GOA BOARD OF SECONDARY AND HIGHER SECONDARY EDUCATION

ALTO BETIM – GOA 403521`

HSSCE

MATHEMATICS AND STATISTICS (606)[effective from March 2015]

MODEL QUESTION PAPER

Time : 2½ hrs. Max Marks : 80

GENERAL INSTRUCTIONS:

- This question paper contains seven main questions.
- All seven questions are compulsory.
- Answer each main question on a fresh page.
- Use of calculator is not allowed.
- Log tables will be supplied on request.
- Graphs should be drawn on the answer paper only.
- For each main questions the subquestions carry the following marks:
  A = 1 mark , B = 2 marks , C = 3 marks , D = 4 marks , E = 5 marks.

Q No 1 (A) Define " Symmetric matrix"

(B) Find area of the triangle whose vertices are (3,8), (-4,2) and (5,1) by using Determinants.

(C) Find $\frac{dy}{dx}$, if $xy = 100(x + y)$

(D) By using properties of Determinants as far as possible , show that

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$$

Q No 2 (A) Select and write the correct alternative from those given below.

A matrix in which all the non-diagonal elements are zero is called ------- matrix

- Scalar
- Diagonal
- Square
- Zero

(B) If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ then find $\frac{dy}{dx}$

(C) If $y = (\log x)^{\cos x} + x^3$ then find $\frac{dy}{dx}$

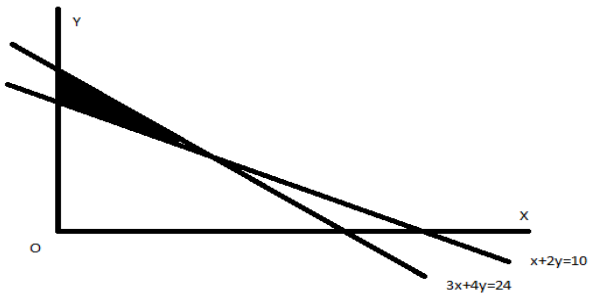
(D) Solve $2x - y + z = 3$, $-x + 2y - z = -4$, $x - y + 2z = 1$ by using matrix method.

Q No 3 (A)) Select and write the correct alternative from those given below.

If R be the relation in the set N given by $R = \{(a, b)/a = b - 2,\ b > 6\}$ then

- (2,6) ∈ R
- (3,8) ∈ R
- (6,8) ∈ R
- (8,7) ∈ R

(B) Write the constraints for the following shaded region



(C) Show that the function $f : R \to R$ given by $f(x) = \frac{2x-1}{3}$, $x \in R$ is bijective

(D) Find the values of a and b such that the function defined by

f(x) = 5 , if $x \leq 2$
= ax + b , if $2 < x < 10$
= 21 , if $x \geq 10$

Is continuous in its domain.

Q No 4 (A) Select and write the correct alternative from those given below.

$\int \frac{1}{x^2-a^2} dx = \_\_\_\_$

- $\frac{1}{2a} \text{Log} \left|\frac{x+a}{x-a}\right| + c$
- $\frac{1}{2a} \text{Log} \left|\frac{x-a}{x+a}\right| + c$
- $\text{Log} \left|\frac{x+a}{x-a}\right| + c$
- $\text{Log} \left|\frac{x-a}{x+a}\right| + c$

(B) Evaluate $\int \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx$

(C) Find $\int_0^2 x\sqrt{2-x}\, dx$

(D) Attempt **any one** of the following:

(i) Evaluate $\int \text{Log}(9+x)dx$

(ii) Evaluate $\int \frac{sec^2 X}{\sqrt{4sec^2 x - 4tanx - 1}} dx$

Q No 5 (A) Select and write the correct alternative from those given below.

The difference between the resources and Liabilities of a firm is called its -----

- Present worth
- Future value
- Profit
- Loss

(B) Evaluate $\int_0^1 \frac{x^2 + 3x + 2}{\sqrt{x}} dx$

(C) Solve the differential equation $x(1+y)\,dx - y(1+x^2)\,dy = 0$ given that y = 0 when x = 1

(D) Maximise z = 5x +3y , subject to the constraints $3x+5y \leq 15$, $5x+2y \leq 10$, $x \geq 0$ $y \geq 0$

Q No 6 (A) Define " Ordinary Annuity"

(B) A black and a red dice are rolled . Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in 5.

(C) Form differential equation representing the given family of curves by eliminating arbitrary constants a and b from $y^2 = a(b^2 - x^2)$.

(D) A man is known to speak truth 3 out of 4 times . He throws a die and then reports that it is a six . Find the probability that it is actually six.

(E) Attempt **any one** of the following:

(i) A house is sold for Rs.50,000 **down** and 10 semi annual payments of Rs. 5000 each, the first due 3 years hence. Find the cash price of the house if the money is worth 6% p.a. compounded semi annually . Given that

$(1.03)^{-5} = 0.8630$, $(1.03)^{-6} = 0.8379$, $(1.03)^{-16} = 0.6242$, $(1.03)^{-15} = 0.6427$

(ii) At the beginning of each month Rs. 5000 is deposited into a savings account in a post office that pays 12% p.a. compounded monthly. What is the future value of amount in the account of post office at the end of 6 years? (Use log tables)

Q No 7 (A) Define " Bill of exchange"

(B) A , B and C were farmers sharing profits in the ratio 4 :3 : 2 . B retires from the firm and A and C decide to share profits in the ratio 3 : 2 . Calculate the gaining ratio.

(C) A , B , and C started a partnership and invested Rs.100000 , Rs. 80,000 and Rs. 1,20,000 respectively . C took loan of Rs 70,000 and paid 9.5% interest to the firm. The firm earned the profit of Rs 135000 in addition to the interest from the loan . Find each partners total earnings if the profits are distributed in the ratio of the capital investments.

(D) The difference between the true discount and bankers discount on a bill due 6 months hence at 4% per annum is Rs 8. Find the present worth , true discount , the bankers discount and the amount of the bill.

(E) Attempt **any one** of the following:

(i) The profit of a monopolist is given by $P(x) = \frac{8000x}{500+x} - x$. Find the value of x for which the P(x) is maximum. Find the maximum profit.

(ii) The cost C(x) , associated with productions and making of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$.

Find (a) The average cost function.

(b) The average cost of output of 10 units.

(c) The marginal cost function.

(d) The marginal cost when 3 units are produced.

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