Math 4.2

IV Semester M.Sc. in Mathematics Examination, June 2014
GRAPH THEORY AND ALGORITHMS

Time : 3 Hours Max. Marks : 80

***Note** : 1) Answer **any five** questions.*
*2) **All** questions carry **equal** marks.*

1. a) State and prove Handshaking property. Hence prove that there is no graph with odd number of odd degree vertices.
   b) Show that every u.v. walk in graph contains a u-v path.
   c) Define a bipartite graph. Show that a graph is bipartite if and only if all its cycler are even. **(6+4+6)**

2. a) Define the following with an example :
      i) composition product
      ii) normal produt
      iii) tensor product
   b) Let G be a labeled graph with adjacency matrix A. Then show that the $(i, j)^{th}$ entry A^n is the number of walks of length n from v_i to v_j.
   c) If X is a cut-set matrix of a connected graph G with e edges and n-vertices, then show that the rank of X = n – 1. **(6+5+5)**

3. a) If G has a Hamiltonian cycle, then show that for each non-empty set $S \subseteq V(G)$, the graph G – S has atmost |S| components.
   b) Explain the following :
      i) Seating problem ii) Travelling salesman problem
   c) Show that a connected graph is a tree if and only if it is minimally connected. **(6+4+6)**

4. a) Show that any graph G is connected if and only if it has a spanning tree.
   b) Show that every cut-set in a connected graph G must contain at least one branch of every spanning tree of G.
   c) Define the following with an example :
      i) Block graph
      ii) Cut-vertex graph **(6+6+4)**

P.T.O.

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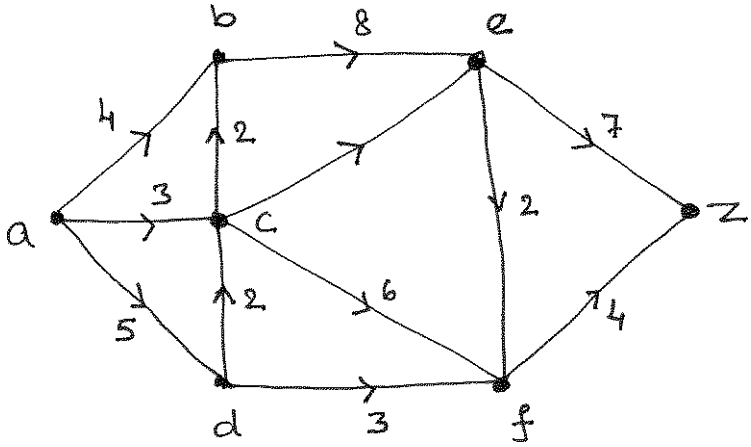
5. a) Show that for any graph G,

$$\kappa(G) \leq \lambda(G) \leq \delta(G)$$

b) Show that a graph with atleast 2n-vertices is n-connected if and only if for any two disjoint sets V_1 and V_2 of n vertices each, there exist n disjoint paths joining these two sets of vertices. **(8+8)**

6. a) Explain the Breadth First Search algorithm with an example.

b) Apply Dijkstra's algorithm to the weighted digraph shown in the following figure in order to find the shortest distance from the vertex a to z.

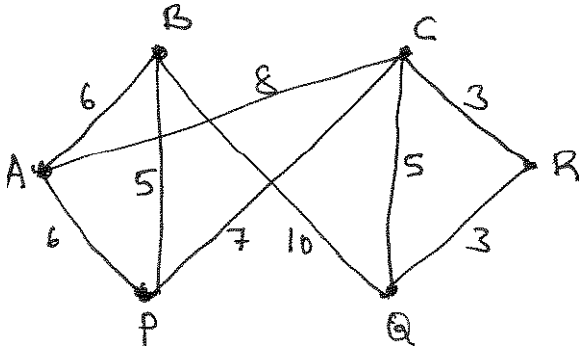


(8+8)

7. a) Explain the minimal spanning tree with an example.

b) Write an prims algorithm with an example

c) Using Kruskal's algorithm, find minimal spanning tree of the weighted graph shown below.



(4+6+6)

8. a) Show that a connected planar graph with n-vertices and e-edges has (e – n + 2) regions.

b) Show that a graph has a dual if and only if it is planar.

c) State and illustrate an algorithm to find a given partition $P = (d_1, d_2, \ldots, d_p)$ with $d_1 \geq d_2 \geq \ldots \geq d_p, p \geq 2, d_1 \geq 1$, is graphical or not. **(5+6+5)**
