

# Solutions

1. (d) Given,  $\frac{14587}{1250} = \frac{14587}{5^4 \cdot 2} = \frac{14587 \cdot 2^3}{5^4 \cdot 2^4}$

\ Power of  $m = n = 4$

Hence, it will terminate after 4 decimal places.

2. (d) Let number of girls be  $x$  and number of boys be  $y$ .  
According to the question,

$$x + y = 100 \quad \dots (i)$$

$$\text{and } 50x + 25y = 3000 \quad \dots (ii)$$

From Eq. (i),  $x = 100 - y$

From Eq. (ii),

$$50(100 - y) + 25y = 3000$$

$$\text{P } 2(100 - y) + y = 120 \text{ [dividing both sides by 25]}$$

$$\text{P } 200 - 2y + y = 120$$

$$\text{P } 200 - y = 120$$

$$\text{P } 200 - 120 = y$$

$$\text{P } y = 80 \text{ and } x = 20$$

\ Number of boys is 80.

3. (c) Let the terms of AP be  $a, a + d, a + 2d, a + 3d, a + 4d, a + 5d$

$$\text{where, } a = 4 \quad \dots (i)$$

$$\text{and } a + 5d = 49 \quad \dots (ii)$$

On subtracting Eq. (i) from Eq. (ii), we get

$$5d = 45$$

$$\text{P } d = 9$$

We have, the 4 inserted term equal to

$a + d, a + 2d, a + 3d, a + 4d$  i.e. 13, 22, 31,

40. Here, 40 is the biggest number.

4. (a) QNumber of oatmeal cookies = 30  
and number of chocolate chip cookies = 42  
Maximum number of cookies, which can be put in the container = HCF of (30, 42) = 6  
Now, number of containers required to pack the cookies

$$= \frac{30 + 42}{6} = \frac{72}{6} = 12$$

5. (b) We have,  $\sqrt{\frac{1 + \sin q}{1 - \sin q}}$

$$\text{Rationalising, we get } \sqrt{\frac{(1 + \sin q)(1 + \sin q)}{(1 - \sin q)(1 - \sin q)}} = \sqrt{\frac{(1 + \sin q)^2}{1 - \sin^2 q}}$$

$$= \frac{1 + \sin q}{\cos q} \quad [1 - \sin^2 q = \cos^2 q]$$

$$= \frac{1}{\cos q} + \frac{\sin q}{\cos q} = \sec q + \tan q$$

6. (a) Since, tyre A has a diameter,  $14 + 2(5.1)$  or 24.2 inch.

$$\text{Circumference} = 2 \cdot \pi \cdot r = \frac{24.2}{2} = 76.06 \text{ inch}$$

$$\text{and tyre B has a diameter} = 15 + 2(5.25) = 25.5 \text{ inch}$$

\ Circumference = 80.14 inch

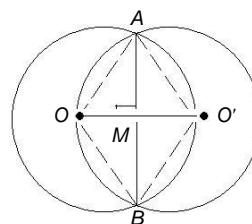
Given, travelled distance = 200 ft

$$= 2400 \text{ inch}$$

$$\text{\ Number of revolutions of tyre A} = \frac{2400}{76.06} = 31.5 = 32$$

$$\text{and number of revolutions of tyre B} = \frac{2400}{80.14} = 29.9 = 30$$

7. (b)



We have, radii of both the circles are equal.

$$\text{P } AO = AO' = x \text{ cm}$$

In  $\Delta AOM$ ,

$$(OA)^2 = (OM)^2 + AM^2$$

$$\text{P } x^2 = \frac{x^2}{4} + AM^2$$

$$\text{P } \frac{\sqrt{3}}{2}x = AM$$

$$\text{P } AB = 2 \cdot AM = 2 \cdot \frac{\sqrt{3}x}{2} = \sqrt{3}x$$

[since, the line joining the centres bisects the common chord]

Now, radius =  $x$

$$\text{P } \text{Diameter} = 2x$$

$$\text{\ Required ratio} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$$

8. (c) Given, list of numbers = 4, 4, 8, 11, 13

Here,  $n = 5$

$$\text{Now, average} = \frac{4 + 4 + 8 + 11 + 13}{5} = \frac{40}{5} = 8$$

$$\text{\ Median} = \frac{a_n + 1}{2} = \frac{5 + 1}{2} = 3 \text{ rd term} = 8$$

and mode = 4 [since, 4 is repeated maximum 2 times]

Now, according to the question,

New list of numbers = 2, 4, 4, 8, 11, 13

Here,  $n = 6$  [even]

$$\text{Now, average} = \frac{2 + 4 + 4 + 8 + 11 + 13}{6} = \frac{42}{6} = 7$$

$$\text{\ Median} = \frac{a_{\frac{n}{2}} + a_{\frac{n}{2} + 1}}{2} = \frac{a_3 + a_4}{2} = \frac{4 + 8}{2} = 6$$

and mode = 4 [since, 4 is repeated maximum 2 times]

Hence, the mode will not change.

9. (b)

$$\frac{x^2 + (3 - \pi)x + \frac{1}{\pi} - 3\pi + \pi^2}{x + \pi} \div \frac{x^2 + 3x + \frac{1}{\pi} + 1}{x + \pi}$$

$$\frac{(3 - \pi)x^2 + \frac{x}{\pi} + 1}{(3 - \pi)x^2 + (3 - \pi)\pi x}$$

$$+ \frac{\left(\frac{1}{\pi} - 3\pi + \pi\right)x + 1}{(3 - \pi)x^2 + (3 - \pi)\pi x}$$

$$+ \frac{\left(\frac{1}{\pi} - 3\pi + \pi\right)x + \pi\left(\frac{1}{\pi} - 3\pi + \pi\right)}{(3 - \pi)x^2 + (3 - \pi)\pi x}$$

$\therefore$  Remainder =  $-\pi^3 + 3\pi^2$

10. (b) Given,  $(p^2 + r^2)x^2 - 2r(p + q)x + r^2 + q^2 = 0$

For equal roots,  $D = 0$

i.e.  $b^2 - 4ac = 0$

$\therefore [2r(p + q)]^2 - 4(p^2 + r^2)(r^2 + q^2) = 0$

$\therefore 4r^2(p + q)^2 - 4(p^2r^2 + p^2q^2 + r^4 + r^2q^2) = 0$

$\therefore 4r^2(p^2 + q^2 + 2pq) - 4(p^2r^2 + p^2q^2 + r^4 + r^2q^2) = 0$

$\therefore r^2(p^2 + q^2 + 2pq) - (p^2r^2 + p^2q^2 + r^4 + r^2q^2) = 0$

$\therefore r^2p^2 + r^2q^2 + 2pr^2q - p^2r^2 - p^2q^2 - r^4 - r^2q^2 = 0$

$\therefore r^4 + p^2q^2 - 2pqr^2 = 0$

$\therefore (r^2 - pq)^2 = 0$

$\therefore r^2 = pq$

11. (a) Given points are  $A(2, k)$ ,  $B(5, 6)$  and  $C(6, 7)$ . Since, the points are collinear.

$\therefore$  Area of  $\Delta ABC = 0$

$\therefore -[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\therefore [(-6 - 7) + 5(7 - k) + 6(k - 6)] = 0$

$\therefore 2(-1) + 35 - 5k + 6k - 36 = 0$

$\therefore k + 35 - 36 - 2 = 0$

$\therefore k = 3$

12. (b) Q Number of letters = 8

and number of vowels = 3

$\therefore P(\text{selecting a vowel}) = \frac{3}{8}$

13. (d) Given,  $\sqrt{\frac{4 + 2\sqrt{3}}{7 + 4\sqrt{3}}} = \sqrt{\frac{1 + 3 + 2\sqrt{3}}{4 + 3 + 4\sqrt{3}}}$

$= \sqrt{\frac{(\sqrt{3} + 1)^2}{(\sqrt{3} + 2)^2}} = \frac{\sqrt{3} + 1}{\sqrt{3} + 2}$

14. (b) Given,  $f(x) = 3x^2 + 13x + 4$

Let  $a$  and  $b$  be zeroes of the given polynomial.

Then, sum of zeroes,  $a + b = \frac{-13}{3} = a + b$

and product of zeroes,  $ab = \frac{4}{3} = ab$

$\therefore \frac{a + b}{b/a} = \frac{a^2 + b^2}{ab} = \frac{(a + b)^2 - 2ab}{ab}$

$\frac{-13/3}{4/3} = \frac{(-13/3)^2 - 2 \cdot 4/3}{4/3}$

$= \frac{169 - 8}{9 \cdot 3} = \frac{169 - 24}{12} = \frac{145}{12}$

$\frac{4}{3}$

15. (d) Since, the possible outcomes are (2, 1), (2, 6), (2, 8), (4, 1), (4, 6), (4, 8), (9, 1), (9, 6) and (9, 8).

$\therefore$  Number of possible outcomes = 9

Number of favourable outcomes = 5

$\therefore$  Required probability =  $5/9$

16. (a) Let original price be  $x$ .

According to the question,

$\frac{300}{x} - \frac{300}{x + 45} = 15$

$\therefore 300(x + 45 - x) = 15(x^2 + 45x)$

$\therefore 20(45) = x^2 + 45x$

$\therefore x^2 + 45x - 900 = 0$

$\therefore x^2 + 60x - 15x - 900 = 0$

$\therefore x(x + 60) - 15(x + 60) = 0$

$\therefore (x - 15)(x + 60) = 0$

$\therefore x = 15, -60$

$\therefore$  Price =  $15$  [neglecting -ve sign]

$\therefore$  New quantity of the commodity bought

$= \frac{300}{15 + 45} = \frac{300}{60} = 5 \text{ kg}$

17. (c) Given,  $a + 3d = 620$  ... (i)

and  $a + 8d = 720$  ... (ii)

From Eqs. (i) and (ii), we get

$5d = 100$

$\therefore d = 20$

and  $a = 620 - 60 = 560$

$\therefore S_{10} = \frac{10}{2} [2a + (10 - 1)d] = \frac{10}{2} [2 \cdot 560 + 9 \cdot 20]$

$= 10 \left[ \frac{2a + (n-1)d}{2} \right]$

$= 10 [560 + 90]$

$= 10 \cdot 650 = 6500$

18. (c) Given, coordinates are  $A(5, 6)$ ,  $B(1, 5)$ ,  $C(2, 1)$  and  $D(6, 2)$ .

Firstly, we show the distance,  $AB = BC = CA = DA$

and diagonal,  $AC = BD$

Now,  $AB = \sqrt{(5 - 1)^2 + (6 - 5)^2} = \sqrt{4^2 + 1^2} = \sqrt{17}$

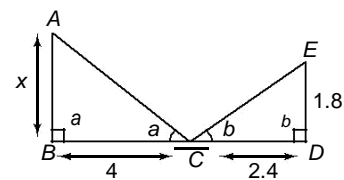
Similarly,  $BC = CA = DA = \sqrt{17}$

Again,  $AC = \sqrt{(5 - 2)^2 + (6 - 1)^2} = \sqrt{34}$

and  $BD = \sqrt{(1 - 6)^2 + (5 - 2)^2} = \sqrt{34}$

Hence, the geometrical figure represents square.

19. (b) We know that,



$\angle A = \angle E$  [image is joined]

$\angle B = \angle D$  [each  $90^\circ$ ]

So,  $\Delta ABC \sim \Delta DEC$  [by AA similarity]

$\therefore \frac{x}{4} = \frac{1.8}{2.4}$

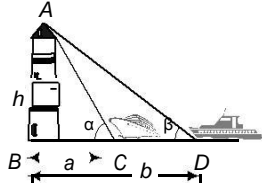
$\therefore x = 3 \text{ m}$

20. (d) All are correctly matched.



$$\begin{aligned} \text{P } (q+21)^2 &= 1089 \\ \text{P } q+21 &= \pm 33 \\ \text{P } q &= -21 \pm 33 \\ \text{P } q &= 12, -54 \\ \text{Commodity sold} &= 12 \text{ units} \\ \text{P } \text{Price, } p &= \frac{400}{16} = 25 \end{aligned}$$

30. (a) Let height of the lighthouse be  $h$ .



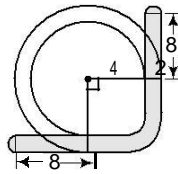
$$\text{In } \triangle ABC, \quad \tan a = \frac{h}{a} \quad \text{P } a = \frac{h}{\tan a}$$

$$\text{In } \triangle ABD, \quad \tan b = \frac{h}{b} \quad \text{P } b = \frac{h}{\tan b}$$

$$\begin{aligned} \text{Now, } b - a &= \frac{h}{\tan b} - \frac{h}{\tan a} = h \cot a - h \cot b \\ &= h \left( \frac{\cos a}{\sin a} - \frac{\cos b}{\sin b} \right) = h \left( \frac{\cos a \sin b - \sin a \cos b}{\sin a \sin b} \right) \\ &= h \frac{\sin(b-a)}{\sin a \sin b} \end{aligned}$$

31. (a) Area of boomerang

$$= 2 (\text{Area of semi-circle}) + 2 (\text{Area of rectangle}) + \text{Area of portion of ring}$$



$$\text{Area of semicircle} = \frac{1}{2} \times \pi \times 4^2$$

$$\text{Area of rectangle} = 8 \times 2$$

$$\text{Area of portion of ring} = \text{Area of inner quadrant} - \text{Area of outer quadrant}$$

$$\begin{aligned} \text{Q Outer radius} &= 4 + 2 = 6 \text{ and inner radius} = 4 \\ &= \frac{1}{4} \times \pi \times 6^2 - \frac{1}{4} \times \pi \times 4^2 = (9\pi - 4\pi) = 5\pi \end{aligned}$$

$$\text{Area of boomerang} = 2 \left( \frac{1}{2} \times \pi \times 4^2 \right) + 2(8 \times 2) + 5\pi$$

$$= (6\pi + 32) \text{ sq units}$$

32. (b) We know that,  $S_n = 1$

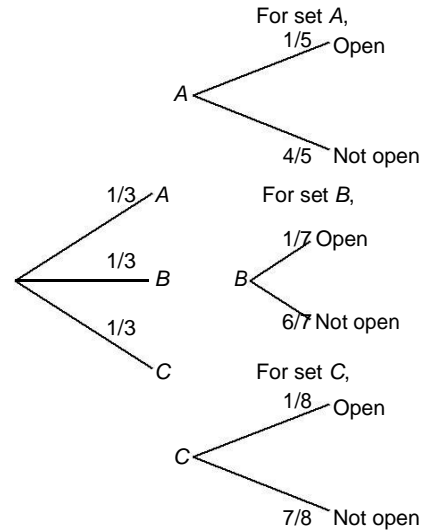
$$\text{P } 0.25 + 0.20 + z + 0.15 + 0.05 = 1$$

$$\text{P } z = 0.35$$

$$\text{P } \frac{x}{100} = 0.35$$

$$\text{P } x = 35$$

33. (c)



$$\text{Required probability} = \frac{1}{3} \times \frac{1}{8} = \frac{1}{24}$$

$$\begin{aligned} \text{34. (d)} \quad & \frac{8x^3 - y^3 + z^3 + 6xy}{a^3 - 8b^3 + 27c^3 + 18abc} \\ &= \frac{(a^2 + 4b^2 + 9c^2 + 2ab + 6bc - 3ac)(4x^2 + y^2 + z^2 + 2xy + yz - 2xz)}{(a - 2b + 3c)(a^2 + 4b^2 + 9c^2 + 2ab + 6bc - 3ac)} \\ &= \frac{2x - y + z}{a - 2b + 3c} \end{aligned}$$

35. (a) Let the initial savings be  $a$ .

$$\text{Given, } S_{20} = 1050000$$

$$\text{and } S_{40} = 4100000$$

$$\text{Now, } S_{20} = \frac{20}{2} [2a + 19d]$$

$$\text{P } S_{20} = 20a + 190d$$

$$\text{Similarly, } S_{40} = 40a + 780d$$

$$\text{P } 20a + 190d = 1050000 \quad \dots(i)$$

$$\text{and } 40a + 780d = 4100000 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$40a + 380d = 2100000$$

$$-40a + 780d = -4100000$$

$$\text{P } -400d = -2000000$$

$$\text{P } d = 5000$$

From Eq. (i), we get

$$20a + 950000 = 1050000$$

$$\text{P } 20a = 100000$$

$$\text{P } a = 5000$$

Hence, the initial savings is 5000.

