INTERNATIONAL Olympiad of Mathematics

Presented by SILVER ZONE ISO 9001: 2000 Certified

Organized by Mathematics Olympiad Foundation New Delhi, India

SILVER ZONE FOUNDATION

Question Paper

CLASS - 11 Duration : 60 Minutes Total Questions : 40 Maximum Marks :100

MATHEMATICS 1st Level

iOM Roll Number

Student's Name

TEST PAPER TYPE AA

INSTRUCTIONS

1. **Write your 12 digit iOM roll number and your name on top of the question paper in the given space.**
2. **Filling up improper roll number may lead to unavailability of 'Result'.**
3. **This question paper consist of 40 questions. Each question carries equal marking of 2.5 marks each.**
4. **Mark your answer (A, B, C, D or E) on the Answer Sheet with HB Pencil or Black/Blue Ball point Pen.**
5. **This question paper contains 4 pages.**
6. **Do not start attempting the test paper till you are asked to do so.**
7. **Time taken by individual student to complete the paper will be one of the criteria for tie-breaker, if any.**

Note: Return this question paper along with answer sheet

1. Find the coordinates of the circumcentre of the triangle whose vertices are (8,6), (8, –2) and (2, –2).

(A) (5,5) (B) (7,2)
(C) (5,2) (D) (2,5)
(E) None of these

2. Find the equation of the circle with centre (4,–5) and radius $\sqrt{53}$.

(A) $x^2 + y^2 - 5x + 10y = 10$
(B) $x^2 + y^2 - 6x + 12y = 12$
(C) $x^2 + y^2 - 7x + 14y = 12$
(D) $x^2 + y^2 - 8x + 10y = 12$
(E) None of these

3. Find the angle between the lines $3y - \sqrt{3}x - 12 = 0$ and $y - \sqrt{3}x + 9 = 0$.

(A) 30° (B) 90°
(C) 150° (D) Both (A) and (C)
(E) None of these

4. If A = {3,5,7,9,11}, B = {7,9,11,13,15} C = {11,13,15,17}, D = {17,19}, then find the value of $(A \cup D) \cap (B \cup C)$.

(A) {7,9,11,15} (B) {7,9,11,17}
(C) {5,7,11,17,19} (D) {3,5,11,19}
(E) None of these

5. In a group of students 200 know Hindi, 125 know English and 75 know both. Each of the students knows either Hindi or English. How many students are there in the group ?

(A) 125 (B) 225
(C) 250 (D) 300
(E) None of these

6. If the set A and B are defined as

$$A = \left\{(x,y) : y = \frac{1}{x}, 0 \neq x \in R\right\}$$

and B = {(x, y): y = – x, x ∈ R}, then

(A) $A \cap B = A$ (B) $A \cap B = B$
(C) $A \cap B = \phi$ (D) All of these
(E) None of these

7. If A, B, C be three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then

(A) A = B (B) B = C
(C) A = C (D) A = B = C
(E) None of these

8. If $\log_7 2 = m$, then $\log_{49} 28$ is equal to

(A) $2(1+2m)$ (B) $\frac{1+2m}{2}$

(C) $\frac{2}{1+2m}$ (D) $1+m$

(E) None of these

9. If $x = 2^{1/3} - 2^{-1/3}$, then $2x^3 + 6x =$

(A) 0 (B) 2

(C) 3 (D) 4

(E) None of these

10. If $\sin\theta + \cos\theta = 1$, then the general value of θ is

(A) $2n\pi$

(B) $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

(C) $2n\pi + \frac{\pi}{2}$

(D) $2n\pi + \frac{2\pi}{2}$

(E) None of these

11. The most general value of θ satisfying the equations $\sin\theta = \sin\alpha$ and $\cos\theta = \cos\alpha$ is

(A) $2n\pi + \alpha$ (B) $2n\pi + \pi/4$

(C) $n\pi + \alpha$ (D) $n\pi - \alpha$

(E) None of these

12. Genral solution of $\tan 5\theta = \cot 2\theta$ is

(A) $\theta = \frac{n\pi}{7} + \frac{\pi}{14}$

(B) $\theta = \frac{n\pi}{7} + \frac{\pi}{5}$

(C) $\theta = \frac{n\pi}{7} + \frac{\pi}{2}$

(D) $\theta = n\pi + (-1)^n \frac{\pi}{3}$, $n \in z$

(E) None of these

13. The equation $\sin x + \cos x = 2$ has

(A) One solution

(B) Two solution

(C) Infinite number of solutions

(D) No solution

(E) None of these

14. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he steps back 40 metres from the bank, he finds the angle to be 30°. The breadth of the river is

(A) 20m (B) 30m

(C) 40m (D) 60m

(E) None of these

15. If $1 = \sqrt{-1}$, then $1 + i^2 + i^3 - i^6 + i^8$ is equal to

(A) $2+i$ (B) $2-i$

(C) $3+i$ (D) -1

(E) None of these

16. If $x + \frac{1}{x} = 2\cos\theta$, then x is equal to

(A) $\cos\theta + i\sin\theta$ (B) $\cos\theta - i\sin\theta$

(C) $\cos\theta \pm i\sin\theta$ (D) $\sin\theta \pm i\cos\theta$

(E) None of these

17. The smallest positive integer n for which $(1+i)^{2n} = (1-i)^{2n}$ is

(A) 1 (B) 2

(C) 3 (D) 4

(E) None of these

18. If $(a+ib)(c+id)(e+if)(g+ih) = A + iB$, then $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) =$

(A) $A^2 + B^2$ (B) $A^2 - B^2$

(C) A^2 (D) B^2

(E) None of these

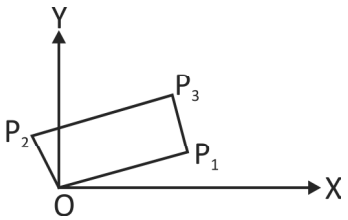
19. If $z = \frac{1 - i\sqrt{3}}{1 + i\sqrt{3}}$, then arg (z) =

(A) 60° (B) 120°

(C) 240° (D) 300°

(E) None of these

20. If the points P_1 and P_2 represent two complex numbers Z_1 amd Z_2, then the point P_3 represents the number



(A) $Z_1 + Z_2$ (B) $Z_1 - Z_2$

(C) $Z_1 \times Z_2$ (D) $Z_1 \div Z_2$

(E) None of these

21. If $x+\frac{1}{x}=2\cos\theta$, then $x^n+\frac{1}{x^n}$ is equal to

(A) $2\cos n\theta$
(B) $2\sin n\theta$
(C) $\cos n\theta$
(D) $\sin n\theta$
(E) None of these

22. If p^{th} term of an A.P. be q and q^{th} term be p, then its r^{th} term will be

(A) p + q + r (B) p + q – r
(C) p + r – q (D) p – q – r
(E) None of these

23. The sum of the series $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+\ldots..$ to 9 terms is

(A) 1 (B) $-\frac{1}{2}$
(C) $-\frac{3}{2}$ (D) $-\frac{5}{6}$
(E) None of these

24. If $(p + q)^{th}$ term of a G.P. be m and $(p – q)^{th}$ term be n, then the p^{th} term will be

(A) m/n (B) $\sqrt{mn}$
(C) mn (D) 0
(E) None of these

25. The value of $4^{1/3}.\ 4^{1/9}.\ 4^{1/27}$ ..... upto ∞ is

(A) 2 (B) 3
(C) 4 (D) 9
(E) None of these

26. If the roots of the equation $ax^2 + bx + c = 0$ be α and β, then the roots of the equation $cx^2 + bx + a = 0$ are

(A) $-\alpha, -\beta$ (B) $\alpha, \frac{1}{\beta}$
(C) $\frac{1}{\alpha}, \frac{1}{\beta}$ (D) $\frac{1}{\alpha}, \beta$
(E) All of these

27. If the roots of $x^2 – bx + c = 0$ are two consecutive integers, then $b^2 – 4c$ is

(A) 1 (B) 2
(C) 3 (D) 4
(E) All of these

28. In how many ways can 5 prizes be distributed among four students when every student can take one or more prizes?

(A) 1024 (B) 625
(C) 120 (D) 720
(E) All of these

29. In how many ways 7 men and 7 women can be seated around a round table such that no two women can sit together?

(A) $(7!)^2$
(B) $7!\times 6!$
(C) $(6!)^2$
(D) 7!
(E) All of these

30. If ${}^{43}C_{r-6} = {}^{43}C_{3r+1}$, then the value of r is

(A) 6 (B) 8
(C) 10 (D) 12
(E) All of these

31. The value of $\left(\sqrt{5}+1\right)^5 - \left(\sqrt{5}-1\right)^5$ is

(A) 252 (B) 352
(C) 452 (D) 532
(E) All of these

32. In the expansion of $\left(x-\frac{1}{x}\right)^6$, the constant term is

(A) –20 (B) 20
(C) 30 (D) – 30
(E) All of these

33. The equation of the line whose slope is 3 and which cuts off an intercept 3 from the positive x-axis is

(A) y = 3x – 9 (B) y = 3x + 3
(C) y = 3x + 9 (D) y = 3x + 12
(E) None of these

34. The acute angle between the lines y = 3 and $y = \sqrt{3}\,x + 9$ is

(A) 30° (B) 45°
(C) 60° (D) 90°
(E) None of these

35. Radius of the circle $x^2+y^2+2x\cos\theta + 2y\sin\theta – 8 = 0$ is

(A) 1 (B) 3
(C) $2\sqrt{3}$ (D) $\sqrt{10}$
(E) None of these

36. The line $y = mx + 1$ is a tangent to the parabola $y^2 = 4x$, if

(A) $m = 1$ (B) $m = 2$
(C) $m = 4$ (D) $m = 3$
(E) None of these

37. In the ellipse, minor axis is 8 and eccentricity is $\frac{\sqrt{5}}{3}$. Then major axis is

(A) 6 (B) 10
(C) 12 (D) 16
(E) None of these

38. $\lim_{x \to 0}\left(\frac{a^x - b^x}{x}\right) =$

(A) $\log\left(\frac{b}{a}\right)$ (B) $\log\left(\frac{a}{b}\right)$
(C) $\frac{a}{b}$ (D) $\log a^b$
(E) None of these

39. The chances of throwing a total of 3 or 5 or 11 with two dice is

(A) $\frac{5}{36}$
(B) $\frac{2}{9}$
(C) $\frac{1}{9}$
(D) $\frac{19}{39}$
(E) None of these

40. If A and B are any two events, then the probability that exactly one of them occur is

(A) $P(A) + P(B) - P(A \cap B)$
(B) $P(A) + P(B) - 2P(A \cap B)$
(C) $P(A) + P(B) - P(A \cup B)$
(D) $P(A) + P(B) - 2P(A \cup B)$
(E) None of these

Space for Rough Work