MATH 3.1

III Semester M.Sc. in Mathematics Examination, May 2015
TOPOLOGY

Time : 3 Hours

Max. Marks : 80

Instructions : *i) Answer **any five** questions.*
*ii) **All** questions carry **equal** marks.*

1. a) Define a topology on a non-empty set. Prove the following hold in $(X, \mathfrak{J})$
   i) $d(\emptyset) = \emptyset$
   ii) $A \subseteq B \Rightarrow d(A) \subseteq d(B)$
   iii) $d(A \cup B) = d(A) \cup d(B)$ **8**

   b) Prove that a set is closed iff it contains all its limit points. **4**

   c) Prove that a point x belongs to the closure of a set A iff every open set G which containing x has a non-empty intersection with A. **4**

2. a) Given a mapping $f : (X, \mathfrak{J}) \to (Y, \mathcal{U})$, prove that the following are equivalent
   i) f is continuous
   ii) B closed in $Y \Rightarrow f^{-1}(B)$ closed in X.
   iii) $f(\overline{A}) \subseteq \overline{f(A)}.\ \forall A \subseteq X$. **12**

   b) Let X be a metric space with metric d. Define $\overline{d}(x, y) = \min\{d(x, y), 1\}$. Then show that $\overline{d}$ is a metric that induces the topology of X. **4**

3. a) If C is a connected subset of $(X, \mathfrak{J})$ and $C \subseteq Y \subseteq \overline{C}$, then prove Y is connected. **5**

   b) If $\{A_i\}$, is the family of connected subsets of X such that $\bigcap_i A_i \neq \emptyset$, then prove that $\bigcup_i A_i$ is connected. **5**

   c) Prove that a closed subset of a compact space is compact. **6**

P.T.O.

4. a) Prove that a continuous image of compact space is compact. 4

   b) Prove that $(X, \mathfrak{I})$ is compact iff every family of closed sets having finite intersection property has a non-empty intersection. 8

   c) Show that every compact space is countably compact. 4

5. a) Prove that second countability is a topological property. 4

   b) Prove that every second axiom space is a Lindel of space. 4

   c) Prove that a countably compact metric space is totally bounded. 8

6. a) Prove that a compact subset of a Hausdorff space is closed. 8

   b) Show that a metric space is normal and hence T_4. 8

7. State and prove Tychonoff theorem. 16

8. Let X be a regular space with a basis $\mathscr{B}$ that is countably locally finite, then prove that X is metrizable. 16

MATH 3.2

III Semester M.Sc. (Mathematics) Examination, May 2015
MEASURE AND INTEGRATION

Time : 3 Hours

Max. Marks : 80

Note : *1) Answer* ***any five*** *questions.*
2) ***All*** *questions carry* ***equal*** *marks.*

1. a) Let A be an algebra of X and {Ai } a sequence of sets in A. Then prove that there is a sequence {Bi } of sets in A such that $B_n \cap B_m = \phi$ for $n \neq m$ and $\bigcup_{i=1}^{\infty} B_i = \bigcup_{i=1}^{\infty} A_i$. 8

   b) Prove that the interval (a, ∞) is Lebesgue measurable. 8

2. a) Let { E_n } be an infinite decreasing sequence of Lebesgue measurable sets, that is a sequence with $E_{n+1} C E_n$, n = 1, 2, 3, ... Let mE_1 be finite. Then prove that

$$m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} mEn$$ 8

   b) If C is a constant and the functions f and g are measurable real-valued functions defined on the same domain, then show that the functions f + c, cf, f + g and g – f are measurable. 8

3. a) If ϕ and ψ are simple functions which vanish outside a set of finite measure, then prove that, $\int(a\phi + b\psi) = a\int\phi + b\int\psi$ and if $\phi \geq \psi$ a.c., then $\int\phi \geq \int\psi$. 10

   b) State and prove Bounded convergence theorem. 6

4. a) If f and g are non-negative measurable functions defined on a measurable set E then show that

   i) $\int_E cf = c\int_E f, c > 0$ ii) $\int_E (f+g) = \int_E f + \int_E g$

   iii) If $f \leq g$ a.e., then $\int_E f \leq \int_E g$ 10

   b) State and prove Lebesgue convergence Theorem for integrable functions. 6

P.T.O.

5. State and prove Vitali Lemma. **16**

6. a) If f is a function of bounded variation on [a, b] then prove the following : **8**

   i) $P - N = f(b) - f(a)$

   ii) $P + N = T$, where, P, N, T are the positive, negative and total variation of f over [a, b].

   b) If f is bounded and measurable on [a, b] and $F(x) = \int_a^x f(t)\,dt + F(a)$, then prove that $F'(x) = f(x)$ almost every where in [a, b]. **8**

7. a) A function F is an indefinite integral if and only if it is absolutely continuous. **8**

   b) Let (X, B, μ) be a measure space. If $E_i \in B$ for $i = 1, 2, 3, \ldots,$ $\mu E_1 < \infty$ and $E_i \supset E_{i+1}$ $i = 1, 2, 3, \ldots$, then prove that $\mu\left[\bigcap_{i=1}^{\infty} E_i\right] = \lim_{n \to \infty} \mu E_n$. **8**

8. a) State and prove Hahn Decomposition theorem. **10**

   b) If A is an algebra and $A \in \mathscr{A}$, then prove that $\mu A = \mu^* A$. **6**

MATH 3.3

Third Semester M.Sc. (Mathematics) Examination, May 2015
FUNCTIONAL ANALYSIS

Time : 3 Hours Max. Marks : 80

Note : *1) Answer* ***any five*** *questions.*
2) ***All*** *questions carry* ***equal*** *marks.*

1. a) Define a closed set in a metric space. Show that a set F is a closed set in a metric space if and only if the complement of F is open. **6**

   b) If $\overline{A}$ denotes the closure of a subset A of a metric space (X, d), then show that $\overline{A} = \{x \in X : d(x, A) = 0\}$. **4**

   c) Let (X, d) be a complete metric space and (Y, d) be a subspace of (X, d). Then prove that Y is closed if and only if Y is complete. **6**

2. a) Define a contraction mapping. Let (X, d) be a complete metric space and $T : X \to X$ be a contraction mapping. Then prove that T has a fixed point. **6**

   b) Let $f : D \to \mathbb{R}$ be a continuous function of open domain $D \subseteq \mathbb{R}^2$, satisfies Lipschitz's condition : $|f(x, y_1) - f(x, y_2)| \leq M|y_1 - y_2|$ for all $(x, y_1), (x, y_2) \in D$. If X is the space of all solutions of the initial value problem $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$, then show that X is a complete metric space. **6**

   c) Define a everywhere dense subset and nowhere dense subset of a metric space (X, d). Give examples for each. **4**

3. a) State Cantor's intersection theorem and Baire's Category theorem. Making use of Cantor's intersection theorem prove Baire's Category theorem. **8**

   b) Prove that every sequentially compact metric space is both complete and totally bounded. **8**

4. a) Let $T : N \to N'$ be linear operator from a normed linear space N into a normed linear space N'. Then prove that the following are equivalent :
   1) T is a continuous linear operator
   2) T is continuous at x = 0
   3) T is a bounded linear operator. **8**

   b) State Weierstrass theorem and Stone-Weierstrass theorem in real case. Deduce Weierstrass theorem as a simple corollary to Stone-Weierstrass theorem. **8**

P.T.O.

5. a) Let X be a normed linear space. If the closed unit ball $B = \{x \in X : ||x|| \leq 1\}$ in X is compact, then prove that X is finite dimensional. 6

b) State Hahn-Banach theorem for a normed linear space. Prove the theorem by making use of Hahn-Banach theorem for a complex linear space. 10

6. a) State and prove open mapping theorem. 12

b) If $T : X \to Y$ is one to one continuous linear operator from a Banach space X onto Banach space Y, then show that $T^{-1} : Y \to X$ is also a continuous linear operator. 4

7. a) State and prove closed graph theorem. 8

b) State and prove uniform bounded principle. 8

8. a) Prove the following two properties of a Hilbert space H :

i) $||x + y||^2 + ||x - y||^2 = 2(||x||^2 + ||y||^2)$

ii) If $x_n \to x$, $y_n \to y$ as $n \to \infty$, then $(x_n, y_n) \to (x, y)$ as $n \to \infty$. 4

b) Let M be a closed linear subspace of a Hilbert space H. Then prove that $H = M \oplus M^{\perp}$. 8

c) If T is an operator on a Hilbert space H, then prove that T is self adjoint if and only if $(T(x), x)$ is real for all $x \in H$. 4

MATH 3.4

Third Semester M.Sc. (Mathematics) Examination, May 2015
MATHEMATICAL MODELING

Time : 3 Hours Max. Marks : 80

***Note** : 1) Answer **any five** questions.*
*2) **All** questions carry equal marks.*

1. a) Explain any four characteristics of Mathematical modeling. 8
   b) What are the limitation of Mathematical modeling ? 4
   c) A particle of mass m is stationary at time t = 0 and subject to a force $F(t) = F_0 \sin^2 wt$. Set up a differential equation to describe the motion. 4

2. a) Consider the below differential equation $\frac{dx}{dt} = 2\cos(\pi x)$. Find all equilibra and determine the stability of those equilibria. 8
   b) Construct a model for rectilinear motion of string. 8

3. a) Explain the construction of spring and dashpot system. 8
   b) Describe a model for the detection of diabetes. 8

4. a) Draw some trajectories for the completion model $\frac{dx}{dt} = x(1-0.1y); \frac{dy}{dt} = -y(1-0.1x)$. Also discuss its stability of equilibrium points. 8
   b) Show that the force required to make a particle of mass 'm' move in a circular orbit of radius with velocity 'v' is $\frac{mv^2}{a}$ directed towards the centre. 8

5. a) Describe a model for glacier flow. 8
   b) Describe a partial differential equation model for birth-death-immigration process. 8

P.T.O.

6. a) Solve initial value problem for Burger's equation $u_t + vu_x = vu_{xx}$, $x \in R$, $t > 0$ and $u(x, 0) = F(x)$. 8

b) Show that in cylindrical polar co-ordinates $\nabla^2\psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{\partial^2\psi}{\partial z^2}$. 8

7. a) What are the composition of nitrogen, oxygen, carbon-di-oxide and Argan-inter gas pollution ? 4

b) Distinguish between primary and secondary air pollutants. 6

c) Identify the various pollution sources of the following air contaminants. 6

i) Hydrocarbons

ii) Hydrogen sulphide

iii) Sulphur dioxide.

8. Using gradient diffusion model, derive an expression for air pollution and hence derive conservation of mass equation. 16

III Semester M.Sc. Examination, May 2015
MATHEMATICS
Computer Programming

Time : 3 Hours | Max. Marks : 80

Instructions : *Answer **any five full** questions. **All** questions carry **equal** marks.*

1. a) What is microprocessor ? Explain briefly.
   b) What are the differences between analog and digital computer ?
   c) Explain the following operating systems.
      i) File
      ii) Program
      iii) Directory
      iv) Multilevel directories. **(4+4+8)**

2. a) Write an algorithm and flow chart to compute the circumference of a circle.
   b) Define decision table and explain its different types.
   c) Describe the sequences of steps involved in problem solving of computer programming. **(4+4+8)**

3. a) Define and explain the different types of C operators.
   b) Define storage class and explain the different storage class and its specifications.
   c) Explain the four types of integer constants in C. **(6+6+4)**

4. a) Explain the following along with example.
      i) Loop
      ii) For loop
      iii) While loop
      iv) Do while loop
   b) What is special operator ? Explain briefly.
   c) What is the difference between break and continue statements ? Explain with an example. **(8+4+4)**

5. a) Explain the types of arrays with example.
   b) Write a program to find the largest and smallest number in array.
   c) Write an algorithm for binary searching. **(6+6+4)**

6. a) Explain the categories of functions with an example.
   b) Explain the pointer declaration along with examples.
   c) Explain the following :
      i) Dynamic memory allocation
      ii) Allocating a block of memory
      iii) Allocating multiple blocks of memory. **(4+6+6)**

7. a) Write an algorithm and C-Program for the sum and product of two matrices.
   b) Explain briefly bisection method.
   c) Write an algorithm and C program for Newton-Raphson method. **(6+4+6)**

8. a) Write an algorithm and C program for Weddle's rule.
   b) Write an algorithm and C program for Runge-Kutta 2^{nd} order. **(8+8)**