Interdisciplinary programme in

INDUSTRIAL ENGINEERING & OPERATIONS RESEARCH

INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

Sample Questions for Ph.D. Admissions Entrance Test
(some of which have appeared in the previous written tests)

Candidates are to answer as many questions as possible

Question 1: X, Y, Z and W are jointly distributed Bernoulli random variables; and each of these can assume values 0 or 1 only. It is known that $X = \max\{W, Z\}$ and $Y = \min\{W, Z\}$.

(a) If $E(X) = 0.6$ and $E(Y) = 0.1$, determine the Conditional Expectation $E(X \mid Y = 0)$.

(b) Determine the maximum possible value of $E(Y)$ if it is only known that $E(X) = 0.6$ (and there is no other numerical information available).

Question 2: Solve the following optimization problem using any algorithm:

$$\begin{array}{lrl} \text{Maximize} & x_1 + x_2 & \\ \text{s.t.} & 3\,x_1 + 4\,x_2 & \le 11 \\ & x_1 + 2\,x_2 & \ge 4 \\ & x_1\,,\, x_2 \in \mathbb{Z}^2 & \text{(set of positive integers)} \end{array}$$

Question 3: *Sudoku* is a popular puzzle that appears regularly in the daily newspapers. The puzzle is to fill in the girds in such a manner that every row, every column and every 3x3 box accommodates the digits 1 to 9, without repeating a digit. A sample puzzle is shown on the right side. Now, formulate an optimization model to solve the puzzle. Clearly define the variables, constraints and objective function. DO NOT SOLVE THE PUZZLE.

8	1					7		3
			6		7			8
9		2	3	1		6		
	4			7		5	6	
		7	9		1	2		
	6	3		4			9	
		4		9	2	1		6
6			5		4			
7		8					5	9

Question 4: Weekly demand for an item stocked by a retailer is uncertain. You are given past data (say demands $d(t)$ in week t for $t = 1, 2, ..., T$ – current time, are known),

(1) Explain how you would decide on the stock level that a retailer would keep (assume that the item is ordered and replenished at the beginning every week). You would need to assume relevant parameters and you need to state the decision model clearly. If you were to do this to minimize costs, what would your approach be? If you were to do this to meet customer service requirements, what would your approach be? How would you reconcile different policies that arise from these two approaches?

(2) The previous setting is for a fixed price p of the item. Someone suggests that demand in week t depends on the price $p(t)$ that you charge in week t. Give an example of a function that captures this dependence of demand on price, with appropriate assumptions. What are the ways in which the correct price $p(t)$ can be set in this setting (i.e. suggest an optimization problem to determine the price $p(t)$, as usual with some assumptions).

Question 5: Let X_1, X_2 … X_n, be IID random sample from a normal population having mean μ and variance σ^2. Let $\overline{X}$ and S^2 denote the sample mean and sample variance, respectively. Determine $E[S^2]$. Show all steps in your computation.

Question 6: For the Markov chain $\{X_n, n \geq 0\}$ on $\{1, 2, 3, 4, 5\}$ with P as its transition matrix, find $\lim_{n\to\infty} p_{ij}^n$ for all state-pairs (i, j) with $X_0 = 2$ and

$$P = \begin{bmatrix} .1 & .3 & .3 & .2 & .1 \\ 0 & .8 & .2 & 0 & 0 \\ 0 & .4 & .6 & 0 & 0 \\ 0 & 0 & 0 & .7 & .3 \\ 0 & 0 & 0 & .5 & .5 \end{bmatrix}$$