

GOA BOARD OF SECONDARY AND HIGHER SECONDARY EDUCATION
 ALTO BETIM – GOA 403521
 HSSCE
 MATHEMATICS (754) [effective from March 2015]
 MODEL QUESTION PAPER

Time : 2½ hrs.

Max Marks : 80

GENERAL INSTRUCTIONS:

- This question paper contains seven main questions.
- All seven questions are compulsory.
- Answer each main question on a fresh page.
- Use of calculator is not allowed.
- Log tables will be supplied on request.
- Graphs should be drawn on the answer paper only.
- For each main questions the subquestions carry the following marks:
 A = 1 mark , B = 2 marks , C = 3 marks , D = 4 marks , E = 5 marks.

Q 1 (A) Choose the correct alternative from the given alternatives :

If $y = e^{\log x^2}$, then the value of $\frac{dy}{dx} =$ _____

• $2x$ • $\frac{x}{2}$ • x^2 • 2

(B) Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \frac{2x-1}{3}$, $x \in \mathbb{R}$ is bijective.

(C) $y = \log \left(\tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right)$, then show that $\frac{dy}{dx} - \sec x = 0$

(D) Using integration , prove that

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| + c$$

Q 2 (A) Choose the correct alternative from the given alternatives :

If $\sin^{-1} \frac{1}{5} + \cos^{-1} 2x = \frac{\pi}{2}$ then $x =$ _____

• $\frac{\pi}{5}$ • $\frac{\pi}{10}$ • $\frac{4\pi}{5}$ • $\frac{\pi}{2}$

(B) If $y = \sin^{-1} x$, then prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0$

(C) Find the value of x if the points $(3, 2, 1)$, $(4, x, 5)$, $(4, 2, -2)$ and $(6, 5, -1)$ are coplanar.

(D) Define continuity of the function $f(x)$ at $x = a$. Find the values of a and b if the function $f(x)$ given by

$$\begin{aligned} f(x) &= x + a\sqrt{2} \sin x, & 0 \leq x < \frac{\pi}{4} \\ &= 2x \cot x + b, & \frac{\pi}{4} \leq x < \frac{\pi}{2} \\ &= a \cos 2x - b \sin x, & \frac{\pi}{2} \leq x \leq \pi \end{aligned}$$

Is continuous in $[0, \pi]$.

Q 3 (A) Choose the correct alternative from the given alternatives :

$$\int_{-1}^1 (x^3 + x^4 + x^5) dx = \underline{\hspace{2cm}}$$

- 0
- $\frac{37}{30}$
- $\frac{2}{5}$
- $\frac{5}{12}$

(B) If * is a binary operation on the set of integers \mathbb{Z} defined by $a * b = a + b - 15$ $a, b \in \mathbb{Z}$, then find the (i) identity element. (ii) the inverse of an element in \mathbb{Z} if exists.

(C) Find the differential equation of the family of curves given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are arbitrary constants.

(D) Prove that

$$\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a - x) dx$$

and hence prove that

$$\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx, \quad \text{if } f(2a - x) = f(x)$$

$$= 0, \quad \text{if } f(2a - x) = -f(x)$$

Q 4 (A) Two cards are drawn at random without replacement from a pack of 52 cards. Find the probability that both the cards are black.

(B) A problem in mathematics is given to 3 students whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. What is the probability that the problem is solved?

(C) Solve the differential equation $(1 + e^{2x})dy + (1 + y^2)e^x dx = 0$

(D) Using integration, find the area of the region enclosed between $x^2 + y^2 = 2a^2$ and $y^2 = ax$, where $a > 0$

Q 5 (A) Find X if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

(B) Using determinants, find the equation of the line joining the points (-1,2) and (-3, -4)

(C) An insurance company insured 2000 scooter driver, 4000 car drivers and 6000 truck drivers. The probability of an accident are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

(D) Solve the following equations using matrix method

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$\text{and } x + y - 2z = -3$$

Q 6 (A) Choose the correct alternative from the given alternatives :

The point on the curve $y = x^2 - 2x + 3$ at which the tangent is parallel to $x - axis$ is ___

- (2, 1)
- (1, 1)
- (2, 2)
- (1, 2)

(B) Find a unit vector perpendicular to each of the vectors

$$\hat{i} + \hat{j} + \hat{k} \text{ and } \hat{i} + 2\hat{j} + 3\hat{k}$$

(C) By using properties of determinant as far as possible, prove that

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

(D) A diet is to contain atleast 50 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs 4 per unit of food and F_2 costs Rs 6 per unit of food. One unit of food F_1 contains 3 units of vitamin A and 4 units of mineral. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

(E) Attempt **any one** of the following:

(i) Find the equation of the line passing through the point (3,4) which cuts X and Y axes at the points A and B respectively such that area of ΔABC is minimum.

(ii) Find the intervals in which the function given by

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

is (a) Strictly increasing and (b) Strictly decreasing.

Q 7 (A) Find the equations of the line passing through the point (1, 2, -4) and parallel to

$$\text{the line } \frac{x-2}{5} = \frac{-y+1}{2} = z$$

(B) If $\bar{a}, \bar{b}, \bar{c}$ are the vectors such that $\bar{a} + \bar{b} + \bar{c} = \bar{0}$ and

$$|\bar{a}| = 3, |\bar{b}| = 4 \text{ and } |\bar{c}| = 5, \text{ then find the value of } \bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}$$

(C) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then show that $x + y + z = xyz$

(D) Attempt **any one** of the following:

(i) A variable plane which remains at a constant distance $3p$ from the origin cuts the coordinate axes at the points A, B, C. Show that the locus of the centroid of ΔABC is given by $x^{-2} + y^{-2} + z^{-2} = p^{-2}$

(ii) Find the distance of the point (-1, -5, -10) from the point of the intersection of the line $\bar{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane.

$$\bar{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

(E) Attempt **any one** of the following:

(i) Evaluate $\int \frac{1+\sin x}{\sin x(1+\cos x)} dx$

(ii) Evaluate $\int \frac{\sqrt{x^2+1} [\log(x^2+1) - 2\log x]}{x^4} dx$

END

