KVPY – XI CLASS - STREAM – SA
(Held on 1st November 2015)

PART – A
MATHEMATICS

1. Two distinct polynomials $f(x)$ and $g(x)$ are defined as follows:
$f(x) = x^2 + ax + 2;\ g(x) = x^2 + 2x + a$.
If the equation $f(x) = 0$ and $g(x) = 0$ have a common root then the sum of the roots of the equation $f(x) + g(x) = 0$ is

(A) $-\frac{1}{2}$ (B) 0
(C) $\frac{1}{2}$ (D) 1

1. C

Sol. $f(\alpha) = g(\alpha)$ (where α is common root)
$(a-2)\alpha = (a-2)$
$\alpha = 1$
Now : $f(\alpha) = 0$
$\Rightarrow f(1) = 0 \quad \Rightarrow \quad a = -3$
$f(x) + g(x) = \quad 2x^2 + (a+2)x + (a+2)$
Sum of roots $= \frac{-(a+2)}{2} = \frac{1}{2}$

2. If n is the smallest natural number such that $n + 2n + 3n + ... + 99n$ is a perfect square, then the number of digits in n^2 is

(A) 1 (B) 2
(C) 3 (D) more than 3

2. C

Sol. $n + 2n + 3n + ... + 99n = 99 \times 50 \times n$
$= 9.25.22.n$
For this to be perfect square.
$n = 22$
$n^2 = 484$
Number of digits = 3

3. Let x, y, z be positive reals. Which of the following implies x = y = z?

(I) $x^3 + y^3 + z^3 = 3xyz$ (II) $x^3 + y^2z + yz^2 = 3xyz$
(III) $x^3 + y^2z + z^2x = 3xyz$ (IV) $(x+y+z)^3 = 27xyz$

(A) I, IV only (B) I, II, IV only
(C) I, II and III only (D) All of them

3. B

Sol.

(i) $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x+y+z)\left((x-y)^2 + (y-z)^2 + (z-x)^2\right)$
Since $x + y + z \neq 0$
$\Rightarrow x = y = z$

(ii) $\dfrac{x^3+y^2z+yz^2}{3} \geq \left(x^3y^3z^3\right)^{\frac{1}{3}}$

$\Rightarrow x = y = z$ (since AM = GM)

(iii) $x = 1,\ y = 2,\ z = 1$ satisfies this equation. (Counter example)

(iv) $\dfrac{x+y+z}{3} \geq \left(xyz\right)^{1/3}$

$\Rightarrow x = y = z$ (since AM = GM)

4. In the figure given below, a rectangle of perimeter 76 units is divided into 7 congruent rectangles.

What is the perimeter of each of the smaller rectangles?

(A) 38 (B) 32

(C) 28 (D) 19

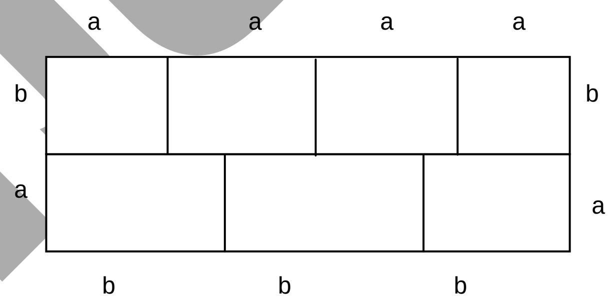
4. C

Sol. $3b = 4a$

$6a + 5b = 76$

Solving a = 6, b = 8

Perimeter = 2a + 2b = 28



5. The largest non – negative integer k such that 24^k divides 13! Is

(A) 2 (B) 3

(C) 4 (D) 5

5. B

Sol. $13! = 2^{10} \times 3^5 \times 5 \times 7 \times 11 \times 13$

$13! = 24^3\left(2.3^2.5^2.7.11.13\right)$

$\therefore$ maximum k can be 3.

6. In a triangle ABC, points X and Y are on AB and AC, respectively, such the XY is parallel to BC. Which of the two following equalities always hold? (Here [PQR] denotes the area of triangle PQR)

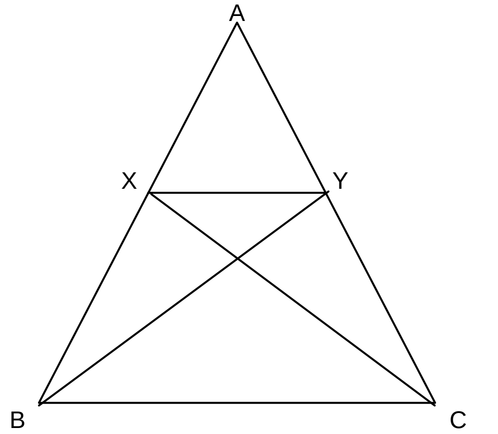
(I) $\left[BCX\right] = \left[BCY\right]$ (II) $\left[ACX\right].\left[ABY\right] = \left[AXY\right]\left[ABC\right]$

(A) Neither (I) nor (II) (B) (I) only

(C) (II) only (D) Both (I) and (II)

6. D

Sol.



(i) $[BCX] = [BCY]$ (same base and height)

(ii) $[ACX][ABY] = [AXY].[ABC]$

$$\frac{[ACX]}{[AXY]} = \frac{[ABC]}{[ABY]}$$

$$\frac{[AXY]+[XYC]}{[AXY]} = \frac{[ABY]+[BYC]}{[ABY]}$$

$$\frac{[XYC]}{[AXY]} = \frac{[BYC]}{[ABY]}$$

7. Let P be an interior point of triangle ABC. Let Q and R be the reflections of P in AB and AC, respectively. If Q, A, R are collinear then $\angle A$ equals

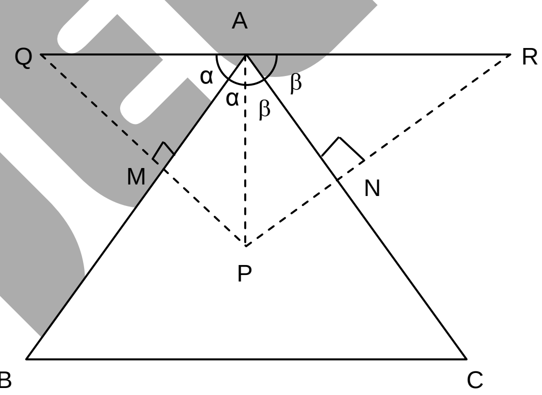
(A) 30^o
(B) 60^o
(C) 90^o
(D) 120^o

7. C

Sol. $2\alpha + 2\beta = 180^o$

$\alpha + \beta = 90^o$

$\angle A = 90^o$



8. Let ABCD be a square of side length 1, and there be a circle passing through B and C, and touching AD. The radius of the circle is

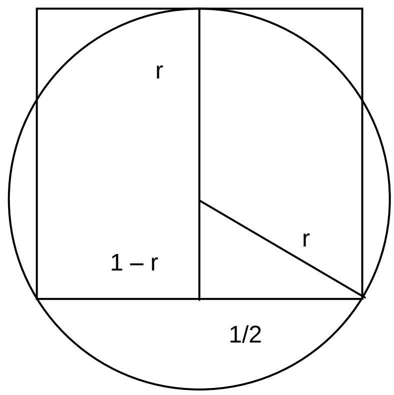
(A) $\frac{3}{8}$
(B) $\frac{1}{2}$
(C) $\frac{1}{\sqrt{2}}$
(D) $\frac{5}{8}$

8. D

Sol. $r^2 - (1-r)^2 = \frac{1}{4}$

$2r - 1 = \frac{1}{4}$

$r = \frac{5}{8}$



9. Let ABCD be a square of side length 1. Let P, Q, R, S be points in the interiors of the sides AD, BC, AB, CD, respectively, such that PQ and RS intersect at right angles. If $PQ = \frac{3\sqrt{3}}{4}$ then RS equals

(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{3\sqrt{3}}{4}$

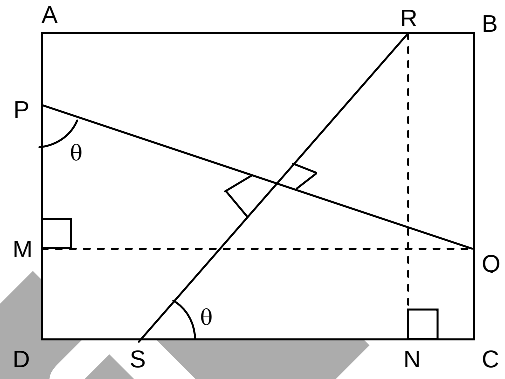
(C) $\frac{\sqrt{2}+1}{2}$ (D) $4-2\sqrt{2}$

9. B

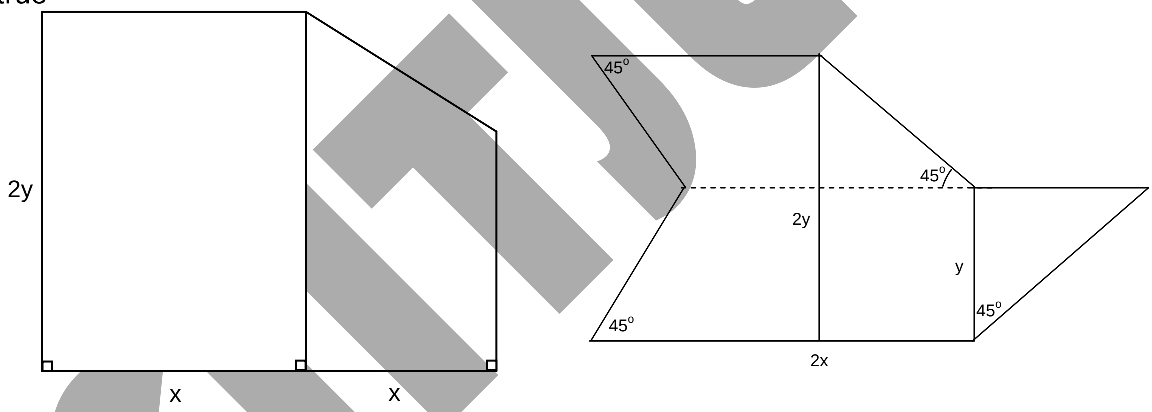
Sol. In ΔPQM, $PQ\sin\theta = 1$

In ΔRSN, $RS\sin\theta = 1$

$\Rightarrow$ $PQ = RS = \frac{3\sqrt{3}}{4}$



10. In the figure given below, if the area of the two regions are equal then which of the following is true



(A) x = y (B) x = 2y
(C) 2x = y (D) x = 3y

10. B

Sol. $A_1 = 2xy + \frac{1}{2}(2y+y)x$

$A_2 = (2x+y)(2y) - \left[3y^2\right]$

$A_1 = A_2$

$2y = x$

11. A man standing on a railway platform noticed that a train took 21 seconds to cross the platform (this means the time elapsed from the moment the engine enters the platform till the last compartment leaves the platform) which is 88 metres long, and that it took 9 seconds to pass him. Assuming that the train was moving with uniform speed, what is the length of the train in metres?

(A) 55 (B) 60
(C) 66 (D) 72

11. C

Sol. Let length of train be x and speed of train be V

$\frac{88+x}{V} = 21$

Since $\frac{x}{V}=9$

$\Rightarrow \frac{88}{V}+9=21$

$\frac{88}{V}=12$

$V=\frac{88}{12}$

$x=9\times\frac{88}{12}$

x = 66

12. The least positive integer n for which $\sqrt[3]{n+1}-\sqrt[3]{n}<\frac{1}{12}$ is

(A) 6 (B) 7
(C) 8 (D) 9

12. C

Sol. $n^{1/3}\left(\left(1+\frac{1}{n}\right)^{1/3}-1\right)<\frac{1}{12}$

$n^{1/3}\left(\left(1+\frac{1}{3n}-h\right)-1\right)<\frac{1}{12},\ (h>0)$

$n^{1/3}\frac{(1-3nh)}{3n}<\frac{1}{12}$

$n^{2/3}>4(1-3nh)$

$n>8(1-3nh)^{3/2}$

$n=8$

13. Let n > 1 be an integer. Which of the following sets of numbers necessarily contains a multiple of 3?

(A) $n^{19}-1, n^{19}+1$ (B) $n^{19}, n^{38}-1$
(C) $n^{38}, n^{38}+1$ (D) $n^{38}, n^{19}-1$

13. B

Sol. Numbers will be of type 3k, (3k +1) & (3k – 1)

If $n=3k \rightarrow$ n^{19} and n^{38} are multiplies of 3

If $n=3k+1 \rightarrow$ $n^{19}-1$ and $n^{38}-1$ are multiplies of 3

If $n=3k-1 \rightarrow$ $n^{19}+1, n^{38}-1$ are multiples

$\therefore n^{19}, n^{38}-1$ necessarily contain multiples of 3.

14. The number of distinct primes dividing $12!+13!+14!$ is

(A) 5 (B) 6
(C) 7 (D) 8

14. A

Sol. $12!+13!+14!$

$=12!(1+13+14\times3)$

$=12!\times14^2$

2, 3, 5, 7, 11 divide this.

$\therefore$ 5 distinct primes.

15. How many ways are there to arrange the letters of the word EDUCATION so that all the following three conditions hold?
– the vowels occur in the same order (EUAIO);
– the consonants occur in the same order (DCTN);
– no two consonants are next to each other

(A) 15 (B) 24
(C) 72 (D) 120

15. A

Sol. Vowels can be arranged in one way. There will be 6 gaps around them we need 4 gaps as arrangement can be done in one way.

$\therefore {}^6C_4 = 15$

PHYSICS

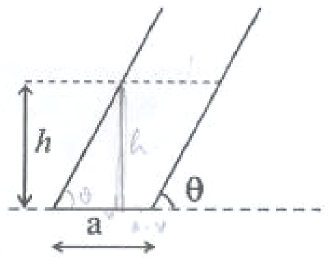
16. In an experiment, mass of an object is measured by applying a known force on it, and then measuring its acceleration. If, in the experiment, the measured values of applied force and the measured acceleration are F = 10.0 $\pm$ 0.2 N and a = 1.00 $\pm$ 0.01 m/s^2, respectively, the mass of the object is

(A) 10.0 Kg (B) 10.0 $\pm$ 0.1 Kg
(C) 10.0 $\pm$ 0.3 Kg (D) 10.0 $\pm$ 0.4 Kg

16. C

Sol. $\frac{dm}{m} = \frac{dF}{F} + \frac{da}{a}$

17. A hollow tilted cylindrical vessel of negligible mass rests on a horizontal plane as shown. The diameter of the base is a and the side of the cylinder makes an angle θ with the horizontal. Water is then slowly poured into the cylinder. The cylinder topples over the when the water reaches a certain height h, given by

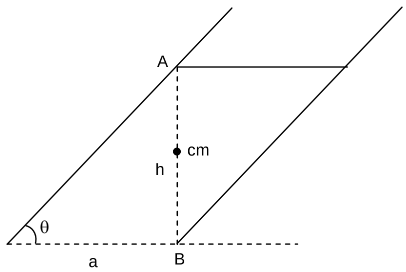


(A) h = 2a tan θ (B) h = 2a tan^2 θ

(C) h = a tan θ (D) $h = \frac{a}{2}\tan\theta$

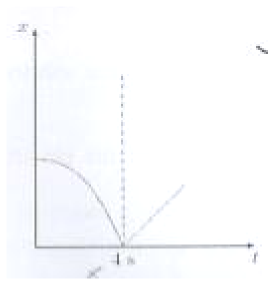
17. C

Sol. The cylinder will topple over when centre of mass of liquid will cross the line AB.



18. An object at rest at the origin begins to move in the +x direction with a uniform acceleration of 1 m/s^2 for 4 s and then it continues moving with a uniform velocity of 4 m/s in the same direction. The x–t graph for object's motion will be

(A)



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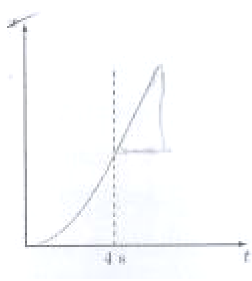
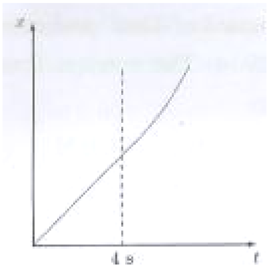
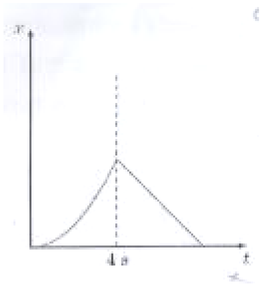


(C)  (D) 

18. B

Sol. Parabolic curve for first 4 sec and straight line afterwards.

19. If the axis of rotation of the earth were extended into space then it would pass close to
(A) the moon
(B) the sun
(C) the pole star
(D) the centre of mass of the all the planets in the solar system

19. C

Sol. Fact based

20. Methane is a greenhouse gas because
(A) it absorbs longer wavelengths of the electromagnetic spectrum while transmitting shorter wavelengths.
(B) it absorbs shorter wavelengths of the electromagnetic spectrum while transmitting longer wavelengths.
(C) it absorbs all wavelengths of the electromagnetic spectrum.
(D) it transmits all wavelengths of the electromagnetic spectrum.

20. A

Sol. Green house gases absorb infrared rays.

21. A parachutist with total weight 75 kg drops vertically onto a sandy ground with a speed of 2 ms^{-1} and comes to a halt over a distance of 0.25 m. the average force form the ground on her is close to
(A) 600 N (B) 1200 N
(C) 1350 N (D) 1950 N

21. C

Sol. Work energy theorem

$$(75)\ (10)\ (0.25) + F(0.25) = 0 - \frac{1}{2}(75)\ (2)^2$$

22. The beta particles of a radioactive metal originate from
(A) the free electrons in the metal.
(B) the orbiting electrons of the metal atoms.
(C) the photons released from the nucleus.
(D) the nucleus of the metal atoms.

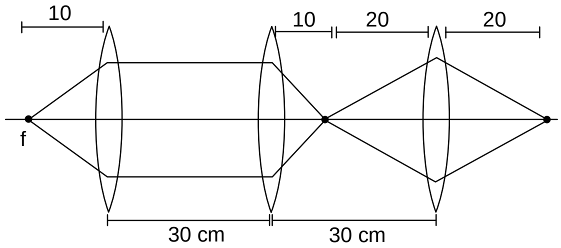
22. D

Sol. Factual

23. An optical device is constructed by fixing three identical convex lenses of focal lengths 10 cm each inside a hollow tube at equal spacing of 30 cm each. One end of the device is placed 10 cm away from a point source. How much does the image shift when the device is moved away from the source by another 10 cm?
(A) 0 (B) 5 cm
(C) 15 cm (D) 45 cm

23. A

Sol. Apply principle of reversibility.



24. An isosceles glass prisms with base angles 40° is clamped over a tray of water in a position such that the base is just dipped in water. A ray of light incident normally on the inclined face suffers total internal reflection at the base. If the refractive index of water is 1.33 then the condition imposed on the refractive index μ of the glass is

(A) $\mu < 2.07$ (B) $\mu > 2.07$

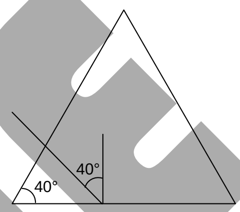
(C) $\mu < 1.74$ (D) $\mu > 1.74$

24. B

Sol. For total internal reflection.

$\mu \sin 40 > (1033) \sin 90$

$$\mu > \frac{1.33}{\sin 40}$$



25. A point source of light is moving at a rate of 2 cm s^{-1} towards a thin convex lens of focal length 10 cm along its optical axis. When the source is 15 cm away from the lens the image is moving at

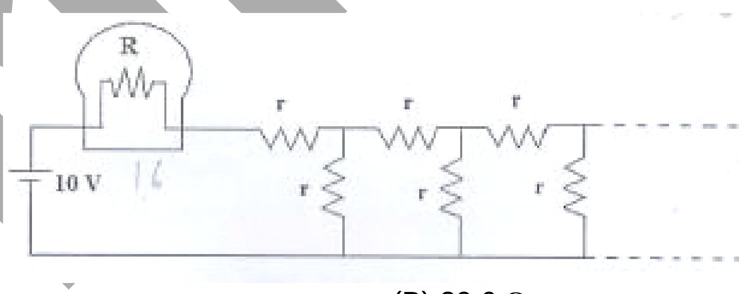
(A) 4 cm s^{-1} towards the lens. (B) 8 cm s^{-1} towards the lens.

(C) 4 cm s^{-1} away from the lens. (D) 8 cm s^{-1} away from the lens.

25. D

Sol. Image velocity = $\left(\frac{v^2}{u^2}\right)\vec{v}_o$

26. A light bulb of resistance r = 16 Ω is attached in series with an infinite resistor network with identical resistance r as shown below. A 10 V battery drives current n the circuit. What should be the value of r such that the bulb dissipates about 1 W of power



(A) 14.8 Ω (B) 29.6 Ω

(C) 7.4 Ω (D) 3.7 Ω

26. A

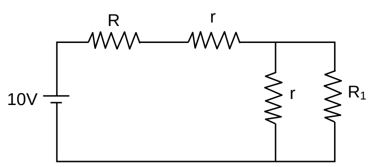
Sol. Current through bulb is I

$P = I^2 R$

$\Rightarrow\ I = \frac{1}{4} A$

$\Rightarrow$ Voltage drop across rest of the circuit is 6 volt.

$\Rightarrow$ If rest of the circuit has equivalent resistance R_1 (where R_1 = 24 Ω)



Then, $R_1 = \left(\frac{R_1 r}{R_1 + r}\right) + r$

$\Rightarrow$ r = 14.8 Ω

27. A ball is launched from the to Mt. Everest which is at elevation of 9000 m. The ball moves in circular orbit around each. Acceleration due to gravity near the earth's surface is g. The magnitude of the ball's acceleration while in orbit is
(A) close to g/2
(B) zero
(C) much greater than g
(D) nearly equal to g

27. D

Sol. $g_h = \dfrac{gR^2}{(R+h)^2}$

$\because$ h << R

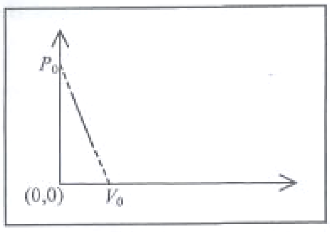
$g_h \approx g$

28. A planet is orbiting the sun in an elliptical orbit. Let U denote the potential energy and K denote the kinetic energy of the planet at an arbitrary point on the orbit. Choose the correct statement.
(A) K < |U| always.
(B) K > |U| always.
(C) K = |U| always
(D) K = |U| for two positions of the planet in the orbit.

28. A

Sol. Total energy is negative always therefore K < |U|.

29. One mole of ideal gas undergoes a linear process as shown in figure below. Its temperature expressed as a function of volume V is,



(A) $\dfrac{P_0V_0}{R}$
(B) $\dfrac{P_0V}{R}$
(C) $\dfrac{P_0V}{R}\left(1-\dfrac{V}{V_0}\right)$
(D) $\dfrac{P_0V_0}{R}\left(1-\left(\dfrac{V}{V_0}\right)^2\right)$

29. C

Sol. Equation of line

$P = -\dfrac{P_o}{V_o}V + P_o$ …(1)

$P = \dfrac{RT}{V}$ …(2)

30. The international space station is maintained in a nearly circular orbit with a mean altitude of 330 km and a maximum of 410 km. An astronaut is floating in the space station's cabin. The acceleration of astronaut as measured from the earth is
(A) zero
(B) nearly zero and directed towards the earth.
(C) nearly g and directed along the line of travel of the station.
(D) nearly g and directed towards the earth.

30. D

Sol. $g_h = \dfrac{gR^2}{(R+h)^2}$

CHEMISTRY

31. The percentage of nitrogen by mass in ammonium sulphate is closest to (atomic masses H = 1, N = 14, O = 16, S = 32)
(A) 21% (B) 24%
(C) 36% (D) 16%

31. A

Sol. Ammonium sulphate is $(NH_4)_2SO_4$
Molecular mass of $(NH_4)_2SO_4$ = 132
Mass of nitrogen present in $(NH_4)_2SO_4$ = 28 gram
Mass % of nitrogen = $\frac{28}{132} \times 100 = 21.21\% \approx 21$

32. Mendeleev's periodic law states that the properties of elements are periodic function of their
(A) reactivity of elements (B) atomic size
(C) atomic mass (D) electronic configuration

32. C

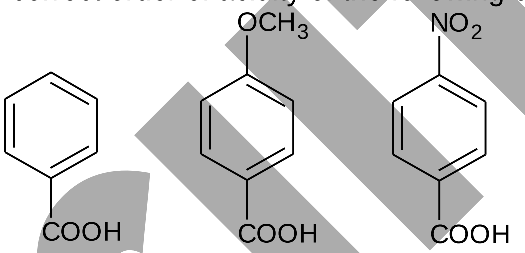
Sol. According to Mendeleev's periodic law, the properties of elements are a periodic function of the atomic mass of the elements.

33. Maximum number of electrons that can be accommodated in the subshell with azimuthal quantum number l = 4, is
(A) 10 (B) 8
(C) 16 (D) 18

33. D

Sol. No. of electron with 'ℓ' = $4\ell + 2$
For $\ell = 4$, the no. of electrons = $4 \times 4 + 2 = 18$

34. The correct order of acidity of the following compound is

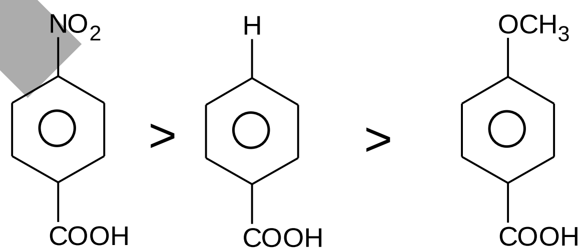


1 2 3

(A) 1 > 2 > 3 (B) 1 > 3 > 2
(C) 3 > 1 > 2 (D) 3 > 2 > 1

34. C

Sol. Acidic Strength:



Electron withdrawing groups (–I, –R) increase acidity and electron releasing groups (+I, +R) decrease acidic strength
NO_2 group exert –I and – R effect
OCH_3 group exert –I and + R effect but its +R effect dominates over –I effect at ortho and para positions.

35. Reaction of 2-butene with acidic $KMnO_4$ gives
(A) CH_3CHO (B) HCOOH

(C) CH_3CH_2OH (D) CH_3COOH

35. D

Sol. $CH_3CH=CHCH_3 \xrightarrow{MnO_4^-/H^+} 2CH_3COOH$

Two moles of acetic acid is formed.

36. The gas released when baking soda is mixed with vinegar is

(A) CO (B) CO_2

(C) CH_4 (D) O_2

36. B

Sol. Baking soda = $NaHCO_3$

Vinegar contains CH_3COOH

$\therefore CH_3COOH + NaHCO_3 \longrightarrow CH_3COONa + CO_2 \uparrow + H_2O$

$\therefore CO_2$ gas is evolved.

37. The element which readily forms an ionic bond has the electronic configuration

(A) $1s^2 2s^2 2p^3$ (B) $1s^2 2s^2 2p^1$

(C) $1s^2 2s^2 2p^2$ (D) $1s^2 2s^2 2p^6 3s^1$

37. D

Sol. The element having $1s^2 2s^2 2p^6 3s^1$ electron configuration has the lowest first ionization energy.

38. The major products of the following reaction

$ZnS(s) + O_2(g) \xrightarrow{heat}$

are

(A) ZnO and SO_2 (B) $ZnSO_4$ and SO_3

(C) $ZnSO_4$ and SO_2 (D) Zn and SO_2

38. A

Sol. $2ZnS(s) + 3O_2(g) \xrightarrow{heat} 2ZnO(s) + 2SO_2(g)$

39. If Avogadro's number is A_0, the number of sulphur atoms present in 200 mL is 1 N H_2SO_4 is

(A) $A_0/5$ (B) $A_0/2$

(C) $A_0/10$ (D) A_0

39. C

Sol. Normality(N) of H_2SO_4 = 1

Molarity(M) of H_2SO_4 = $\frac{1}{2}$

Moles of H_2SO_4 = $\frac{M \times V}{1000} = \frac{\frac{1}{2} \times 200}{1000}$ = 0.1

$\therefore$One mole H_2SO_4 contains one mole S atom

0.1 mole H_2SO_4 contains 0.1 mole S atom

No. of sulphur atoms = 0.1 × A_0 = $\frac{A_0}{10}$

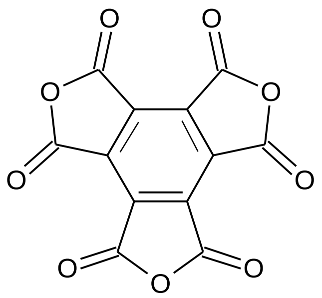
40. The functional group present in a molecule having the formula $C_{12}O_9$ is

(A) carboxylic acid (B) anhydride

(C) aldehyde (D) alcohol

40. B

Sol. $C_{12}O_9$ is mellitic anhydride. Its structure is



41. A sweet smelling compound formed by reacting acetic acid with ethanol in the presence of hydrochloric acid is
(A) $CH_3COOC_2H_5$ (B) C_2H_5COOH
(C) $C_2H_5COOCH_3$ (D) CH_3OH

41. A

Sol. $CH_3COOH + C_2H_5OH \xrightarrow{HCl} \underset{\text{ester}}{CH_3COOC_2H_5} + H_2O$

Esters have sweet smell.

42. Among Mg, Cu, Fe, Zn, the metal that does not produce hydrogen gas in reaction with hydrochloric acid is
(A) Cu (B) Zn
(C) Mg (D) Fe

42. A

Sol. 'Cu' is present below hydrogen in activity series. So, it cannot displace hydrogen either from H_2O or dil.acids.

43. The maximum number of isomeric ethers with the molecular formula $C_4H_{10}O$ is
(A) 2 (B) 3
(C) 4 (D) 5

43. B

Sol. Ethers having formula C_4H_{10} are
$CH_3OCH_2CH_2CH_3$
$CH_3CH_2OCH_2CH_3$
$CH_3OCH(CH_3)_2$

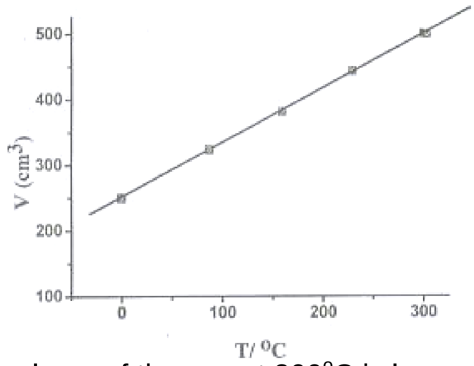
44. The number of electrons required to reduce chromium completely in $Cr_2O_7^{2-}$ to Cr^{3+} in acidic medium is
(A) 5 (B) 3
(C) 6 (D) 2

44. C

Sol. $Cr_2O_7^{2-} + 14H^+ \longrightarrow 2Cr^{3+} + 7H_2O$

The oxidation no. of Cr in $Cr_2O_7^{2-}$ is +6
The oxidation no. of Cr in Cr^{3+} is +3
No. of electrons gained = 2(+6) – 2(+3) = 6

45. At constant pressure, the volume of a fixed mass of a gas varies as a function of temperature as shown in the graph



The volume of the gas at 300^{o}C is larger than that at 0^{o}C by a factor of
(A) 3 (B) 4
(C) 1 (D) 2

45. D

Sol. Volume corresponding to 300^{o}C = 500 cm^3
Volume corresponding to 0^{o}C = 250 cm^3

$$\therefore \frac{V\left(300^oC\right)}{V\left(0^oC\right)} = \frac{500}{250} = 2$$

BIOLOGY

46. Excess salt inhibits bacterial growth in pickles by
(A) endosmosis (B) exosmosis
(C) oxidation (D) denaturation

46. B

47. Restriction endonucleases are enzymes that are used by biotechnologists to
(A) cut DNA at specific base sequences (B) join fragments of DNA
(C) digest DNA from the 3’ end (D) digest DNA from the 5’ end

47. A

48. Enzyme X extracted from the digestive system hydrolyses peptide bonds. Which of the following are probable candidates to be enzyme X?
(A) Amylase (B) Lipase
(C) Trypsin (D) Maltase

48. C

49. A person with blood group AB has
(A) antigen A and B on RBCs and both anti-A and anti-B antibodies in plasma
(B) antigen A and B on RBCs, but neither anti-A nor anti-B antibodies in plasma
(C) no antigen on RBCs but both anti-A and anti-B antibodies present in plasma
(D) antigen A on RBCs and anti-B antibodies in plasma

49. B

50. Glycolysis is the breakdown of glucose to pyruvic acid. How many molecules of pyruvic acid are formed from one molecule of glucose?
(A) 1 (B) 2
(C) 3 (D) 4

50. B

51. The process of transfer of electrons from glucose to molecular oxygen in bacteria and mitochondria is known as
(A) TCA cycle (B) Oxidative phosphorylation
(C) Fermentation (D) Glycolysis

51. B

52. Which one of the following cell types is a part of innate immunity?
(A) Skin epithelial cells (B) B cells
(C) T lymphocytes (D) Liver cells
52 A

53. Deficiency of which one of the following vitamins can cause impaired blood clotting?
(A) Vitamin B (B) Vitamin C
(C) Vitamin D (D) Vitamin K
53. D

54. Which one of the following is detrimental to soil fertility?
(A) Saprophytic bacteria (B) Nitrosomes
(C) Nitrobacter (D) Pseudomonas
54. D

55. In which one of the following phyla is the body segmented?
(A) Porifera (B) Platyhelminthes
(C) Annelida (D) Echinodermata
55. C

56. Widal test is prescribed to diagnose
(A) Typhoid (B) Pneumonia
(C) Malaria (D) Filaria
56. A

57. Which among grass, goat, tiger and vulture in a food chain, will have the maximum concentration of harmful chemicals in its body due to contamination of pesticides in the soil?
(A) Grass since it grows in the contaminated soil
(B) Goat since it eats the grass
(C) Tiger since it feeds on the goat which feeds on the grass
(D) Vulture since it eats the tiger, which it turn eats the goat, which eats the grass
57. D

58. Considering the average molecular mass of a base to be 500 Da, what is the molecular mass of a double stranded DNA of 10 base pairs?
(A) 500 Da (B) 5 kDa
(C) 10 kDa (D) 1 kDa
58. C

59. Which of the following pairs are both polysaccharides?
(A) Cellulose and glycogen (B) Starch and glucose
(C) Cellulose and fructose (D) Ribose and sucrose
59. A

60. Which one of the following is modified leaf?
(A) Sweet potato (B) Ginger
(C) Onion (D) Carrot
60. C

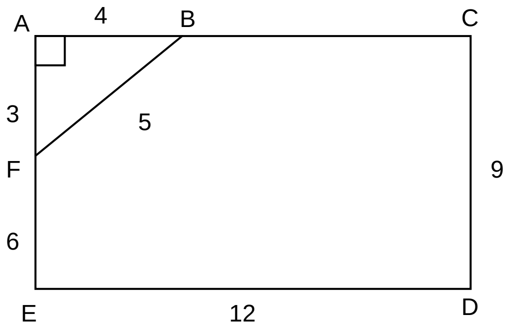
PART – B
MATHEMATICS

61. A triangular corner is cut from a rectangular piece of paper and the resulting pentagon has sides 5, 6, 8, 9, 12 in some order. The ratio of the area of the pentagon to the area of the rectangle is
(A) $\frac{11}{18}$ (B) $\frac{13}{18}$

(C) $\frac{15}{18}$ (D) $\frac{17}{18}$

61. D

Sol. $\frac{\text{area}(BCDEF)}{\text{area}(ACDE)} = \frac{108-6}{108} = \frac{17}{18}$



62. For a real number x, let [x] denote the largest integer less than or equal to x, and let $\{x\} = x - [x]$. The number of solutions x to the equation [x] {x} = 5 with $0 \le x \le 2015$ is

(A) 0 (B) 3

(C) 2008 (D) 2009

62. D

Sol. $\{x\} = \frac{5}{[x]}$

$0 \le \{x\} < 1$

$\therefore [x] = 6, 7, 8, ...2015$

Since $x \le 2015$, therefore

$[x] = 6, 7, 8, ..., 2014$

$\therefore 2009$ solutions.

63. Let ABCD be a trapezium with AD parallel to BC. Assume there is a point M in the interior of the segment BC such that AB = AM and DC = DM. Then the ratio of the area of the trapezium to the area of triangle AMD is

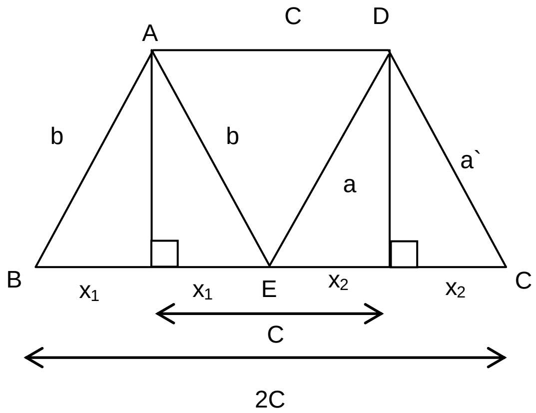
(A) 2 (B) 3

(C) 4 (D) not determinable from the data

63. B

Sol. Area of Trapezium $= \frac{1}{2}(3C)(H)$

Area of $\Delta AED = \frac{1}{2}(C)H$



64. Given are three cylindrical buckets X, Y, Z whose circular bases are of radii 1, 2, 3 units, respectively. Initially water is filled in these buckets upto the same height. Some water is then transferred from Z to X so that they both have the same volume of water. Some water is then transferred between X and Y so that they both have the same volume of water. If h_y h_z denote the height of water at this stage in the Y, Z, respectively, then the ratio $\frac{h_y}{h_z}$ equals

(A) $\frac{4}{9}$ (B) 1

(C) $\frac{9}{4}$ (D) $\frac{81}{40}$

64. D

Sol. Let initial height in all buckets be h

<u>After first transfer</u> :

Let height in Z be h_z

Volume transferred $= 9\pi(h - h_z)$

New volume in X = $\pi h + 9\pi(h - h_z)$

$$= 10\pi h - 9\pi h_z$$

Volume in X = volume in Z

$$10\pi h - 9\pi h_z = 9\pi h_z$$

$$\frac{h}{h_z} = \frac{9}{5}$$

<u>After second transfer:</u>

Let height in X be h_x

Volume transferred $= 10\pi h - 9\pi h_z - \pi h_x$

New volume in Y $= 4\pi h + 10\pi h - 9\pi h_z - \pi h_x$

$$= 14\pi h - 9\pi h_z - \pi h_x$$

Volume in X = volume in Y

$$\pi h_x = 14\pi h - 9\pi h_z - \pi h_x$$

$$\Rightarrow \frac{h_x}{h_z} = 14\frac{h}{h_z} - 9 - \frac{h_x}{h_z}$$

$$\Rightarrow \frac{h_x}{h_z} = \frac{81}{10}$$

Let height in Y be h_y

$$\Rightarrow h_y = \frac{\text{volume in Y}}{4\pi}$$

$$\Rightarrow h_y = \frac{14\pi h - 9\pi h_z - \pi h_x}{4\pi}$$

$$\Rightarrow \frac{h_y}{h_z} = \frac{14}{4} \times \frac{h}{h_z} - \frac{9}{4} - \frac{1}{4} \times \frac{4h_x}{h_z}$$

$$= \frac{14}{4} \times \frac{9}{5} - \frac{9}{4} - \frac{1}{4} \times \frac{81}{10}$$

$$= \frac{81}{40}$$

65. The average income of the people in two villages are P and Q, respectively. Assume that $P \neq Q$. A person moves from the first village to the second village. The new average incomes are P' and Q' respectively. Which of the following is not possible?

(A) P' > P and Q' > Q (B) P' > P and Q' < Q

(C) P' = P and Q' = Q (D) P' < P and Q' < Q

65. C

Sol. Let Number of people in two villages be x and y respectively

Average Income	Number of People	Total Income	Total income after person moves	New Population	New Average Income

P	x	Px	Px – P_1	x – 1	$P' = \frac{Px - P_1}{x - 1}$
Q	y	Qy	$Qy + P_1$	y + 1	$Q' = \frac{Qy + P_1}{y + 1}$

In option C,
If $P' = P$ and $Q' = Q$

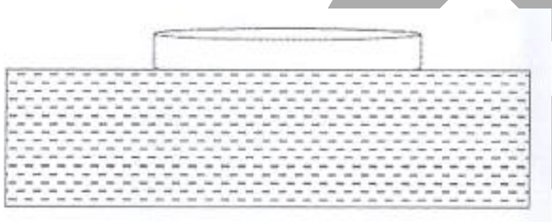
$P' = P \quad \Rightarrow \qquad P = \frac{Px - P_1}{x - 1} \Rightarrow P_1 = P$

$Q' = Q \quad \Rightarrow \qquad Q = \frac{Qy + P_1}{y + 1} \Rightarrow P_1 = Q$

$\Rightarrow$ P = Q which is a contradiction.

PHYSICS

66. A girl sees through a circular glass slab (refractive index 1.5) of thickness 20 mm and diameter 60 cm to the bottom of a swimming pool. Refractive index of water is 1.33. The bottom surface of the slab is in contact with the water surface.



The depth of swimming pool is 6 m. The area of the bottom of swimming pool that can be seen through the slab is approximately

(A) 100 m^2 (B) 160 m^2
(C) 190 m^2 (D) 220 m^2

66. B

Sol. (1.5) sin θ = (1) sin (90) …(1)

(1.33) sin θ = (1.5) sin θ

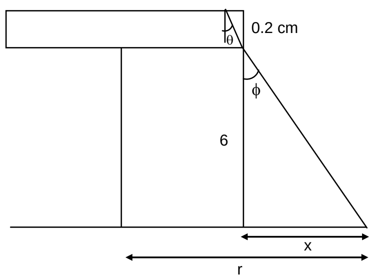
$\sin\theta = \frac{1}{1.33}$

Radius of circular area at bottom of pool

r = (30 + x)

Area A = πr^2

A = 158.4 cm^2



67. 1 Kg of ice at –20°C is mixed with 2 Kg of water at 90°C. Assuming that there is no loss of energy to the environment, what will be the final temperature of the mixture? (Assume latent heat of ice = 334.4 kJ/Kg, specific heat of water and ice are 4.18 kJ/(kg.K) and 2.09 kJ/(kg.K), respectively.)

(A) 30°C (B) 0°C
(C) 80°C (D) 45°C

67. A

Sol. Principle of Calorimetry

Heat lost = heat gained.

68. A rigid body in the shape of a "V" has two equal arms made of uniform rods. What must the angle between the two rods be so that when the body is suspended from one end, the other arm is horizontal?

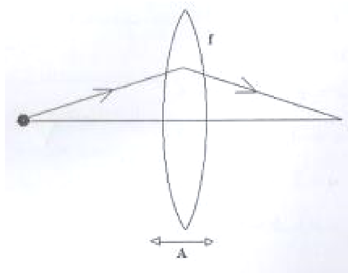
(A) $\cos^{-1}\left(\frac{1}{3}\right)$ (B) $\cos^{-1}\left(\frac{1}{2}\right)$

(C) $\cos^{-1}\left(\frac{1}{4}\right)$ (D) $\cos^{-1}\left(\frac{1}{6}\right)$

68. A

Sol. Centre of mass of the system will be vertically below point of suspension.

69. A point object is placed 20 cm left of a convex lens of focal length f = 5 cm (see the figure). The lens is made to oscillate with small amplitude A along the horizontal axis. The image of the object will also oscillate along the axis with



(A) amplitude A/9, out of phase with the oscillations of the lens.
(B) amplitude A/3, out of phase with the oscillations of the lens.
(C) amplitude A./3, in phase with the oscillations of the lens.
(D) amplitude A/9, in phase with the oscillations of the lens.

69. A

Sol. $v_p = \left(\frac{v^2}{u^2}\right)v_o$; $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

When lens will move towards right image will move towards left.

70. Stoke's law states that the viscous drag force F experienced by a sphere of radius a, moving with a speed ν through a fluid with coefficient of viscosity η, is given by

$F = 6\pi\eta a\nu$

If this fluid is flowing through a cylindrical pipe of radius r, length ℓ and a pressure difference of P across its two ends, then the volume of water V which flows through the pipe in time t can be written as

$$\frac{\nu}{t} = k\left(\frac{p}{\ell}\right)^a \eta^b r^c$$

where k is a dimensionless constant. Correct values of a, b and c are

(A) a = 1, b = –1, c = 4
(B) a = –1, b = 1, c = 4
(C) a = 2, b = 1, c = 3
(D) a = 1, b = –2, c = –4

70. None of the option are correct.

Sol. $LT^{-2} = [ML^{-2}T^{-2}]^a [ML^{-1}T^{-1}]^b [L]^c$

$a + b = 0$

$-2a - b + c = 1$

$-2a - b = -2.$

CHEMISTRY

71. When 262 g of xenon (atomic mass = 131) reacted completely with 152 g of fluorine(atomic mass = 19), a mixture of XeF_2 and XeF_6 was produced. The molar ratio $XeF_2 : XeF_6$ is

(A) 1 : 2
(B) 1 : 4
(C) 1 : 1
(D) 1 : 3

71. C

Sol. Moles of xenon = $\frac{262}{131} = 2$

Moles of F_2 = $\frac{152}{(2\times19)} = 4$

$Xe + F_2 \longrightarrow XeF_2$

$Xe + 3F_2 \longrightarrow XeF_6$

In the reaction one mole of XeF_2 and one mole of XeF_6 are formed.

72. Reaction of ethanol with conc. sulphuric acid at 170°C produces a gas which is then treated with bromine in carbon tetrachloride. The major product obtained in this reaction is

(A) 1, 2-dibromoethane
(B) ethylene glycol
(C) bromoethane
(D) ethyl sulphate

72. A

Sol. $CH_3CH_2OH \xrightarrow[170^{o}C]{Conc.H_2SO_4} CH_2 = CH_2 + H_2O$

$CH_2 = CH_2 + Br_2 \xrightarrow{CCl_4} BrCH_2 - CH_2Br$
1, 2-dibromoethane

73. When 22.4 L of C_4H_8 at STP is burnt completely, 89.6 L of CO_2 gas at STP and 72 g of water are produced. The volume of the oxygen gas at STP consumed in the reaction is closest to
(A) 89.6 L
(B) 112 L
(C) 134.4 L
(D) 22.4 L

73. C

Sol. Moles of CO_2 = $\frac{89.6}{22.4} = 4$

Moles of H_2O = $\frac{72}{18} = 4$

1 mole of CO_2 contains one mole of O_2 gas
4 moles of CO_2 will contain 4 moles of O_2 gas

1 mole H_2O contains $\frac{1}{2}$ mole of O_2 gas

4 mole of H_2O contains $\frac{1}{2} \times 4$ = 2 mole of O_2 gas

$\therefore$ Total moles of O_2 needed = 4 + 2 = 6
Volume of six moles of O_2 at STP = 6 $\times$ 22.4 = 134.4 L

74. The amount of Ag(Atomic mass = 108) deposited at the cathode when a current of 0.5 amp is passed through a solution of $AgNO_3$ for 1 hour is closest to
(A) 2 g
(B) 5 g
(C) 108 g
(D) 11 g

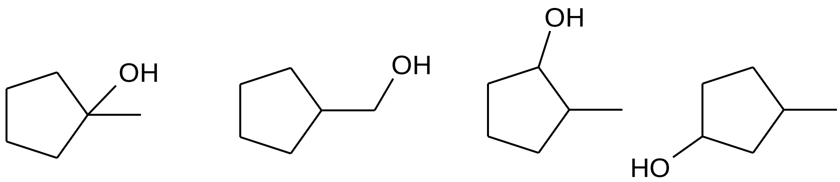
74. A

Sol. Charge passed = 0.5 amp $\times$ (60 $\times$ 60) sec = 1800 coulomb
96500 coulomb = 1 Faraday

1800 coulomb = $\frac{1800}{96500}$ = 0.018 F

IF deposits 108 g of silver (Eq. mass = Molar mass)
$\therefore$ 0.018 F will deposit = 108 $\times$ 0.018 = 1.94 $\simeq$ 2g

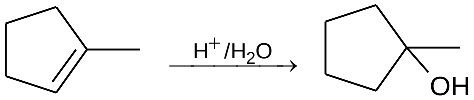
75. The major product of the reaction is



I II III IV

(A) I
(B) II
(C) III
(D) IV

75. A

Sol.


Markownikoff addition of water takes place.

BIOLOGY

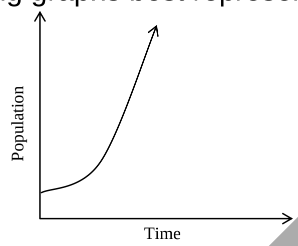
76. Genomic DNA is digested with AIU I, a restriction enzyme which is a four base – pair cutter. What is the frequency with which it will cut the DNA assuming a random distribution of bases in the genome?
(A) 1/4 (B) 1/24
(C) 1/256 (D) 1/1296

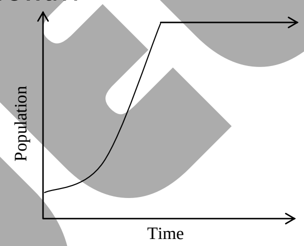
76. C

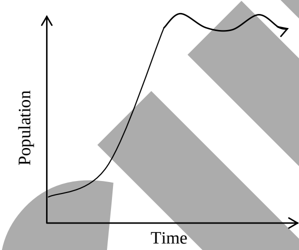
77. If rice is cooked in a pressure cooker on the Siachen glacier, at sea beach, and on Deccan plain, which of the following is correct about the time taken for cooking rice?
(A) Gets cooked faster on the Siachen glacier
(B) Gets cooked faster at sea beach
(C) Gets cooked faster on Deccan plain
(D) Gets cooked at the same time at all the three places

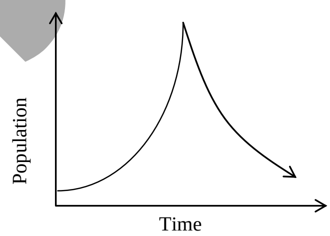
77. D

78. A few rabbits are introduced in an – inhabited island with plenty of food. If these rabbits breed in the absence of any disease, natural calamity and predation, which one of the following graphs best represents their population growth?

(A) 

(B) 

(C) 

(D) 

78. C

79. What is the advantage of storing glucose as glycogen in animals instead of as monomeric glucose?
(A) Energy obtained from glycogen is more than that from the corresponding glucose monomers.
(B) Glucose present as monomers within the cell exerts more osmotic pressure than a single glycogen molecule, resulting in loss of water from the cells.
(C) Glucose present as monomers within the cell exerts more osmotic pressure than a single glycogen molecule, resulting in excess water within the cells.
(D) Glycogen gives more rigidity to the cells

79. C

80. A line id drawn from the exterior of an animal cell to the centre of the nucleus, crossing through one mitochondrion. What is the minimum number of membrane bilayers that the line will cross?
(A) 4 (B) 3
(C) 5 (D) 6

80. D