# NEST - 2012



## NATIONAL ENTRANCE SCREENING TEST (NEST – 2012)

#### Total Marks: 200

#### Time: 3 hours

#### <u>General instructions</u>

- 1. This question booklet contains five sections. Each section carries 50 marks.
- 2. First section is a general section and it is **compulsory**.
- 3. Section 2 to 5 are subject sections (biology, chemistry, maths and physics). Choose any three, that is, omit any one of the four subject sections.
- 4. Pocket calculators, cell phones, log tables etc. are **NOT** permitted in the examination hall.
- 5. Answers to the questions are to be marked in the supplied OMR sheet.
- 6. Please make sure that question booklet code (A or B) matches with OMR sheet code (A or B) respectively. In case of discrepancy please inform the invigilator immediately.
- 7. Rough work may be done on separate blank sheets provided.
- 8. Return the OMR sheet to the invigilator at the end of examination.
- 9. Read the instructions given at the beginning of each section carefully.

#### Instructions for writing on OMR sheet

- 1. Read and follow the instructions given on OMR sheet.
- 2. Write your name, roll number and other required information with ball point pen in appropriate boxes provided. Sign your name with ball point pen in the box provided.
- 3. In the remaining part of OMR sheet, use **HB pencil only (as instructed). Make** sure the bubbles are filled properly (as indicated in OMR sheet).
- 4. As far as possible, fill in the answers only after you are sure that you do not need to change them. In case you have to change the answer after filling, erase the mark properly so that no black spot is left inside the bubble.
- 5. Ensure that you are filling the bubbles corresponding to correct sections and answers.
- 6. Your roll number (as given in the admit card) MUST BE ENTERED COR-RECTLY. If entered wrongly or not entered, the OMR sheet will be invalid and will not be graded.

## Section 1: GENERAL

## Marks for Section 1: 50

This section contains 19 questions.

For each question, only one of the four options is a correct answer. For questions, 1.1 to 1.12, a correct answer will earn 3 marks. For questions, 1.13 to 1.19, a correct answer will earn 2 marks. For this **GENERAL** section, a wrong answer or an unattempted question will earn 0 marks.

Read the following passage carefully and answer questions 1.1 to 1.3.

In mathematics and computer science, graph theory is the study of graphs, mathematical structures used to model pairwise relations between objects from a certain collection. A *graph* in this context is a collection of *vertices* or nodes and a collection of *edges* that connect pairs of vertices. An edge may be undirected, meaning that there is no distinction between the two vertices associated with it, or directed, in which it is directed from one vertex to another. The graph, in addition to vertices, may contain only undirected edges or only directed edges or a combination of both. The graphs studied in graph theory should not be confused with the graphs of functions or other kinds of graphs.

Graphs can be used in modelling and treatment of many problems of practical interest in physical, biological and social systems. In computer science, graphs are used to represent networks of communication, data organization, computational devices, flow of computation, etc. In chemistry a graph makes a natural model for a molecule, where vertices represent atoms and edges represent bonds. This approach is especially used in computer processing of molecular structures. Likewise, graph theory is useful in conservation biology where a vertex can represent regions where certain species exist and the edges represent migration paths, or movement between the regions. This information is important when looking at breeding patterns or tracking the spread of diseases or how changes in the movement of one species can affect the other species.

- **1.1** Some situations from daily life are described below. Which one of them **cannot** be represented as a problem in graph theory?
  - (A) An electrician wants to connect 10 computers in the school to the Internet.
  - (B) A Web-master wants to design a website with 10 web-pages linked to each other.
  - (C) A salesman wants to represent annual sale of computers over last 10 years.
  - (D) A postman has to deliver letters to 10 households taking the shortest path.

#### 1.2 Among the following statements, the **false** statement is

- (A) In the nervous system, sensory receptors can be taken as vertices.
- (B) In a study of interacting particles, mutual forces can be taken as edges.
- (C) In a study of a bus network, zonal boundaries can be taken as edges.
- (D) In a study of government aid programme, aid recipients can be taken as vertices.

**1.3** Three points P, Q and R lie in the plane. Which of these is **not** a "graph"?

- (A) The triangle formed by P, Q and R.
- (B) A line segment between Q and R.
- (C) A vector from P to Q.
- (D) A circle centred at P.

Read the following passage carefully and answer questions 1.4 to 1.6.

Silicon is a tetravalent metalloid, comprising about 28% of the earth's crust and is the eighth most common element in the universe by mass. However, it rarely occurs in the free form in nature. It is combined with oxygen and exists in various forms. Silicate minerals are the most common minerals on the earth, and include quartz, feldspar, mica, amphibole, etc. The silicates contain  $SiO_4^{4-}$  units.

Purification of silicon is of paramount importance and is achieved by multi step processes. In one such process, metallurgical grade silicon is produced by treating silica with coke at very high temperature (Equation 1). The product consists of about 98% silicon and about 1-2% of impurities in the form of metals and metal oxides. Further purification is done by converting silicon to trichlorosilane (Equation 2). The trichlorosilane is purified by distillation. Finally, pure silicon is obtained by reducing the trichlorosilane by hydrogen (Equation 3).

 $SiO_2 + 2C \rightarrow Si + 2CO$  (1)

$$Si + 3HCl \rightarrow SiCl_3H + H_2$$
 (2)

$$SiCl_3H + H_2 \rightarrow Si + 3HCl$$
 (3)

- **1.4** In the first step (Equation 1), which of the following oxide impurity would also reduce to the corresponding metal?
  - (A) Aluminium oxide.

(B) Iron oxide.

(C) Calcium oxide.

- (D) Sodium oxide.
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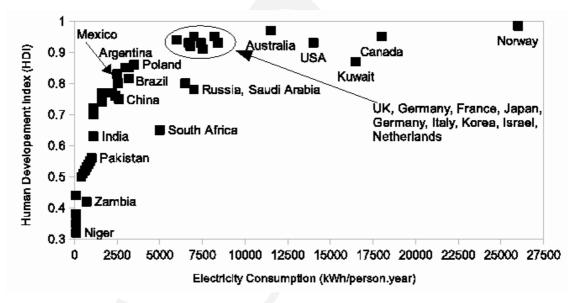
**1.5** In step 2 (Equation 2), silicon is converted to trichlorosilane as the trichlorosilane

- (A) has a lower boiling point than that of the solid impurities.
- (B) is further reduced by the hydrogen formed in the process.
- (C) can be separated by dissolving in water.
- (D) can be oxidised easily.

## 1.6 The most likely geometry for a silicate unit is

- (A) square planar.
- (C) octahedral.

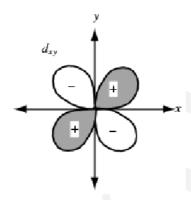
- (B) square pyramidal.
- (D) tetrahedral.
- 1.7 In the graph below, the scale on x-axis shows consumption of electric energy per person per year and y-axis shows human development index (HDI) indicating average living conditions for the citizens. The data corresponds to the year 2003-2004. Choose the statement which is not consistent with the information presented in the graph.



- (A) India's position on the graph indicates both low electric consumption as well as low HDI.
- (B) A cluster of countries that includes Germany roughly indicates optimal electric consumption for high HDI.
- (C) USA's position is significantly away from the overall trend curve.
- (D) As per the overall trend, Saudi Arabia shows excess electric consumption in relation to its HDI.
- 1.8 Which one of the following numbers is a perfect square?
  - (A) 436928. (B) 452929. (C) 469232. (D) 478637.

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**1.9** Following is the  $d_{xy}$  orbital diagram. The X and Y axes are shown, while Z-axis is directed perpendicular to the plane of the paper. Under which of the following operations would the orbital remain identical?



- (A) Rotation of  $180^{\circ}$  about Y axis.
- (B) Rotation of  $180^{\circ}$  about X axis.
- (C) Reflection in YZ plane.
- (D) Reflection in XY plane.
- **1.10** Consider the polynomial,  $(x a)m^2 + (a + x)m 2x$ . One of the factors of this polynomial is
  - (A) m + 1. (B) m 1. (C) x a. (D) x + a.
- 1.11 Marks of 20 students in a class were written in descending order in a mark sheet. However, the marks of first 2 and last 2 students got erased due to a mistake. Which of the following will definitely remain unaffected?
  - (A) Mean. (B) Mode. (C) Median. (D) none of these.
- 1.12 Mr. A and Mrs. A have two daughters. Mr. A's paternal grandmother has a genetic disorder related to the X-chromosome, whereas Mrs A's family lineage has no history for the same disorder. The probability that at least one of the daughters of the couple will show the same disorder is
  - (A) 0. (B) 0.0625. (C) 0.125. (D) 0.33.

For rest of the questions in this section, each correct answer will earn 2 marks.

1.13 Which of the following group of diseases is water-borne?

- (A) Jaundice, dysentery, cholera.
- (B) Jaundice, cholera, tetanus.
- (C) Cholera, malaria, filaria.
- (D) Malaria, diarrhea, tetanus.

1.14 Recent studies using DNA sequencing reveal that *Homo sapiens* originated in

- (A) Middle East. (B) East Asia.
- (C) Central Africa. (D) Indian subcontinent.
- 1.15 The number of moles of solute present in 1 kg of a solvent is called its
  - (A) molarity. (B) molality. (C) normality. (D) formality.

1.16 The year 2011 was celebrated as the International Year of Chemistry and it marked the centenary of Nobel prize in Chemistry for

- (A) Marie Curie's discovery of Radium & Polonium.
- (B) Otto Hahn's discovery of fission of heavy elements.
- (C) Dmitri Mendeleev's discovery of the periodic table.
- (D) Svante August Arrhenius's electrolytic theory of dissociation.
- **1.17** Recently, India has been declared free of Polio by the World Health Organisation. The causative organism for Polio is a
  - (A) bacterium. (B) virus. (C) protozoan. (D) parasitic worm.

1.18 In the context of computer networks, the acronym LAN stands for

- (A) Large Area Network. (B) Linked Area Network.
- (C) Limited Area Network.
- (D) Local Area Network.
- 1.19 India has a major Antarctic programme under which 30 scientific expeditions have been carried out. The research centre set up in Antarctica by India in 1989 and operating till today is named
  - (A) Dakshin Gangotri. (B) Rohini. (C) Megha. (D) Maitri.

## Section 2: BIOLOGY

Marks for Section 2: 50

This section contains 14 questions.

For questions 2.1 to 2.10 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 marks.

2.1 In an *in vitro* transcription experiment, the transcript and both the strands of DNA were separated and analyzed for their base composition. The result obtained is shown in the following table. Determine which DNA strand is serving as the template for mRNA synthesis.

	G	Α	Т	С	U
DNA Strand 1	23.8	30	26	20	0
DNA Strand 2	20	26.2	30	24.1	0
mRNA	24	29.6	0	20	25.8

- (A) Strand 1.
- (C) Both strands 1 and 2.

(B) Strand 2.

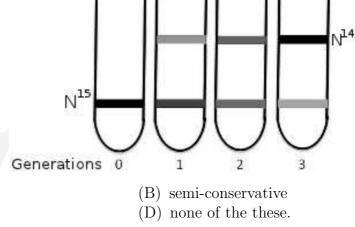
(D) Neither strand 1 nor 2.

- 2.2 Desert animals need to conserve water. Kidneys regulate the concentration of salt and water in blood through the formation and excretion of urine. A kidney is composed of approximately one million units called nephrons. The kidneys of desert animals have modified nephrons which help them survive long periods without water. Which one of these options would be the expected modification?
  - (A) A very short collecting duct.
  - (C) A very large Bowman's capsule.
- (B) A very short distal tubule.
- (D) A very long loop of Henle.

2.3 The results obtained after following the experimental design of Meselson and Stahl to study DNA replication are shown in the adjacent figure. These results suggest that the mode of DNA replication is

(A) conservative.

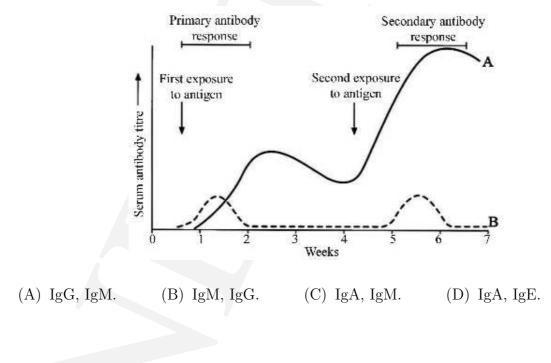
(C) dispersive



- 2.4 Insectivorous plants have evolved an ability to capture and digest insects in addition to their ability to photosynthesize. Which of the following nutritional requirements is most likely being partially met by this "carnivorous" ability of plants?
  - (A) Hydrogen. (B) Carbon. (C) Nitrogen. (D) Oxygen.
- 2.5 The phenomenon of "linked genes" contradicts which of the following fundamental principles of genetics?
  - (A) Law of segregation.
  - (B) Incomplete dominance.
  - (C) Principle of dominance and recessivity.
  - (D) Principle of independent assortment.
- 2.6 Telomeres are nucleotide sequences at the ends of DNA molecules of eukaryotic cells. Since linear chromosomal DNA gets shortened naturally after each round of replication, telomeres protect the genes near the ends of DNA from being eroded. An enzyme, telomerase, catalyses the lengthening of telomeres. In which of the following cells do you expect the activity of telomerase to be the highest?
  - (A) Actively respiring cells.
- (B) Differentiated cells.

(C) Stem cells.

- (D) Aging cells.
- 2.7 The graph below depicts serum antibody levels in a child after vaccination. Which antibody types do the curves A and B represent, respectively?

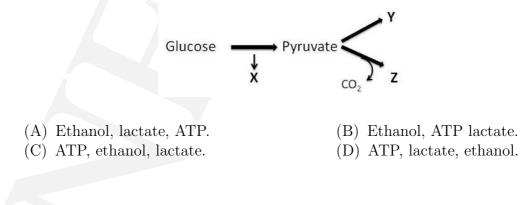


- 2.8 Flowering plants can reproduce either sexually (through fusion of haploid gametes that are produced following meiotic divisions in reproductive organs) or asexually (from diploid vegetative organs). The following statements represent a mix of correct and incorrect general features of both these modes of reproduction.
  - 1. In sexual reproduction, progeny are genetically different from each other.
  - 2. In asexual mode of reproduction, progeny are genetically identical to each other but different from the parent.
  - 3. Development of greater adaptive ability is possible in case of progeny derived by sexual reproduction.
  - 4. A minor change in the habitat may adversely affect all offspring derived by asexual reproduction.
  - 5. A bisexual plant grown in isolation is incapable of sexual reproduction.

Which of the following combinations of the above statements correctly represents general features of both sexual and asexual reproduction of plants?

(A) 1, 3 and 4. (B) 1, 2 and 5. (C) 1, 4 and 5. (D) 1, 2 and 3.

- 2.9 The endosymbiotic theory of evolution is based on a phenomenon in which one organism engulfs another but does not digest it. Both organisms continue to live together in an arrangement that is mutually beneficial. It is considered to be one of the key evolutionary events that led to the development of higher eukaryotic organisms including plants and animals. The organelles of a eukaryotic cell that are believed to have originated through (such) early endosymbiotic events are
  - (A) chloroplasts and endoplasmic reticulum.
  - (B) mitochondria and chloroplasts.
  - (C) mitochondria and lysosomes.
  - (D) Golgi bodies and chloroplasts.
- **2.10** Below is a generalized representation of the process of anaerobic cellular respiration. Which molecules represent X, Y, Z, respectively?



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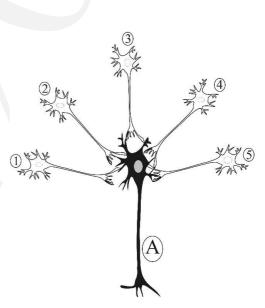
For questions 2.11 to 2.14 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 marks.

- 2.11 *E. coli* is a bacterium that is routinely used in cloning experiments and can be grown on artificially synthesized media in the laboratory. One of the strains of this bacterium, XL-1 Blue, is naturally resistant to the antibiotic, tetracycline. A genetic transformation experiment was performed to introduce a plasmid that encodes an ampicillin resistance gene into this strain. Following transformation, an equal number of cells were grown independently on two types of growth media - (A) growth medium with tetracycline alone and (B) growth medium with tetracycline and ampicillin. The number of colonies that grew on medium A was much higher than that obtained on medium B. Which of the following statement(s) explain the above observation?
  - (A) The transformation efficiency of cells is lower than their viability.
  - (B) Tetracycline was inhibitory to the growth of bacterial cells.
  - (C) Untransformed cells also grew on growth medium "A".
  - (D) Ampicillin did not allow transformed cells to grow on growth medium "B".
- 2.12 An operon comprises of regulatory DNA sequences such as activator and repressor binding sites near the promoter and controls the transcription of structural genes as a single mRNA. Lac operon is a classical example of gene regulation related to the primary utilization of carbon source for energy in the gut bacterium, *Escherichia coli*. The transcription of structural genes is governed by availability of different concentration of glucose and/or lactose in the medium. Which of the following statements related to the Lac operon is/are true.
  - (A) Repressor binds to the RNA polymerase.
  - (B) Lac operon works on the principle of negative regulation.
  - (C) cAMP level will be low in the absence of glucose.
  - (D) If glucose is absent and lactose is present, high levels of lac mRNA will be synthesized.

2.13 Birth records of 4 babies were mixed up in a hospital. The ABO blood groups of the four babies were known to be A, B, AB and O. To determine their respective parents, blood groups of all parents, except the father of baby III were tested. Choose the correct statements:

		Blood Type
Parents I	father	AB
	mother	0
Parents II	father	А
	mother	Ο
Parents III	father	Unknown
	mother	А
Parents IV	father	Ō
	mother	0

- (A) The blood group of Father II could be O and baby III A.
- (B) The blood group of the Father III could be A and baby III is A.
- (C) The blood group of Father III could be B and baby III is AB.
- (D) The blood group of Father III could be either AB or B.
- 2.14 Presynaptic neurons generate nerve impulses (action potentials) which are transmitted to the postsynaptic neuron via synapses (junctions). Presynaptic neurons release neurotransmitters in the synapse which bind to the specific receptors on the postsynaptic neuron. A diagrammatic figure showing connectivity of neurons in a neural circuit is given below. The dendrites and cell body of a postsynaptic neuron (A) receive synaptic contacts from 5 different presynaptic neurons (1, 2, 3, 4, and 5). The presynaptic neurons 1 and 5 release an excitatory neurotransmitter, acetylcholine (Ach) to act on the postsynaptic neuron A, whereas presynaptic neurons 2, 3 and 4 release an inhibitory neurotransmitter,  $\gamma$ -amino butyric acid (GABA) to act on 'A'. The ability of the postsynaptic neuron to



generate the action potential depends upon the relative contribution from the excitatory and inhibitory neurotransmitters. Choose the correct answer(s) from the options given below.

- (A) Neuron A will generate an action potential if only neurons 1 and 5 release Ach on it.
- (B) Neuron A will not generate an action potential but instead gets inhibited if neuron 1 releases Ach and at the same time neurons 2, 3, and 4 release GABA on it.
- (C) Neuron A will generate an action potential if neuron 1 releases Ach and at the same time neurons 2, 3, and 4 release GABA on it.
- (D) All of the above.

#### Section 3: CHEMISTRY

Marks for Section 3: 50

This section contains 14 questions.

For questions 3.1 to 3.10 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 marks.

**3.1** The probability distribution for the speed (v) of molecules for an ideal gas at temperature T, is given by

$$f(v) = 4\pi v^2 \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right)$$

where m is the mass of a molecule and  $k_{\rm B}$  is the Boltzmann constant. Among the following, the correct statement is

- (A) The most probable velocity is proportional to  $T^{3/2}$ .
- (B) The most probable velocity is higher than the average velocity.
- (C) For two different gases A and B, the average velocity of the molecules of A is same as that of B, at a given temperature.
- (D) For two different gases A and B, the average kinetic energy of the molecules of A is same as that of B, at a given temperature.

**3.2** The correct order of acidity of the following compounds is

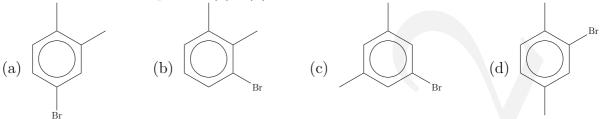
- (A)  $HIO_4 \approx HClO_4 < HClO_2 < HOCl$
- (B)  $HOCl < HClO_2 < HClO_4 \approx HIO_4$
- (C)  $HOCl < HClO_2 < HIO_4 < HClO_4$
- (D)  $HOCl < HClO_2 < HClO_4 < HIO_4$

**3.3** The relative stabilities of  $N_2$ ,  $N_2^+$ , and  $N_2^-$  are

- (A)  $N_2 < N_2^+ \approx N_2^-$
- (B)  $N_2 > N_2^+ \approx N_2^-$
- (C)  $N_2 \approx N_2^+ > N_2^-$
- (D)  $N_2 \approx N_2^+ < N_2^-$
- **3.4** For the complete electrolysis of 1 mole of water using a voltage of 100V across two electrodes, the minimum amount of electrical energy (in Joule) to be spent is (F is the Faraday constant)
  - (A) F/100 (B) F/50 (C) 100F (D) 200F

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**3.5** Consider the four compounds (a) - (d)



Each of them is subjected to the treatment of the following sequence of reagents: (i) Mg, ether (ii)  $CO_2$  (iii) H<sup>+</sup>, H<sub>2</sub>O (iv) hot KMnO<sub>4</sub>. The two compounds that give the same product are

(A) a and b.	(B) a and d
(C) b and c.	(D) c and d.

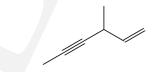
- **3.6** A mixture of two isomeric pentoses on reaction with hydroxylamine gives 5 millimoles of a mixture of oximes. The same mixture of pentoses on treatment with bromine water produces a mixture with one of the pentoses unchanged (0.45g). The other product reacts with NaHCO<sub>3</sub>. The mixture of the pentoses consists of
  - (A) 0.3g aldose and 0.45g ketose.
  - (B) 0.45g aldose and 0.45g ketose.
  - (C) 0.45g ketose and 0.40g aldose.
  - (D) 0.45g aldose and 0.25g ketose.

**3.7** Which of the following pentachlorides does not exist?

(A)  $SbCl_5$  (B)  $PCl_5$  (C)  $BiCl_5$  (D)  $AsCl_5$ 

- **3.8** The number and type of interstices present in a cubic closed-pack structure of N atoms are
  - (A) N, tetrahedral.(C) 2N, octahedral.

- (B) 2N, tetrahedral.
- (D) 4N, tetrahedral.
- 3.9 The IUPAC name of the following compound is



(A) 3-methylhex-4-yn-1-ene.

(C) 3-methylhex-1-en-4-yne.

- (B) 4-methylhex-2-yn-5-ene.
- (D) 4-vinyl-2-pentyne.

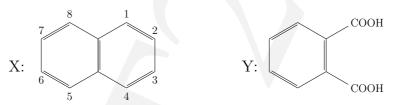
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**3.10** The structure of the most predominant species present in the aqueous solution of serine (a natural amino acid) at pH 1 is



For questions 3.11 to 3.14 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 marks.

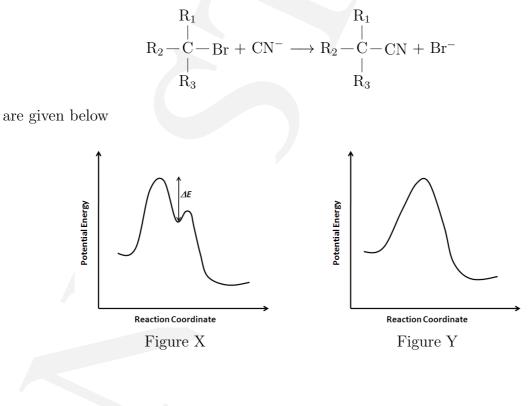
**3.11** Naphthalene (X) is an aromatic compound and is the largest single component of coal tar.



To prove the structure of naphthalene, Graebe (1869) carried out oxidation studies on it. X on oxidation gives phthalic acid (Y). X can be shown as a resonance hybrid of three principal structures (assumed to be equally contributing). Given the bond lengths (Å):  $C_{sp^3}-C_{sp^3} = 1.54$ ;  $C_{sp^3}-C_{sp^2} = 1.50$ ;  $C_{sp^2}-C_{sp^2} = 1.48$ ;  $C_{sp^2}=C_{sp^2} = 1.34$ , choose the correct statement(s) from the following.

- (A) The 1,2-bond in naphthalene has a bond length of 1.39 Å.
- (B) There are ten isomers of dichloronaphthalene.
- (C) 2-Nitronaphthalene on oxidation gives nitrophthalic acid, while 2-aminonaphthalene gives phthalic acid.
- (D) Complete reduction of naphthalene gives decalin  $(C_{10}H_{18})$ . There are two isomeric decalins.

- **3.12** Boron is a fairly rare element. However, it has many important applications. It is a nonmetal and has a very high melting point of 2180 °C. It is extracted from borax and it forms a large number of electron deficient compounds. Choose the correct statement(s).
  - (A)  $B_2O_3$  is a basic oxide.
  - (B) Boron can form  $BF_6^{3-}$  ion.
  - (C) Crystalline boron may be obtained by thermal decomposition of diborane  $(B_2H_6)$ .
  - (D) Amorphous boron may be obtained by the reduction of  $B_2O_3$  at high temperature using sodium or magnesium.
- **3.13** A reaction A + B  $\rightarrow$  C, follows first order kinetics with respect to each reactant, with an overall rate constant,  $k = 2.0 \times 10^{-3} \text{ M}^{-1} \text{ s}^{-1}$ . The initial concentrations of A and B are 0.10M and 6.93M, respectively. Given ln2 = 0.693, choose the correct statement(s).
  - (A) The half-life for A is about 50 seconds.
  - (B) The half-life for B is about 50 seconds.
  - (C) The concentration of A remaining after 100 seconds is approximately 2.5  $\times$   $10^{-2}{\rm M}.$
  - (D) The concentration of B remaining after 100 seconds is approximately 1.73M.
- 3.14 The energy profiles for the two possible mechanisms of the following reaction,





Consider the following substrates (I to IV) for the above reaction I:  $R_1 = R_2 = R_3 = H$ II:  $R_1 = C_2H_5$ ;  $R_2 = R_3 = H$ III:  $R_1 = C_2H_5$ ;  $R_2 = CH_3$ ;  $R_3 = H$ IV:  $R_1 = C_2H_5$ ;  $R_2 = R_3 = CH_3$ 

Choose the correct statement(s).

- (A) The reaction for substrate I follows  $2^{nd}$  order kinetics and that for IV follows  $1^{st}$  order kinetics.
- (B) Considering the mechanism corresponding to Figure X for substrates I to IV, the correct order of  $\Delta E$  is:  $\Delta E_{\rm I} < \Delta E_{\rm II} < \Delta E_{\rm IV}$
- (C) The reaction mechanism changes from the one shown in Figure Y to that in Figure X, as one changes the substrate from I to IV.
- (D) If CN<sup>-</sup> is taken in large excess, it is difficult to ascertain if the reaction follows mechanism corresponding to energy profile in Figure X or that in Figure Y.

### Section 4: MATHEMATICS

Marks for Section 4: 50

This section contains 14 questions.

For questions 4.1 to 4.10 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 marks.

**4.1** If f is a polynomial function which is odd, then

- (A) there is no real root.
- (B) the number of real roots is always more than 1.
- (C) f + 1 is an odd function.
- (D) the derivative f' is an even function.

**4.2** The domain and range of the function  $\log(|\log x|)$  are respectively,

(A)  $\mathbb{R}$  and  $\mathbb{R}$ .(B)  $(0, \infty)$  and  $\mathbb{R}$ .(C)  $\mathbb{R}$  and  $(0, \infty)$ .(D)  $(0, 1) \cup (1, \infty)$  and  $\mathbb{R}$ .

**4.3** If 
$$\begin{pmatrix} \cot \theta & -1 \\ 1 & \cot \theta \end{pmatrix} \begin{pmatrix} \cot \theta & 1 \\ -1 & \cot \theta \end{pmatrix}^{-1} = \begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix}$$
 then  
(A)  $\alpha = \cos \theta$  and  $\beta = \sin \theta$ .  
(B)  $\alpha = \cos 2\theta$  and  $\beta = \sin 2\theta$ .  
(D)  $\alpha = 1$  and  $\beta = 0$ .

**4.4** For  $x \in \mathbb{R}$  with |x| < 1, the value of  $\sum_{n=0}^{\infty} (1+n)x^n$  is

(A) 
$$\frac{x^2 + x + 1}{(1 - x)^2}$$
. (B)  $\frac{x^2 - x + 1}{(1 - x)^2}$ . (C)  $\frac{1}{(1 - x)^2}$ . (D)  $\frac{1}{1 - x}$ .

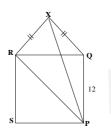
**4.5** Let S denote the set  $\{1, \{1, 2\}, \{1, 2, 3\}\}$ . Then,

 $\begin{array}{ll} \text{(A)} & \{1,2\} \cup \{1\} \subseteq S. \\ \text{(C)} & \{1,2,3\} \cup \{1,2\} \in S. \end{array} \end{array} \\ \begin{array}{ll} \text{(B)} & S \setminus \{1,2,3\} = \{1,\{1,2\}\}. \\ \text{(D)} & \{1,2,3\} \setminus \{2,3\} \in S. \end{array} \\ \end{array}$ 

**4.6** If 
$$f(x) = \int_0^x t e^{-t} dt$$
 then

- (A) it has a minimum but not a maximum.
- (B) it has a maximum but not a minimum.
- (C) it has neither a maximum nor a minimum.
- (D) the function f is not defined.

4.7 In the figure, PQRS is a square of side length 12 and XPQ is a traingle constructed on QR outwardly such that XQ = XR. If area of the figure PQXRS is 192, then area of the figure (PRX) lies between



(A) 80 and 90.	(B) 70 and 80.
(C) 60 and 70.	(D) $50 \text{ and } 60.$

- **4.8** The number of ways of choosing 2 boys and 2 girls in a class for a game of mixed doubles is 1620. Then, the number of ways of choosing 2 students in the class is
  - (A) 153. (B) 171. (C) 182. (D) 212.
- **4.9** The examination co-ordinator of NEST-2011 said "it is not true that some candidates of NEST-2011 neither attempted Mathematics nor attempted Physics". Then we deduce that
  - (A) some candidates attempted both Mathematics and Physics.
  - (B) some candidates attempted Mathematics or Physics.
  - (C) all candidates attempted Mathematics or Physics.
  - (D) all candidates attempted both Mathematics and Physics.
- 4.10 A fair coin is repeatedly tossed. Let X be the event that we get at least 2 heads in 4 tosses and Y be the event that we get at least 3 heads in 6 tosses. Then
  - (A) X is a more likely event than Y.

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- (B) Y is a more likely event than X.
- (C) both events are equally likely.
- (D) given data is insufficient to draw any such conclusion.

For questions 4.11 to 4.14 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 marks.

- **4.11** For any real number x, let [x] denote the largest integer less than or equal to x and  $\langle x \rangle = x [x]$ . If x, y, z are real numbers, then which of the following statements are correct?
  - (A)  $[xyz] = [xy][z\langle xy\rangle].$
  - (B) If 0 < z < 1, then  $\langle xyz \rangle < \langle xy \rangle$ .
  - (C)  $[x + y + z] = [x + y] + [z + \langle x + y \rangle].$
  - (D)  $\langle x + y + z \rangle = \langle \langle x + y \rangle + z \rangle.$

**4.12** The composite function  $g \circ f$  is a constant function if

- (A)  $f(x) = (\cos^{-1} x, \sin^{-1} x)$  and g(x, y) = x + y.
- (B) f(x) = [x] and g(x) = x [x], where [x] denotes the largest integer less than or equal to x.

(C) 
$$f(x) = \frac{x}{|x|}$$
 and  $g(x) = \cos(\frac{\pi}{2}x)$ .  
(D)  $f(x) = |\sin x|$  and  $g(x) = x - [x]$ 

- 4.13 Let X, Y be points on the same side of a straight line  $\ell$  in a plane. Suppose P is a point on  $\ell$  such that XP + PY is the minimum. Then
  - (A) XP and YP are equally inclined to  $\ell$ .
  - (B) The ellipse with X and Y as foci passing through P has  $\ell$  as its tangent at P.
  - (C) The line joining P and the midpoint of XY is perpendicular to  $\ell$ .
  - (D) The circumcircle of triangle XYP has  $\ell$  as its tangent at P.
- **4.14** For any real number t belonging to the open interval  $(0, \frac{1}{2})$  consider the quadratic equation

$$p_t(x) = (t - t^2)x^2 - (1 - t)x + t^2 = 0$$

in x. Let  $\alpha_t, \beta_t$  denote the roots of  $p_t$ ., Then

- (A)  $\alpha_t$  and  $\beta_t$  are always imaginary.
- (B) If  $t \to (\frac{1}{2})$ -, then  $|\alpha_t \beta_t| \to 0$ .
- (C)  $\alpha_t$  and  $\beta_t$  are real and positive; one of  $\alpha_t$  and  $\beta_t$  is greater than 1 and other is less than 1.
- (D) For infinitely many values of t, the corresponding root  $\alpha_t$  is rational and for infinitely many values of t,  $\alpha_t$  is irrational.

## Section 5: PHYSICS

Marks for Section 5: 50

This section contains 14 questions.

For questions 5.1 to 5.10 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 marks.

5.1 A massless rod of length L is hanging from a horizontal ceiling with help of two wires, of equal length and diameter, attached to the ends of the rod. The Young's modulus of the first wire is Y and the second wire is 1.2Y. A mass M is attached to the rod at a distance X from the first wire. If the stretching of the two wires is equal then X/L is

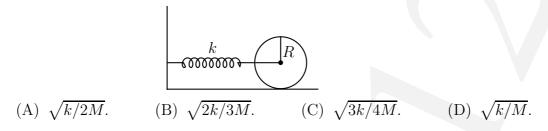
(A) 
$$\frac{5}{11}$$
. (B)  $\frac{6}{11}$ . (C)  $\frac{7}{11}$ . (D)  $\frac{9}{11}$ .

- 5.2 A tape is wound tightly around a cylinder of radius 1 cm. After winding, the radius of the cylinder is found to be 2 cm. If the length of the tape is 10 m then the thickness of the tape should be about
  - (A) 0.001 mm. (B) 0.01 mm. (C) 0.1 mm. (D) 1.0 mm.
- 5.3 A hot-air balloon is rising vertically with a constant velocity of 10 m/sec. An object is dropped from the balloon when it is 60 m above the ground. The angle of elevation of the Sun is 45°. When the falling object is 33.75 m from the ground, the rate at which the shadow of the object is traveling along the ground is (take  $g = 10 \text{ m/sec}^2$ )
  - (A) 30 m/sec. (B) 25 m/sec. (C) 20 m/sec. (D) 15 m/sec.
- 5.4 A thin ring, a disk and an annular cylinder, of same mass M, are released from a point 3.6 m from the ground up an inclined plane of  $30^{\circ}$  inclination. The ring and the disk have the same radius R. For the annular cylinder, the outer radius is R and the inner radius is R/2. Times taken by the ring, disk and annular cylinder, respectively, to reach the ground are in the ratio,

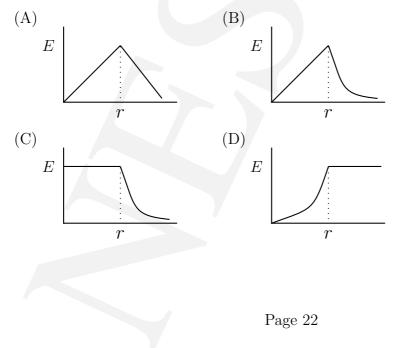
$$\begin{array}{ll} (A) & \sqrt{1.4} : \sqrt{1.5} : \sqrt{2}. \\ (C) & \sqrt{2} : \sqrt{1.5} : \sqrt{1.6}. \end{array} \\ \begin{array}{ll} (B) & \sqrt{2} : \sqrt{1.5} : \sqrt{1.5}. \\ (D) & \sqrt{1.4} : \sqrt{1.5} : \sqrt{1.6}. \end{array} \end{array}$$

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5.5 A disk of mass M and radius R is attached to a spring of constant k as shown in the figure below. The disk rolls back and forth without slipping. The angular frequency of the motion of the disk is,



- 5.6 Water in a dam is 15 m deep. A horizontal pipe of 4 cm diameter passes through the wall of the dam 6 m below the surface of the water. A plug secures the pipe opening. The frictional force between the plug and the pipe wall is about (Take density of water to be  $10^3 \text{ Kg/m}^3$  and g is  $10 \text{ m/s}^2$ .)
  - (A) 80 N. (B) 75 N. (C) 70 N. (D) 65 N.
- 5.7 Two coherent point radio sources  $S_1$  and  $S_2$ , separated by a distance d = 2.3 cm, are radiating radio waves of wavelength  $\lambda = 1$  cm. A detector is moved in the plane of the sources along a circular path of radius R with center at midway between the sources and R >> d. The number of interference maxima that will be detected as the detector completes one circle is
  - (A) 14. (B) 12. (C) 10. (D) 8.
- **5.8** A uniformly charged sphere, of total charge Q, is expanding uniformly about its center. The magnitude of the electric field E at a point r from the center of the sphere as a function of radius of the sphere varies as,



- **5.9** Mean free path of cosmic ray proton in water at  $4^{\circ}$ C is  $\lambda$ . If water is cooled to  $0^{\circ}$ C, then the change in mean free path in water
  - (A) will be positive.(C) will be negative.
- (D) cannot be determined.

(B) will be zero.

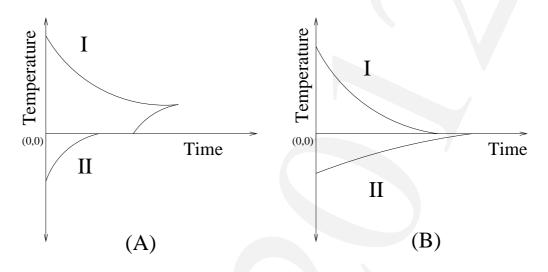
**5.10** Consider a 3-level system with energies  $E_1$ ,  $E_2$  and  $E_3$  in ascending order.  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  are the wavelengths of radiation corresponding to the transitions  $E_2 \rightarrow E_1$ ,  $E_3 \rightarrow E_2$  and  $E_3 \rightarrow E_1$  respectively. The wavelengths are related by,

(A)  $\lambda_2 = \lambda_3 - \lambda_1$ . (B)  $\lambda_1 = \lambda_2 \lambda_3 / (\lambda_3 - \lambda_2)$ . (C)  $\lambda_2 = \lambda_1 \lambda_3 / (\lambda_1 + \lambda_3)$ . (D)  $\lambda_3 = \lambda_1 \lambda_2 / (\lambda_1 + \lambda_2)$ .

For questions 5.11 to 5.14 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 marks.

- **5.11** Two concentric, coplanar rigid circular rings of radii r and 2r carry currents of magnitude 4i and i respectively, in opposite directions. Choose the correct statement(s) from the following.
  - (A) The net torque on outer coil is zero.
  - (B) The net magnetic field at center is zero.
  - (C) The net magnetic dipole moment of the coils is zero.
  - (D) The net force exerted by outer coil on inner coil is zero.
- **5.12** Light of intensity  $I_0$  is incident on air-water interface with an incident angle  $\theta$ . The angle between the refracted and the reflected beams is 90°. A polarization analyser is inserted in water at a distance of 5cm from the interface. The axis of the analyser makes an angle 45° with respect to the normal to the plane of incidence. Choose the correct statement(s) from the following.
  - (A) Intensity of the unpolarized light transmitted through the analyser is less than  $I_0/2.$
  - (B) Intensity of the light polarized along the normal to the plane of incidence transmitted through the analyser is less than  $I_0/2$ .
  - (C) Intensity of the light polarized along the plane of incidence transmitted through the analyser is less than  $I_0/2$ .
  - (D) Intensity of the light polarized along the plane of incidence reflected from the interface is less than  $I_0/2$ .

5.13 In two experiments A and B, the container I contains ice at  $-30^{\circ}$ C and is kept in thermal contact with another container II containing water at  $40^{\circ}$ C. The contents, however, are not allowed to mix. The variation of temperature T with time in the two experiments is shown below. Choose the correct statement(s) from the following.



- (A) In experiment A, water freezes after some time.
- (B) In experiment A, ice melts completely.
- (C) In experiment B, water freezes partially.
- (D) In experiment B, the total mass of ice at P is more than the initial amount.
- **5.14** A and B are two radioactive substances. The half-life of A is same as the average life of B. The decay constant of B is 3.33 per day. (assume  $\ln 2 = 0.70$ )
  - (A) 85.7% of A will disintegrate in about 3 days.
  - (B) If the initial amount of B is twice that of A, then the number of nuclei of A and B will be the same after 0.7 days.
  - (C) If the initial amount of A and B are the same, then the ratio of the number of nuclei of A to that of B will grow exponentially.
  - (D) If initially A and B have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be 1/e after 1 day.