Math 2.1

II Semester M.Sc., Mathematics Examination, January - 2016

Linear Algebra

Time: 3 hours **Max. Marks:80**

Instructions: i) Answer any five full questions.
ii) All questions carry equal marks

1. a) Define a subspace. Let V be the vector space of all real valued continuous functions over the field $\mathfrak{R}$ of all real numbers. Show that the set S of solutions of the differential equation $2\frac{d^2y}{dx^2}-9\frac{dy}{dx}+2y=0$ is a subspace of V.

   b) Express $v=(1,-2,5)$ in $\mathfrak{R}^3$ as a linear combination of the following vectors: $v_1=(1,1,1)$, $v_2=(1,2,3)$, $v_3=(2,-1,1)$.

   c) If the mapping $T:\mathfrak{R}^2\rightarrow\mathfrak{R}^2$ defined by $T(x,y)=(4x+5y,\ 6x-y)$, then show that T is linear transformation.

   (6+5+5)

2. a) State and prove dimension theorem for linear transformation.

   b) Show that product of two linear transformations is a linear transformation.

   c) If V is finite dimensional vector space over a field F, then prove that $T\in A(V)$ is non-invertible (singular) if and only if there exists a $v\neq 0$ in V such that $T(v)=0$.

   (7+5+4)

3. a) If T has matrix $A=[a_{ij}]_{m\times n}$ relative to the bases $\boldsymbol{A}$ of U and $\boldsymbol{B}$ of V. If Q is the matrix of transition from $\boldsymbol{A}$ to the basis $\boldsymbol{A}'$ of U and P is the matrix of transition from $\boldsymbol{B}$ to the basis $\boldsymbol{B}'$ of V, then prove that the matrix of T relative to $\boldsymbol{A}'$ and $\boldsymbol{B}'$ is $\boldsymbol{P^{-1}AQ}$.

   b) Prove that the two $m\times n$ matrices A and B represent the same linear transformation T of U into V if and only if A and B are equivalent.

   c) Relative to the basis $B=\{v_1,v_2\}=\{(1,1),(2,3)\}$ of $\mathfrak{R}^2$, find the co-ordinate matrix of $i)\ v=(4,-3)$ and $ii)\ v=(a,b)$

   (6+5+5)

4. a) If $A=[a_{ij}]_n$, then prove that $det(A^T)=det(A)$

   b) Show that for any matrix $A=[a_{ij}]$, $(A)=\sum_{(j)}(-1)^s a_{1j_1}a_{2j_2}\cdots\cdots\cdots a_{nj_n}$, where $\sum_{(i)}$ denotes the sum over all possible permutations $i_1, i_2,\ldots\ldots i_n$ of $1,2,\ldots\ldots n$, and s is the number of interchanges used to carry $i_1, i_2,\ldots\ldots i_n$ into the natural ordering.

   c) Explain the Crammer's rule with an example.

   (4+8+4)

P.T.O

5. a) Show that a square matrix A has 0 as eigen values if and only if A is not invertible.
   b) Let T be a linear operator on a finite dimensional vector space V and let $f(t)$ be the characteristic polynomial of T. Then show that $f(T) = T_0$, the zero transformation.
   c) Let V be an inner product space over F, then prove the following:

      $|\langle x, y\rangle| = \|x\|.\|y\|$

      $\|x + y\| \leq \|x\| + \|y\|$ for all x,y$\in$V. **(5+5+6)**

6. a) Let $w_1 = (1,0,1,0)$, $w_2 = (1,1,1,1)$ and $w_3 = (0,1,2,1)$ in$\Re^4$. If $\{w_1, w_2, w_3\}$ is linearly independent. Use the Gram - Schmidt Orthogonalization process to compute the orthogonal vectors v_1, v_2 and v_3 and then find the normalize of these vectors to obtain an orthonormal set.
   b) Let C[-π,π] be the inner product space of all continuous functions defined on [-π, π] with the inner product defined by $\langle f, g\rangle = \int_{-\pi}^{\pi} f(t).g(t).dt$. Prove that $\sin t$ and $\cos t$ are orthogonal functions in C [-π,π].
   c) State and prove Bessel's inequality **(6+4+6)**

7. a) Let V be a finite - dimensional inner product space, and let T be a linear operator on V. Then show that there exists a unique function $T^* : V \rightarrow V$ such that $\langle T(x), y\rangle = \langle x, T^*(y)\rangle$ for all x, y $\in$V. Furthermore, T^* is linear.
   b) Find the adjoint of linear transformation $T : R^2 \rightarrow R^2$ given by $T(x,y) = (x+2y, x-y)$ for all $(x,y) \in R^2$.
   c) Let V be an inner product space and let T be a linear operator on V. Then prove that T is an orthogonal projection if and only if T has an adjoint T^* and $T^2 = T = T^*$.

(6+4+6)

8. a) If the matrix $A \in F_n$ has all its eigen values in F, then prove that there is a matrix $C \in F_n$ such that CAC^{-1} is a triangular matrix.
   b) Let $V = F_3[x]$ be the space of all polynomials of degree atmost 3 and let $T: V \rightarrow V$ be the linear transformation given by $T: V \rightarrow V$ be the linear transformation given by $T(f) = f'$ Find the minimal polynomial of T.

c) *If* $T \in A(V)$ has a minimal polynomial $p(x) = q(x)^e$, where $q(x)$ is a monic, irreducible polynomial in $F[x]$, then show that a basis of V over F can be found in which the matrix of T is of the form;

$$\begin{pmatrix} C\left(q(x)^{e_1}\right) & & \\ & C\left(q(x)^{e_2}\right) & \\ & & C\left(q(x)^{e_r}\right) \end{pmatrix}$$

Where, $e = e_1 \geq e_2 \geq \ldots\ldots\ldots\ldots \geq e_r$. **(6+4+6)**

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MATH 2.2

II Semester M.Sc. in Mathematics Examination, January, 2016

REAL ANALYSIS-II

Time : 3 Hours **Max. Marks : 80**

Note : 1) Answer any **Five** Full Questions.

2) All Questions Carry Equal Marks.

1.a) Define a converging sequence. If f and g are real valued functions defined on $A \subseteq \mathbb{R}$, *and* $x_o \in \mathbb{R}$, then prove that $\lim_{x\to xo}(f+g)(x) = \lim_{x\to xo} f(x) + \lim_{x\to xo} g(x)$.

b) Let $A \subseteq \mathbb{R}$ *and* $f{:}A \to \mathbb{R}$. .Then Prove that f is continuous at $x_o \in A$ if and only if for every sequence $\{x_n\}$ in A with $x_n \neq x_o$ for all n, $\lim_{n\to\infty} x_n = x_o \Rightarrow \lim_{n\to\infty} f(x_n) = f(x_o)$.

C) Prove that a continuous image of a connected space is connected. **(6+6+4)**

2.a) Define a monotonic function. If f is monotonic on (a,b), then prove that the set of points at which f is discontinuous is atmost countable.

b) State and prove L'Hospital's rule. **(8+8)**

3.a) State and prove the necessary and sufficient condition for the Riemann-Steiltje's integrability of a bounded function.

b) Show that $f(x) = \begin{cases} 1, if\ x\ is\ a\ rational \\ 0, if\ x\ is\ irrational \end{cases}$ is not Riemann-Steiltje's integrable on [a,b] for any monotonically increasing function $\alpha(x)$.

c) Let f be monotonic and α be monotonic and continuous on $[a,b]$. Then prove that $f \in R(\alpha)$. **(7+3+6)**

4.a) State and prove the second mean value theorem for Riemann-Stieltjes integrals.

b) Prove that product of two functions of bounded variation functions is also of bounded variation.

c) Evaluate $\int_0^3 x\, d([x] - x)$, where [x] is the largest integer not greater than x.

(7+6+3)

P.T.O

5.a) Define uniform convergence of a sequence of functions. Show that the sequence $\{f_n\}$ where $f_n(x)=\frac{nx}{1+n^2x^2}$, $x \in \mathbb{R}$, is not uniformly convergent on any interval containing zero.

b) State and prove the *M – Test* for uniform convergence of sequence of functions.

c) State and prove *Cauchy criterion* for uniform convergence of a sequence of functions.

(4+5+7)

6.a) If $\{f_n\}$ is a decreasing sequence of continuous functions converging pointwise to a continuous function f on a compact subset A of $\mathbb{R}$, then prove that $\{f_n\}$ converges uniformly on A.

b) State and prove the theorem on uniform convergence and continuity.

(8+8)

7.a) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

b) State and prove Weirstrass approximation theorem.

(8+8)

8.a) Define the radius of convergence of a power series. Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

b) Define the exponential function $E(x)$. Prove the following

(i) E is strictly increasing on $\mathbb{R}$ and has range equal to $\{ x \in \mathbb{R}: x>0 \}$.

(ii) $\lim_{x\to\infty} E(x) = \infty$

(iii) $\log_{x\to-\infty} E(x) = 0$

c) Define the trigonometric functions C and S. Prove that C and S are periodic with period 2π.

(3+7+6)

Math 2.3

II Semester M.Sc. Mathematics Examination, January - 2016

Complex Analysis - II

Time: 3 hours **Max. Marks:80**

Instructions: i) Answer any five full questions.
ii) All questions carry equal marks

1. a) State and prove argument principle.
   b) State and prove Rouch's Theorem. Deduce fundamental theorem of algebra from Rouche's theorem.

(8+8)

2. a) Evaluate the given integral by residue method. $\int_0^\infty \frac{x^{\frac{1}{6}} \log x}{(1+x)^2} dx$
   b) Obtain the polar form of the Laplace's equation.

(8+8)

3. a) State and prove Poisson integral formula for a harmonic function *u(z)*.
   b) State and prove Harnack's inequality theorem.

(8+8)

4. a) State and prove Taylor's theorem for an analytic function *f(z)* in a region *D* about a point Z_0 in *D*.
   b) Determine the Laurent series representation of $f(z) = \frac{\sin \pi z}{(z-1)^3}$ in the ring $D = \{z : 0 < |z-1| < \infty\}$.

(8+8)

5. a) State and prove Mittag - Leffler's theorem on a meromorphic function.
   b) Show that, $\frac{\pi^2}{\sin^2 \pi z} = \sum_{n=-\infty}^{\infty} \frac{1}{(z-n)^2}$

(10+6)

P.T.O.

6. a) Prove that an infinite product $\Pi(1+a_n)$ is convergent if and only if the series $\Sigma \log(1+a_n)$ is convergent, provided the principle value of the logarithm being taken in the each case.

   b) State and prove Weierstrass Factorization theorem.

(8+8)

7. a) State and Prove Jensen's Formula.

   b) State and prove Hadamard's theorem.

(8+8)

8. a) Define Riemann's zeta function $\zeta(z)$, prove that the zeta function $\zeta(z)$ is a meromorphic function on the complex plane with only one pole, a simple pole at $z = 1$ with residuce 1.

   b) Prove that,

$$\zeta(1-z) = 2^{1-z}\pi^{-z}\cos\left(\frac{\pi z}{2}\right)\Gamma(z)\zeta(z) .$$

(8+8)

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MATH 2.4

II Semester M.Sc. in Mathematics Examination, January, 2016

NUMERICAL ANALYSIS

Time : 3 Hours **Max. Marks : 80**

Instructions : Answer any **Five** Full Questions.
All Questions Carry Equal Marks.

1.

(a) A real root of the equation $f(x) = x^3 - 5x + 1 = 0$ lies in the interval (0,1). Perform four iterations of the Secant method and the Regula-Falsi method to obtain this root.

(b) Perform two iterations of the Muller method to find the smallest positive root of the equation $f(x) = x^3 - \frac{1}{2} = 0\,; x_0 = 0, x_1 = 1, x_2 = \frac{1}{2}$.

(8+8)

2.

(a) Explain rate of convergence of Muller method and Chebyshev method.

(b) Explain the system of nonlinear equations using Newton –Raphson method.

(8+8)

3.

(a) Perform two iterations of the linear iteration method followed by one iteration of the Aitken Δ^2 method to find the root of the equation $f(x) = x - e^{-x} = 0, x_0 = 1$.

(b) Perform two iterations of Bairstow method to extract a quadratic factor $x^2 + px + q$ from the Polynomial $x^3 + x^2 - x + 2 = 0$.

(8+8)

4.

(a) Given that $f(0) = 1, f(1) = 3, f(3) = 55,$ find the unique polynomial of degree 2 or less, which fits the given data. Find the bound on the error.

(b) Define and explain Piecewise and Spline interpolations. **(8+8)**

P.T.O

5.

(a) Describe Lagrange's interpolation method. Obtain the truncation error formula.

(b) Given the following values of $f(x)$ and $f'(x)$

x	$f(x)$	$f'(x)$
-1	1	-5
0	1	1
1	3	7

Estimate the values of $f(-0.5)$ and $f(0.5)$ using the Hermite interpolation. The exact values are $f(-0.5)=\frac{33}{64}$ and $f(0.5)=\frac{97}{64}$. **(8+8)**

6.

(a) Derive the uniform polynomial approximation.

(b) Define a numerical differentiation method of order '*p*'. Obtain numerical differentiation methods based on finite difference operators. **(8+8)**

7.

(a) Evaluate the integral $\int_0^1 \frac{dx}{1+x}$ using (i) composite Trapezoidal rule, (ii) Composite Simpson's rule, with 2, 4 and 8 equal subintervals.

(b) Solve the difference equation $\Delta^2 y_j+3\Delta y_j-4y_j=j^2$ with the initial conditions $y_0=0, y_2=2$.

(8+8)

8.

(a) Illustrate **P(E C)^{M}E** method for the equation $u'=\lambda u$ and P-C set.

(b) Explain the non-linear second order differential equations using boundary conditions.

(8+8)

Math 2.5

Second Semester M.Sc. (Mathematics) Examination, January – 2016

OPERATION RESEARCH

Time: 3 hours **Max Marks: 80**

Note: 1) Answer any five full questions.
2) All questions carry equal marks.

1. a) What is an Operations Research? Describe the Characteristics of Operation Research. (6)

   b) Explain the following terms related to linear programming model.

   a) Decision variables b) Objective function c) Constraints (5)

   c) A company produces two types of hats. Each hat of the first type requires twice as much labor time as the second type. If all hats are of the second type only, the company can produce a total of 500 hats a day. The market limits daily sales of the first and second type to 150 and 250 hats. Assuming that profit per hat are Rs. 8 for type A and Rs. 5 for type B. Formulate the problem as a Linear Programming Model in order to determine the number of hats to be produced of each type so as to maximize the profit. (5)

2. a) Solve the LPP by Graphical Method: Maximize $z = 5x_1 + 4x_2$ subjected to constraints: $6x_1 + 4x_2 \le 24,\ x_1 + 2x_2 \le 6,\ -x_1 + x_2 \le 1,\ x_2 \le 2,\ x_1, x_2 \ge 0$. (4)

   b) Write an algorithm of simplex method. Using the method solve the following LPP: Maximize $z = 3x_1 + 8x_2$, subject to constraints: $3x_1 - 5x_2 \ge -10,\ 2x_1 - x_2 - x_3 \le 20,$ $x_1 + 2x_2 \le 15\ \ x_1, x_2 \ge 0$. (12)

3. a) Solve by Two-Phase Simplex method: Maximize $z = 5x_1 + 8x_2$, subjected to constraints: $3x_1 + 2x_2 \ge 3;\ x_1 + 4x_2 \ge 4;\ x_1 + x_2 \le 5;\ x_1,\ x_2 \ge 0$. (10)

   b) If either the primal or the dual linear programming problem has an unbounded objective function value, then show that the other problem has no feasible solution. (6)

4. a) State and prove Duality theorem. (6)

   b) By using Dual Simplex method to solve the LPP - Maximize : $z = -2x_1 - 2x_3$; subject to constraints : $x_1 + x_2 - x_3 \ge 5;\ x_1 - 2x_2 + 4x_3 \ge 8;\ x_1,\ x_2,\ x_3 \ge 0$. (10)

5. a) Explain mathematical formulation of the transportation problems (4)

b) Show that the number of basic variables in a basic solution of a transportation problem are at the most $m+n-1$. (6)

c) Determine IBFS to the transportation problem by using Row Minimum method.

	d_1	d_2	d_3	Availability
O_1	50	30	220	1
O_2	90	45	170	3
O_3	250	200	50	4
Requirement	4	2	2	

(6)

6. a) Find the optimum solution for the following transportation problems.

	To			Available
From	2	7	4	5
	3	3	1	8
	5	4	7	7
	1	6	2	14
Required	7	9	18	34

(10)

b) Consider problem of assigning 5 jobs to five persons, the assignment cost are as give below.

	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Determine the optimum assignment schedule and minimum arrangement cost. (6)

7. a) What are the characteristics of Game theory? Write a flow chart for the systematic application. (8)

b) Solve the following (2 × 3) game graphically:

Player B

$$\text{Player } A \quad \begin{bmatrix} 8 & 5 & 2 \\ 1 & 3 & 11 \end{bmatrix}$$

(8)

8. a) Explain the Kendall's notion for representing the Queuing Model. (4)

b) Explain the limitations and applications of Queuing model (4)

c) On an average 96 patients per 24-hour day require the service of an emergency clinic. Also an average, a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs 100 per patient treated to obtain an average serving time of 10 minutes, and that each minute of decrease in this average time would cost Rs. 10 per patient treated. How much would have to be budgeted by the clinic to decrease the average size of the queue from $1\frac{1}{3}$ patients to $\frac{1}{2}$ patient. (8)