INTERNATIONAL Olympiad of Mathematics

Presented by SILVER ZONE ISO 9001: 2000 Certified

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SILVER ZONE FOUNDATION

Question Paper

CLASS - 12 Duration : 60 Minutes Total Questions : 40 Maximum Marks :100

MATHEMATICS 1st Level

iOM Roll Number

Student's Name

TEST PAPER TYPE AA

INSTRUCTIONS

1. **Write your 12 digit iOM roll number and your name on top of the question paper in the given space.**
2. **Filling up improper roll number may lead to unavailability of 'Result'.**
3. **This question paper consist of 40 questions. Each question carries equal marking of 2.5 marks each.**
4. **Mark your answer (A, B, C, D or E) on the Answer Sheet with HB Pencil or Black/Blue Ball point Pen.**
5. **This question paper contains 4 pages.**
6. **Do not start attempting the test paper till you are asked to do so.**
7. **Time taken by individual student to complete the paper will be one of the criteria for tie-breaker, if any.**

Note: Return this question paper along with answer sheet

1. **If $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - \lambda M - I_2 = 0$, then $\lambda =$**

(A) -2 (B) 2
(C) -4 (D) 4
(E) None of these

2. **If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then $A^2 =$**

(A) Unit matrix (B) Null matrix
(C) A (D) $-A$
(E) None of these

3. **If O(A) = 2 × 3, O(B) = 3 × 2, and O(C) = 3 × 3, which one of the following is not defined?**

(A) CB+A' (B) BAC
(C) C(A+B')' (D) C(A+B')
(E) None of these

4. **If A and B are two matrices and (A+B) (A –B) = $A^2 - B^2$, then**

(A) AB = BA (B) $A^2 + B^2 = A^2 - B^2$
(C) A'B' = AB (D) All of these
(E) None of these

5. **The inverse of $\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$ is**

(A) $\frac{-1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$

(B) $\frac{-1}{8}\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$

(C) $\frac{1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$

(D) $\frac{1}{8}\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$

(E) None of these

6. **If $A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $AX = B$, then $X =$**

(A) $[5 \quad 7]$ (B) $\frac{1}{3}\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

(C) $\frac{1}{3}[5 \quad 7]$ (D) $\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

(E) None of these

7. If $A = \begin{bmatrix} 2 & 4 & 5 \\ 4 & 8 & 10 \\ -6 & -12 & -15 \end{bmatrix}$, then rank of A is equal to

(A) 0 (B) 1
(C) 2 (D) 3
(E) None of these

8. If $\cos^{-1}\left(\frac{1}{x}\right) = \theta$, then $\tan\theta$ =

(A) $\frac{1}{\sqrt{x^2-1}}$ (B) $\sqrt{x^2+1}$
(C) $\sqrt{1-x^2}$ (D) $\sqrt{x^2-1}$
(E) None of these

9. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is

(A) $\frac{-2\pi}{3}$ (B) $\frac{-\pi}{3}$
(C) $\frac{4\pi}{3}$ (D) $\frac{5\pi}{3}$
(E) None of these

10. If $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, then x =

(A) –1 (B) $\frac{1}{6}$
(C) –1, $\frac{1}{6}$ (D) $\frac{1}{3}$
(E) None of these

11. $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$ =

(A) 0 (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$ (D) π
(E) None of these

12. If the distance between the points (a,2) and (3,4) be 8, then a =

(A) $2+3\sqrt{15}$ (B) $2-3\sqrt{15}$
(C) $2\pm3\sqrt{15}$ (D) $3\pm2\sqrt{15}$
(E) None of these

13. Three vertices of a parallelogram taken in order are (–1, – 6), (2, – 5) and (7, 2). The fourth vertex is

(A) (1, 4) (B) (4, 1)
(C) (1, 1) (D) (4, 4)
(E) None of these

14. If the position vectors of P and Q are (i + 3j – 7k) and (5i – 2j + 4k), then $|\overrightarrow{PQ}|$ is

(A) $\sqrt{158}$ (B) $\sqrt{160}$
(C) $\sqrt{161}$ (D) $\sqrt{162}$
(E) None of these

15. If $\hat{a}$ and $\hat{b}$ are unit vectors such that $\left[\hat{a}\ \hat{b}\ \hat{a}\times\hat{b}\right] = \frac{1}{4}$, then angle between $\hat{a}$ and $\hat{b}$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$
(E) None of these

16. The direction cosines of the line $4x - 4 = 1 - 3y = 2z - 1$ are

(A) $\frac{3}{\sqrt{56}}, \frac{-4}{\sqrt{56}}, \frac{6}{\sqrt{56}}$
(B) $\frac{3}{\sqrt{29}}, \frac{-4}{\sqrt{29}}, \frac{6}{\sqrt{29}}$
(C) $\frac{3}{\sqrt{61}}, \frac{-4}{\sqrt{61}}, \frac{6}{\sqrt{61}}$
(D) $\frac{4}{\sqrt{29}}, \frac{-3}{\sqrt{29}}, \frac{2}{\sqrt{29}}$
(E) None of these

17. Perpendicular distance of the point (3, 4, 5) from the y–axis, is

(A) $\sqrt{34}$ (B) $\sqrt{41}$
(C) 4 (D) 5
(E) None of these

18. The angle between the straight lines $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ and $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$ is

(A) 45° (B) 30°
(C) 60° (D) 90°
(E) None of these

19. If two planes intersect, then the shortest distance between the planes is

(A) $\cos\theta$ (B) $\cos 90°$
(C) $\sin 90°$ (D) $\tan 90°$
(E) None of these

20. The line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is parallel to the plane

(A) $3x+4y+5z=7$ (B) $2x+y-2z=0$
(C) $x+y-z=2$ (D) $2x+3y+4z=0$
(E) None of these

21. $\frac{d}{dx}\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}=$

(A) $\sec^2 x$ (B) $-\sec^2\left(\frac{\pi}{4}-x\right)$
(C) $\sec^2\left(\frac{\pi}{4}+x\right)$ (D) $\sec^2\left(\frac{\pi}{4}-x\right)$
(E) None of these

22. $\frac{d}{dx}\left(\frac{\sec x+\tan x}{\sec x-\tan x}\right)=$

(A) $\frac{2\cos x}{(1-\sin x)^2}$ (B) $\frac{\cos x}{(1-\sin x)^2}$
(C) $\frac{2\cos x}{1-\sin x}$ (D) $\frac{2\cos x}{(1+\sin x)^2}$
(E) None of these

23. If $y=\sqrt{\frac{1+e^x}{1-e^x}}$, then $\frac{dy}{dx}=$

(A) $\frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$ (B) $\frac{e^x}{(1-e^x)\sqrt{1-e^x}}$
(C) $\frac{e^x}{(1-e^x)\sqrt{1+e^{2x}}}$ (D) $\frac{e^x}{(1-e^x)\sqrt{1+e^x}}$
(E) None of these

24. If f(x) = sin log x, then the value of $f(xy)+f\left(\frac{x}{y}\right)-f(x)\cdot\cos\log y$ is equal to

(A) 1
(B) 0
(C) –1
(D) sin log x. cos log y
(E) None of these

25. If f(x) = cos (log x), then $f(x)\,f(y)-\frac{1}{2}[f(x/y)+f(xy)]=$

(A) –1 (B) $\frac{1}{2}$
(C) –2 (D) 0
(E) None of these

26. If $f(x)=\frac{x}{x-1}=\frac{1}{y}$, then f(y)=

(A) x (B) x + 1
(C) x – 1 (D) 1 – x
(E) None of these

27. If $f(x)=\begin{cases}\frac{\sin 2x}{5x}, \text{when } x\neq 0\\ k, \text{when } x=0\end{cases}$ is continuous at x = 0, then the value of k will be

(A) 1 (B) $\frac{2}{5}$
(C) $-\frac{2}{5}$ (D) All of these
(E) None of these

28. If $f(x)=\frac{x}{1+|x|}$ for $x\in R$, then f'(0) =

(A) 0 (B) 1
(C) 2 (D) 3
(E) None of these

29. $\int\frac{dx}{\tan x+\cot x}=$

(A) $\frac{\cos 2x}{4}+c$ (B) $\frac{\sin 2x}{4}+c$
(C) $-\frac{\sin 2x}{4}+c$ (D) $-\frac{\cos 2x}{4}+c$
(E) None of these

30. The value of $\int\frac{1}{(x-5)^2}dx$ is

(A) $\frac{1}{x-5}+c$ (B) $-\frac{1}{x-5}+c$
(C) $\frac{2}{(x-5)^3}+c$ (D) $-2(x-5)^3+c$
(E) None of these

31. If $\int x \sin x dx = -x \cos x + A$, then A =

(A) sin x + constant (B) cos x + constant
(C) Constant (D) All of these
(E) None of these

32. $\int_0^{\pi/2} e^x \sin x \, dx =$

(A) $\frac{1}{2}(e^{\pi/2} - 1)$ (B) $\frac{1}{2}(e^{\pi/2} + 1)$
(C) $\frac{1}{2}(1 - e^{\pi/2})$ (D) $2(e^{\pi/2} + 1)$
(E) None of these

33. The order and degree of the differential equation $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$ **are**

(A) 1 and $\frac{1}{2}$ (B) 2 and 1
(C) 1 and 1 (D) 1 and 2
(E) None of these

34. The differential equation of all straight lines passing through the origin is

(A) $y = \sqrt{x\frac{dy}{dx}}$
(B) $\frac{dy}{dx} = y + x$
(C) $\frac{dy}{dx} = \frac{y}{x}$
(D) All of these
(E) None of these

35. Solution of differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ **is**

(A) $x^3 + y^2 = px^2$
(B) $\frac{x^2}{2} + \frac{y^3}{x} = y^2 + P$
(C) $x^2 + y^3 = px^2$
(D) $x^2 + y^2 = px^3$
(E) None of these

36. A problem of mathematics is given to three students whose chances of solving the problem are $\frac{1}{3}$, $\frac{1}{4}$ **and** $\frac{1}{5}$ **respectively. The probability that the question will be solved is**

(A) $\frac{2}{3}$ (B) $\frac{3}{4}$
(C) $\frac{4}{5}$ (D) $\frac{3}{5}$
(E) None of these

37. The probability that A speaks truth is $\frac{4}{5}$, **while this probability for B is** $\frac{3}{4}$. **The probability that they contradict each other when asked to speak on a fact is**

(A) $\frac{4}{5}$ (B) $\frac{1}{5}$
(C) $\frac{7}{20}$ (D) $\frac{3}{20}$
(E) None of these

38. Let A and B be two events such that P(A) = 0.3 and P (A $\cup$ B) = 0.8. If A and B are independent events, then P(B) =

(A) $\frac{5}{6}$ (B) $\frac{5}{7}$
(C) $\frac{3}{5}$ (D) $\frac{2}{5}$
(E) None of these

39. Two dice are thrown. What is the probability that the sum of the numbers appearing on the two dice is 11, if 5 appears on the first?

(A) $\frac{1}{36}$ (B) $\frac{1}{6}$
(C) $\frac{5}{6}$ (D) All of these
(E) None of these

40. 8 coins are tosssed simultaneously. The probability of getting at least 6 heads is

(A) $\frac{57}{64}$ (B) $\frac{229}{256}$
(C) $\frac{7}{64}$ (D) $\frac{37}{256}$
(E) None of these