IV Semester M.Sc. in Mathematics Examination, January 2016

NUMBER THEORY

Time : 3 Hours **Max. Marks : 80**

Note : Answer any **FIVE** Full Questions.

All Questions Carry Equal Marks.

1. a) Show that the square of any integer is of the form $3k$ or $3k+1$.

   b) Let a and b be any two integers, not both zero. Then prove that a and b are relatively prime if and only if there exist integers x and y such that $ax + by = 1$.

   c) Prove that there is no polynomial $f(n)$ with integer coefficients that will produce primes for all integers n.

   (3+5+8)

2. a) Prove that $ax \equiv b \pmod{n}$ has a solution if and only if $d|b$, where $d = gcd(a,n)$. If $d|b$, then it has d mutually incongruent solutions $mod\ n$.

   b) State and prove Chinese Remainder Theorem.

   (8+8)

3. a) State and Prove Wilson's theorem. What about the converse? Justify.

   b) If the integer $n > 1$ has the prime factorization $n = p_1^{k_1} p_2^{k_2} \ldots p_r^{k_r}$, then prove that
   $$\varphi(n) = (p_1^{k_1} - p_1^{k_1-1})(p_2^{k_2} - p_2^{k_2-1}) \ldots (p_r^{k_r} - p_r^{k_r-1}).$$

   c) Prove that for $n > 2$, $\varphi(n)$ is an even integer.

   (7+5+4)

4. a) If $n = p_1^{k_1} p_2^{k_2} \ldots p_r^{k_r}$, is the prime factorization of $n > 1$, then prove that the positive divisors of n are precisely those integers d of the form $d = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_r^{\alpha_r}$, where $0 \le \alpha_i \le k_i$, $(i = 1, 2, \ldots, r)$.

   b) If F is a multiplicative function and $f(n) = \sum_{d/n} F(d)$, then prove that f is also multiplicative.

   c) Let f and F be number theoretic functions such that $F(n) = \sum_{d/n} f(d)$. Then prove that for any positive integer N, $\sum_{n=1}^{N} F(n) = \sum_{k=1}^{N} f(k) \left[\frac{N}{k}\right]$.

   (6+4+6)

P.T.O

5. a) For $n > 1$, prove that the sum of the integers less than n and relatively prime to n is $\frac{n\varphi(n)}{2}$.

b) Using the Ceaser cipher, encipher the following: "ALL IS WELL THAT ENDS WELL".

c) Let f and g be arithmetic functions. If g and $f \bullet g$ are multiplicative then prove that f is also multiplicative.

(4+6+6)

6. a) Show that every odd prime divisor of $n^2 + 1$ is of the form $4k + 1$ for some $k \in \mathbb{Z}$.

b) If a is a primitive root *modulo m* then prove that $\{1, a, a^2, \ldots, a^{\varphi(m)-1}\}$ is a reduced residue system *modulo m*.

c) If p is a prime number and $d|(p-1)$, then prove that the congruence $x^d - 1 \equiv 0 (mod\, p)$ has exactly d incongruent solutions *mod p*.

(3+6+7)

7. a) Prove that 2^n has no primitive root for $n \geq 3$.

b) State and prove the quadratic reciprocity law.

(8+8)

8. a) Prove that an odd prime p is expressible as a sum of two squares if and only if $p \equiv 1 (mod\ 4)$

b) Prove that $F_{m+n} = F_{m-1}F_n + F_mF_{n+1}$.

c) Express $\sqrt{15}$ as infinite simple continued fraction.

(6+5+5)

Math 4.2

IV Semester M.Sc., in Mathematics Examination, January 2016

GRAPH THEORY AND ALGORITHMS

Time : 3 Hours **Max. Marks : 80**

Note:

i. Answer any **five full** questions.
ii. All questions carry equal marks.

1. a) Prove that the sum of the degree of the vertices of a graph G is twice the number of edges. Further prove that every cubic graph has even number of vertices.

   b) Prove with usual notation $\delta(G) \leq \frac{2m}{n} \leq \Delta(G)$.

   c) Define spanning and induced subgraphs. Distinguish between edge-disjoint and vertex disjoint subgraphs.

(6+6+4)

2. a) If $B(G)$ is an incidence matrix of a connected graph G with n vertices, then prove that rank of $B(G)$ is $(n-1)$.

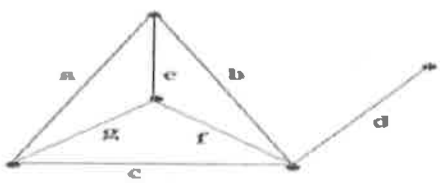
   b) Prove that a simple graph G with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

   c) Let G be a labeled graph with adjacency matrix A. Then show that the $(i, j)^{th}$ entry of A^n is the number of walks of length n from vertex v_i to v_j.

(6+5+5)

3. a) Explain complementary and self-complementary graphs using suitable examples. If a simple graph G of order n is self-complementary, then show that either n or (n-1) must be a multiple of 4.

   b) Find the cut-set matrix of the graph given below.



   c) Define a bipartite graph. Show that a graph is bipartite if and only if all its cycles are even.

(6+4+6)

P.T.O

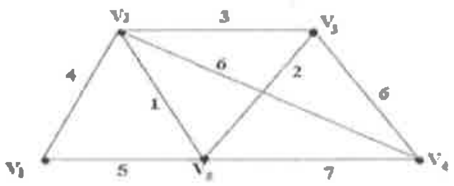
4. a) Prove that a connected graph G has an Euler circuit if and only if all vertices of G are of even degree.
   b) Prove that if G is a simple (n, m) graph with at least three vertices and $\delta(G) \geq \frac{n}{2}$ then G is Hamiltonian.

(8+8)

5. a) Prove that for a sequence $d_1, d_2, \ldots, d_n$ $(n \geq 2)$ of positive integers is the degree sequence of a tree of order n if and only if $\sum_{i=1}^{n} d_i = 2n - 2$.
   b) If T is a tree of order k, $(k \geq 1)$ and if G is a non-trivial graph with $\delta(G) \geq k$. Then prove that T is a subgraph of G.
   c) Prove that G is connected if and only if it has a spanning tree.

(7+5+4)

6. a) Explain Breadth First Search algorithm with an example.
   b) Using Prim's algorithm, find a minimal spanning tree for the weighted graph.

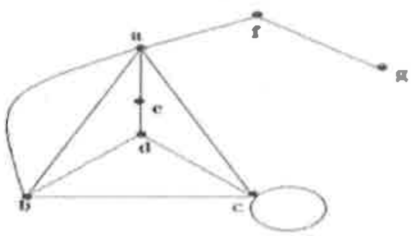


(10+6)

7. a) Define planar graph. Verify whether the complete bipartite graph $K_{3,3}$ is planar.
   b) Prove that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane.

(8+8)

8. a) Draw the geometric dual of the graph.



   b) State and ~~prove~~ Havel-Hakimi theorem.

(6+10)

MATH 4.3

IV Semester M.Sc. in Mathematics Examination, January 2016

FLUID MECHANICS

Time : 3 Hours **Max. Marks : 80**

Note : Answer any **FIVE** Full Questions.
All Questions Carry Equal Marks.

1. (a). Define the terms;
   (i) Compressible and incompressible fluids
   (ii) Viscous fluid and shearing stress
   (iii) Laminar and turbulent flow
   (iv) Steady and unsteady flow.

   (b). Derive the relation between stress and rate of strain Component.

   (8+8)

2. (a) Write short note on thermal conductivity and generalized law of heat Conduction.
   (b) The velocity components of a flow in cylindrical polar Co-ordinates are $(r^2 z\cos\theta,\ r\,z\sin\theta,\ z^2 t)$ determine the components of acceleration of a fluid particle.

   (8+8)

3. (a) Derive the Navier-stoke's equation for a viscous incompressible fluid in the form:

   $$\rho\left(\frac{\partial q}{\partial t} + q\,\nabla\, q\right) = -\nabla P + \rho\vec{g} + \mu\,\nabla^2 q.$$ Explain the meaning of each term.

   (b) A mass of a fluid moves in such a way that each particle describes a circle in one plane about a fixed axis, show that the equation of continuity is $\frac{\partial f}{\partial t} + \frac{\partial(\rho w)}{\partial \theta} = 0$ where W is the angular velocity of a particle whose azimuthal angle is θ at time t.

   (8+8)

4. With usual notation prove $\frac{D\vec{\Omega}}{Dt} = \gamma\nabla^2\vec{\Omega}$ and derive the expression for circulation in a viscous incompressible fluid motion. **(16)**

P.T.O

5. (a) Explain the application of Buckingham π-theorem to viscous compressible fluid motion.

(b) Define and give the physical importance of the following non-dimensional parameters.

(i) Reynolds number
(ii) Froude number
(iii) Mach number
(iv) Prandtl number
(v) Grashoff number

(6+10)

6. (a) An oil of specific gravity 0.85 is flowing through a pipe of 5cm diameter at the rate of 3 litres/sec. Find the type of flow, if the visocity for the oil is 3.8 poise.

(b) Obtain the velocity distribution for a generalized plane Coutte flow in the form $u = \frac{y}{h} + p\frac{y}{h}\left(1 - \frac{y}{h}\right)$. **(6+10)**

7. (a) Write a short note on Karman flow.

(b) Define Poiseuille flow. For such a flow obtain the velocity distribution, average velocity mass flow rate and skin friction. **(6+10)**

8. (a) Prove that for a plane Coutte flow Solution for heat conduction equation can be obtained in the form,

$$\frac{T - T_0}{T_1 - T_0} = \frac{Y}{h} + \frac{E_c}{2} P_r \frac{Y}{h}\left(1 - \frac{Y}{h}\right).$$

(b) In a plane poiseuille flow water at 20^0c flows between two large plates at a distance 1.5mm apart. If the average velocity is 0.15m/sec. Evaluate

(i) The maximum velocity
(ii) The pressure drop
(iii) The wall shearing stress
(iv) The frictional coefficient .

(8+8)

Fourth Semester M.Sc. (Mathematics) Examination, June – 2016

MATHEMATICAL STATISTICS

Time: 3 hours **Max Marks: 80**

Note: 1) Answer *any five* questions.
2) All questions carry equal marks.
3) Use of *Scientific calculator* is *permitted.*

1. a) Define a σ –field and a Borel σ –field on $\mathbb{R}_1$. Prove that a Borel σ –field on $\mathbb{R}_1$ includes Single point set $\{a\}$, $a \in \mathbb{R}$ and the set of all rational numbers. (6)

   b) A pair of two fair dice is rolled. Write the sample space. Find the probability that the sum of the two faces shown is 9 or more. (4)

   c) State Baye's theorem. Illustrate the use of Baye's theorem. To solve the following problem.
   In a factory, machines *A, B, C* produce respectively 25%, 35% and 40% of the total production. Of which 5%, 4% and 2% are defective. An item is drawn at random was found to be defective. Find the probability that
   i) it was manufactured by *A*. ii) it was manufactured by *B* or *C*. (6)

2. a) Define a random variable and elementary random variable. If the selection for a job is based on the pooled percentage of marks in the entrance test and viva, there is a weightage of 0 if the score is in [0, 35); 1 if it is in [35, 50); 3 if it is in [50, 60) and 6 if it is in [60, 100]. Write the sample space and give the associated random variable. (5)

   b) Prove that, any arbitrary random variable can be obtained as a limit of a sequence of elementary random variables. (5)

   c) Prove that, every distribution function F has at most a countable set of discontinuity points. Moreover, show that $F = F_c + F_d$, where F_c is a continuous function and F_d is a pure step function. (6)

3. a) Define expectation (EX) of a random variable X. Let X be a random variable with p.d.f.
   $$f(x) = \begin{cases} \dfrac{1}{2|x|^2} & \text{if } |x| \geq 1 \\ 0 & \text{if } |x| < 1 \end{cases}$$
   . Then, show that EX fails to exist. (6)

   b) Define k^{th} moment of a random variable X. Find first and second moments of a Binomial random variable. (6)

   c) State and prove Jensen's inequality. (4)

P.T.O

4. a) The probability that a pen manufacture by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that; a) exactly two will be defective b) at least two will be defective c) none will be defective. (6)

b) Let X be a Normal random variable with probability density function $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$, Show that the mean of the distribution is μ and the variance is σ^2. (10)

5. a) Let X, Y, Z be independent random variables with unit exponential distribution. Find the distributions of the random variables $U = X + Y + Z$, $V = \frac{X+Y}{X+Y+Z}$ and $W = \frac{X}{X+Y}$. (8)

b) If $\{X_n, n \geq 1\}$ is a sequence of independent and identically distributed random variables, then prove that, the random variable $\frac{S_n}{n} = \frac{X_1 + X_2 + \cdots + X_n}{n}$ converges in probability to $E(X_1)$ provided $E(X_1^2) < \infty$. (8)

6. a) State and prove Neyman factorization theorem. (6)

b) Write a brief note about Maximum Likelihood Estimation (M.L.E.). (6)

c) Define UMP test and state Neyman-Pearson lemma. (4)

7. a) Describe two sample T-test. (10)

b) What is the test statistic of Mann-Whitney-Wilcoxon rank sum test? (6)

8. a) Explain confidence interval for difference of two means for large sample test. (6)

b) State and prove any two properties of regression coefficients. (4)

c) The quantity of oxygen dissolved in water is used as a measure of water pollution. Samples are taken at four locations in a lake and the quantity of dissolved oxygen is recorded as follows (lower reading corresponds to greater pollution)

Location	Quantity of dissolved Oxygen
A	7, 8, 6, 4, 8, 2, 6, 9
B	6, 7, 6, 8, 7, 1, 6, 9, 7, 3
C	7, 2, 7, 4, 6, 9, 6, 4, 6, 5
D	6, 0, 7, 4, 6, 5, 6, 9, 7, 2, 6, 8

Do the data indicate a significant difference in the average amount of dissolved oxygen for the four locations? (6)