P. (c) H.P. (d) none of these
15. The point  $A$  on the parabola  $y^2 = 4ax$  for which  $|AC - AB|$  is maximum, where  $B \equiv (0, a)$  and  $C \equiv (-a, 0)$  is  
 (a)  $(a, 2a)$  (b)  $(4a, 4a)$  (c)  $(a, -2a)$  (d) none of these
16. The sum of all the even divisors of 420 is  
 (a) 860 (b) 192 (c) 1344 (d) 1152
17. For the curve  $|z - 2i| = 1$ , which one is true  
 (a) the maximum value of  $\arg z = \frac{2\pi}{3}$  (b) the maximum value of  $|z| = 2$   
 (c) the maximum value of  $\arg z = \frac{\pi}{2}$  (d) none of these
18. If in a triangle  $ABC$ ,  $CD$  is the angular bisector of the angle  $ACB$ , then  $CD$  is equal to  
 (a)  $\frac{a+b}{2ab} \cos \frac{C}{2}$  (b)  $\frac{a+b}{ab} \cos \frac{C}{2}$  (c)  $\frac{2ab}{a+b} \cos \frac{C}{2}$  (d) none of these
19. India and Sri Lanka play one day international series until one team wins 4 matches. No match ends in a draw. The number of ways in which India can win the series is  
 (a) 35 (b) 70 (c) 40 (d) none of these
20. In a triangle  $ABC$ , the minimum value of  $\cot^2 A + \cot^2 B + \cot^2 C$  is equal to  
 (a) 0 (b) 1 (c) 2 (d) none of these

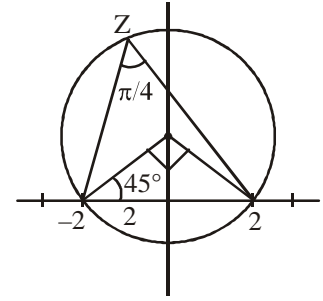


**Sol.: 1 (c):** Centre will lie on perpendicular bisector of  $-2$  and  $2$  i.e.  $x = 0$ .

since chord joining  $-2$  and  $2$  subtends an angle of  $\frac{\pi}{4}$

at circumference, it will subtend an angle of  $\frac{\pi}{2}$  at the centre.

so centre of circle =  $2i$



**Sol.: 2. (b):**  $(2x - a)(2x - b) - 1 = 0$

$$\Rightarrow 4x^2 - 2x(a + b) + ab - 1 = 0$$

$$\Rightarrow \text{Product of roots} = \frac{ab - 1}{4} < 0$$

so one root is positive, other is negative.

**Sol.: 3. (a):** The two given lines are parallel. Also origin does not lie between the lines as  $S_1$  and  $S_2$  have same sign.

Now  $\Delta OPD_1 \sim \Delta OQD_2$

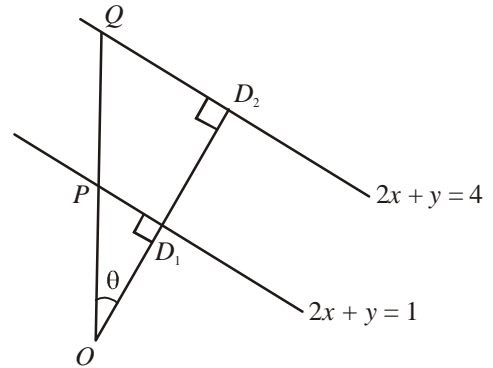
$$\Rightarrow \frac{OP}{PQ} = \frac{OD_1}{D_1D_2}$$

$$\text{Now } OD_1 = \left| \frac{2 \times 0 + 0 - 1}{\sqrt{2^2 + 1^2}} \right| = \frac{1}{\sqrt{5}}$$

$$OD_2 = \left| \frac{2 \times 0 + 0 - 4}{\sqrt{2^2 + 1^2}} \right| = \frac{4}{\sqrt{5}}$$

$$\Rightarrow D_1D_2 = OD_2 - OD_1 = \frac{3}{\sqrt{5}}$$

$$\text{So } \frac{OP}{PQ} = \frac{OD_1}{D_1D_2} = \frac{1}{3}$$



**Sol.: 4. (b):** Parametric point of  $xy = c^2$  is  $\left( ct, \frac{c}{t} \right)$

Let  $P, Q, R, S$  be  $\left( ct_1, \frac{c}{t_1} \right), \left( ct_2, \frac{c}{t_2} \right), \left( ct_3, \frac{c}{t_3} \right)$  and  $\left( ct_4, \frac{c}{t_4} \right)$  respectively.

$$\text{slope of } PQ = \frac{\frac{c}{t_1} - \frac{c}{t_2}}{c(t_1 - t_2)} = \frac{-1}{t_1 t_2}$$

$$\text{similarly slope of } RS = \frac{-1}{t_3 t_4}$$

As  $PQ$  and  $RS$  are perpendicular,  $\frac{-1}{t_1 t_2} \cdot \frac{-1}{t_3 t_4} = -1$

$$\Rightarrow t_1 t_2 t_3 t_4 = -1$$

$$\text{slope of } OP = \frac{\frac{c}{ct_1}}{t_1^2} = \frac{1}{t_1^2}$$

similarly slopes of  $OQ$ ,  $OR$  and  $OS$  are  $\frac{1}{t_2^2}$ ,  $\frac{1}{t_3^2}$ ,  $\frac{1}{t_4^2}$  respectively.

$$\text{so product of slopes of } OP, OQ, OR \text{ and } OS = \frac{1}{t_1^2} \cdot \frac{1}{t_2^2} \cdot \frac{1}{t_3^2} \cdot \frac{1}{t_4^2} = \frac{1}{(t_1 t_2 t_3 t_4)^2} = 1$$

**Sol.: 5. (a):**

$$(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1 x + {}^{10}C_2 x^2 + \dots + {}^{10}C_{10} x^{10}$$

$$(1+x)^{31} = {}^{31}C_0 + {}^{31}C_1 x + {}^{31}C_2 x^2 + \dots + {}^{31}C_{31} x^{31}$$

$$\sum_{i=0}^{10} {}^{10}C_i {}^{31}C_{20-i} = {}^{10}C_0 {}^{31}C_{20} + {}^{10}C_1 {}^{31}C_{19} + {}^{10}C_2 {}^{31}C_{18} + \dots + {}^{10}C_{10} {}^{31}C_{10}$$

$$= \text{coefficient of } x^{20} \text{ in } (1+x)^{10} \cdot (1+x)^{31}$$

$$= \text{coefficient of } x^{20} \text{ in } (1+x)^{41}$$

$$= {}^{41}C_{20}$$

**Sol.: 6. (a):**

$$(t_1 t_2 t_3)^{1/3} = [(1-t_1)(1-t_2)(1-t_3)]^{1/3} \leq \frac{(1-t_1) + (1-t_2) + (1-t_3)}{3}$$

$$\leq 1 - \frac{(t_1 + t_2 + t_3)}{3}$$

$$(t_1 t_2 t_3)^{1/3} \leq 1 - (t_1 t_2 t_3)^{1/3}$$

$$\Rightarrow 2(t_1 t_2 t_3)^{1/3} \leq 1$$

$$\Rightarrow t_1 t_2 t_3 \leq \frac{1}{8}$$

$$\text{Max } (t_1 t_2 t_3) = \frac{1}{8}$$

**Sol.: 7. (a):** Let equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

as this circle cuts the circle  $x^2 + y^2 - 4 = 0$  orthogonally

$$2(+g) \cdot 0 + 2(+f) \cdot 0 = c - 4$$

$$\Rightarrow c = 4$$

$$\Rightarrow x^2 + y^2 + 2gx + 2fy + 4 = 0$$

circle passing from (1, 2)

$$\Rightarrow 2g + 4f + 9 = 0$$

$$\Rightarrow \text{for locus of centre } g = -x, f = -y$$

$$\Rightarrow 2x + 4y - 9 = 0$$

**Sol.: 8. (b):** Obviously  $\alpha$  and  $\beta$  are the roots of

$$2r^2 + (2x - 1)r + x^2 - 2x + 2 = 0$$

$$\text{i.e., } x^2 + 2x(r - 1) + 2r^2 - r + 2 = 0$$

$$\Rightarrow \alpha\beta = 2r^2 - r + 2$$

$$= 2\left(r - \frac{1}{4}\right)^2 + \frac{15}{8} \geq \frac{15}{8}$$

$$\text{So min } (\alpha\beta) = \frac{15}{8}$$

**Sol.: 9. (a):** Let  $(h, k)$  be the general point of deflected curve.

$$\text{Mirror image of } (h, k) \text{ in the line } y = 2x \text{ is } \left(\frac{4k - 3h}{5}, \frac{4h + 3k}{5}\right)$$

$$\Rightarrow \left(\frac{4k - 3h}{5}\right)\left(\frac{4h + 3k}{5}\right) = 1$$

$$\Rightarrow 12x^2 - 12y^2 - 7xy + 25 = 0$$

$$\Rightarrow r = -7$$

**Sol.: 10. (d):** Obviously  $t_2 = -t_1 - \frac{2}{t_1}$

$$\Rightarrow t_1 t_2 = -t_1^2 - 2$$

$$\text{Area of } \triangle OPQ = \frac{1}{2} \begin{vmatrix} t_1^2 & 2t_1 & 1 \\ t_2^2 & 2t_2 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= |t_1 t_2 (t_1 - t_2)|$$

$$= |(t_1^2 + 2)(2t_1 + 2/t_1)| \geq \frac{12}{|t_1|}$$

$$\Rightarrow (t_1^2 + 2)(t_1^2 + 1) \geq 6$$

$$\Rightarrow t_1^4 + 3t_1^2 - 4 \geq 0$$

$$\Rightarrow (t_1^2 - 1)(t_1^2 + 4) \geq 0$$

$$\Rightarrow t_1^2 \geq 1$$

$$\Rightarrow |t_1| \geq 1$$

So  $P$  lies on the right of latus rectum.

**Sol.: 11. (a):**  $P_2 = 101^{100} = (100 + 1)^{100}$

$$= 100^{100} + 100 \cdot 100^{99} + \frac{100 \cdot 99}{2} 100^{98} + \dots + \underbrace{100 \cdot 100 + 1}_{< 100^{100}}$$

The expansion contains 101 terms with sum of last two terms  $< 100^{100}$  and all other terms  $\leq 100^{100}$ .

$$\text{So } P_2 < 100 \cdot 100^{100} = 100^{101} = P_1$$

Similarly,

$$\begin{aligned} P_4 &= 100^{99} = (99 + 1)^{99} \\ &= 99^{99} + 99 \cdot 99^{98} + \frac{99 \cdot 98}{2} 99^{97} + \dots + \underbrace{99 \cdot 99 + 1}_{< 99^{99}} \end{aligned}$$

The expansion contains 100 terms with sum of last two terms  $< 99^{99}$  and all other terms  $\leq 99^{99}$ .

$$\text{So } P_4 < 99 \cdot 99^{99} = 99^{100} = P_3$$

Obviously  $P_2 > P_3$

$$\text{So, } P_1 > P_2 > P_3 > P_4$$

**Sol.: 12. (a):** The point  $(-1, 4)$  lies on the directrix of  $y^2 = 4x$ , so the two tangents from it will be perpendicular. So chord of contact will be the diameter of the circumcircle. So circumcentre will be the mid-point of chord of contact.

Let circumcentre be  $(h, k)$ .

The equation of chord of contact is  $T = S_1$

$$\Rightarrow ky - 2(x + h) = k^2 - 4h$$

$$\Rightarrow ky = 2x + k^2 - 2h \quad \dots(\text{i})$$

Also equation of chord of contact of  $(-1, 4)$  is  $T = 0$

$$\Rightarrow 4y = 2(x - 1) \quad \dots(\text{ii})$$

(i) & (ii) represent same line, so

$$\frac{k}{4} = \frac{2}{2} = \frac{k^2 - 2h}{-2}$$

$$\Rightarrow k = 4, h = 9$$

So circumcentre =  $(9, 4)$

**Sol.: 13. (d):** Chord of contact of  $(h, k)$  is  $T = 0$

$$\Rightarrow \frac{xh}{a^2} + \frac{yk}{b^2} = 1$$

This line can never pass through  $(0, 0)$

So no such point  $(h, k)$  exists.

**Sol.: 14. (b):**  $y, x, z$  are in A.P.

$$\Rightarrow -y, -x, -z \text{ are in A.P.}$$

$$\Rightarrow x + z, y + z, x + y \text{ are in A.P.}$$

$$\Rightarrow 2^{x+z}, 2^{y+z}, 2^{x+y} \text{ are in G.P.}$$

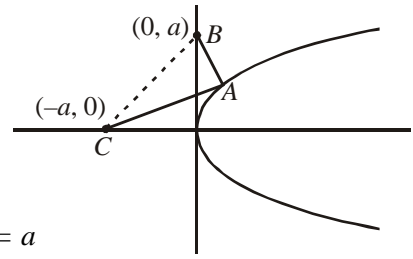
**Sol.: 15. (a):**  $AB + BC \geq AC$



$$\Rightarrow |AC - AB| \leq BC$$

So maximum value is  $BC$ , which will be achieved when  $A, B, C$  are collinear.

So  $A$  is point of intersection of  $BC$  and  $y^2 = 4ax$ .



$$\Rightarrow \text{equation of } BC \text{ is } \frac{x}{-a} + \frac{y}{a} = 1 \quad \Rightarrow \quad y - x = a$$

$$\text{If } A = (at^2, 2at)$$

$$\Rightarrow 2at - at^2 = a$$

$$\Rightarrow t = 1$$

$$\Rightarrow A = (a, 2a)$$

**Sol.: 16. (d):**  $420 = 2^2 \cdot 3 \cdot 5 \cdot 7$

$$\begin{aligned} \text{Sum of even divisors} &= (2 + 2^2)(1 + 3)(1 + 5)(1 + 7) \\ &= 1152 \end{aligned}$$

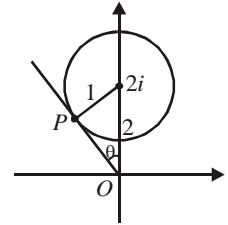
**Sol.: 17. (a):**  $|z - 2i| = 1$  represents a circle with centre  $(0, 2)$  and radius 1

$P$  is the point of circle with maximum argument.

$$\sin \theta = 1/2$$

$$\Rightarrow \theta = 30^\circ$$

So maximum value of  $\arg Z = \pi/2 + \pi/6 = 2\pi/3$



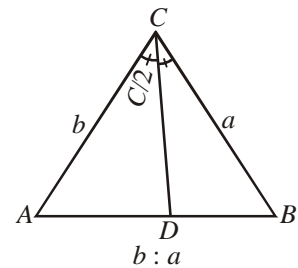
**Sol.: 18. (c):**  $AD : DB = b : a$

$$\text{Let } AD = bk, DB = ak$$

$$\text{Then } bk + ak = c$$

$$\Rightarrow k = \frac{c}{b + a}$$

$$\text{So } AD = \frac{bc}{b + a}$$



$$\text{Now in } \triangle ACD, \frac{\sin C/2}{AD} = \frac{\sin A}{CD}$$

$$\Rightarrow \frac{\sin C/2}{bc/(b + a)} = \frac{\sin A}{CD}$$

$$\Rightarrow CD = \frac{bc \sin A}{(b + a) \sin(C/2)} = \frac{ba \sin C}{(b + a) \sin(C/2)} = \frac{2ab}{a + b} \cos \frac{C}{2}$$

**Sol.: 19. (a):** Obviously maximum number of matches required is 7. Let us assume India wins 4<sup>th</sup> time in  $i^{\text{th}}$  match,  $4 \leq i \leq 7$ . If we assume that India gets defeated in remaining  $(7 - i)$  matches, then India will win exactly 4 out of the 7 matches and there is a one-one correspondence between the given question and India winning 4 out of 7 matches.

$$\text{So no. of ways} = {}^7C_4 = 35$$

**Sol.: 20. (b):**  $(\cot A - \cot B)^2 + (\cot B - \cot C)^2 + (\cot C - \cot A)^2 \geq 0$   
 $\Rightarrow \cot^2 A + \cot^2 B + \cot^2 C \geq \cot A \cot B + \cot B \cot C + \cot C \cot A$   
But in any triangle,  $\tan A + \tan B + \tan C = \tan A \tan B \tan C$   
 $\Rightarrow \cot B \cot C + \cot C \cot A + \cot A \cot B = 1$   
So  $\cot^2 A + \cot^2 B + \cot^2 C \geq \cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

### Mathematics Class-XI Answers

- |         |         |         |         |         |
|---------|---------|---------|---------|---------|
| 1. (c)  | 2. (b)  | 3. (a)  | 4. (b)  | 5. (a)  |
| 6. (a)  | 7. (a)  | 8. (b)  | 9. (a)  | 10. (d) |
| 11. (a) | 12. (a) | 13. (d) | 14. (b) | 15. (a) |
| 16. (d) | 17. (a) | 18. (c) | 19. (a) | 20. (b) |