NATIONAL BOARD FOR HIGHER MATHEMATICS
AND
INDIAN STATISTICAL INSTITUTE

Pre-Regional Mathematical Olympiad 2015
West Bengal Region

November 21, 2015
OR
November 22, 2015

QUESTIONS & ANSWERSCRIPT

Fill in the following. Write name, school name, and test centre in UPPERCASE.

Name: _____

Registration No. PRMO–2015–_____ **Class:** ☐ VIII ☐ IX ☐ X ☐ XI

Date of Birth: _____ **Gender:** ☐ Male ☐ Female ☐ Others

SCHOOL NAME, CITY/TOWN & PINCODE:

Test Centre: _____ **Signature:** _____

Signature of Centre Supervisor: _____

FOR OFFICIAL USE ONLY

Number of Correct Responses	
Marks Obtained	
Signature of Examiner	

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This is a TWO-HOUR test.
2. There are 16 questions in this question paper. Each question carries 5 marks.
3. Answer all questions. Please use a **BALL-POINT PEN WITH BLUE INK** to write the answer
4. (a) Each question has exactly one correct answer. No mark will be awarded if more than one answer is given to a question.
   (b) For each answered question, you'll get full credit if and only if the answer is complete and fully correct. Otherwise, you'll get no credit.
   (c) There is no negative marking and no credit for unanswered question. There is no partial marking.
5. Please report just the answer to a question in the space appearing immediately after it.
6. If you wish to change an answer just strike it off with your pen and then write your new answer legibly. Needless to say that the new answer should be written in pen.
7. (a) All rough work **MUST** be done in the blank pages and nowhere else. No extra paper will be provided.
   (b) Rough work will **NOT** be considered for any credit.
8. Use of any calculators, protractors, log tables, trigonometric tables is not allowed.
9. Mobile phones or any other electronic devices are strictly prohibited in the exam hall.

ATTEMPT OF ANY MALPRACTICE
WOULD RESULT IN
CANCELLATION OF CANDIDATURE!

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1. The sum of two positive integers is 52 and their LCM is 168.
   Find the GCD of the numbers.

2. Find the sum of the last two digits of the number $7^{100} - 3^{100}$.

3. Let A, B, C be distinct digits of a 3-digit number such that

$$\begin{array}{cccc} & A & B & C \\ + & B & C & A \\ + & C & A & B \\ \hline A & B & B & C \end{array}$$

   Then $A + B + C$ equals

4. Let M be the maximum value of $4x - 3y - 2z$ subject to $2x^2 + 3y^2 + 4z^2 = 1$.
   Then M^2 equals

5. Consider the set A of positive integers defined by

$$A := \left\{ p^2 - q^2 \ : \ 3 < q < p \ \text{ and } \ p, q \text{ are primes} \right\}.$$

   Then the GCD of the integers in A equals

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6. Consider all the 5-digit numbers containing each of the digits 1, 2, 3, 4, 5 exactly once, and not divisible by 6. The sum of all these numbers equals

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7. The maximum value of $\sum_{i=1}^{20} i^2\,(21-i)\,a_i$
subject to $a_1, \ldots, a_{20} \geq 0$ and $a_1 + \cdots + a_{20} = 1$ equals

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8. In how many ways can you distribute 100 identical chocolates among 10 children so that the number of chocolates everyone gets is a multiple of 3, allowing some chocolates to be undistributed? [You should consider 0 to be a multiple of 3.]

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9. The number $23104 * 791$ is divisible by 63. The missing digit $(*)$ equals

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10. For $n \geq 1$, let $A_n = p_n! + 1$, where p_n is the n-th prime number. Let P denote the proportion of prime numbers in the set $\{A_n + 1, A_n + 2, \ldots, A_n + n\}$. Then nP equals

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11. The sum of all 3-digit numbers, whose digits are all odd, is

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12. Let ΔABC be an equilateral triangle, each side having length 2 units. Each of the sides AB, BC and CA is divided in the ratio $[p : 1-p]$ where $0 < p < 1$. Thus, there are points P, Q, R on the sides BC, CA, and AB such that $BP : CP = CQ : AQ = AR : BR = p : 1-p$. Consider all the triangles ΔPQR formed in this manner, for varying values of p. Denote by A_0 the area of the triangle which has the smallest perimeter among all these triangle. Then $64A_0^2$ equals

13. The number of positive integers which divide 10^{15} but do not divide 15^{10} equals

14. If the number of ways of choosing 2 boys and 2 girls in a class for a game of mixed doubles is 1620, the number of ways of choosing 2 students from the class equals

15. The number of diagonals of a convex octagon equals

16. Let $\triangle ABC$ with $AB = BC$ and $\angle BAC = 30^\circ$. Let A' be the reflection of A across the line BC; B' be the reflection of B across the line CA; C' be the reflection of C across the line AB. Then $\angle A'B'C'$ equals