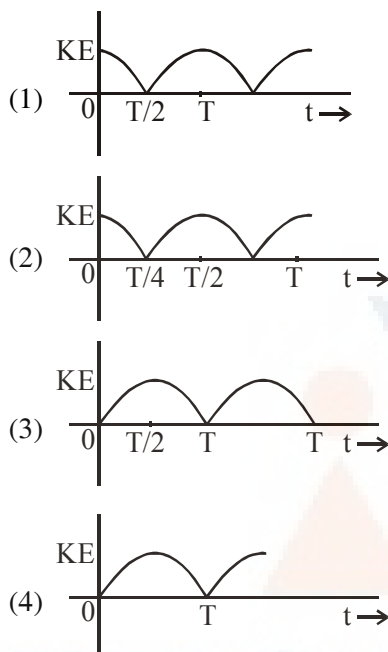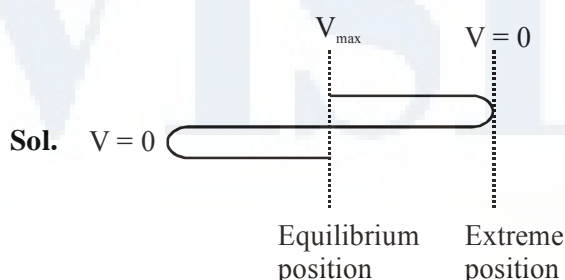**VISION40**

JEE(MAIN) – 2017 TEST PAPER WITH SOLUTION
(HELD ON SUNDAY 02nd APRIL, 2017)
PART A – PHYSICS

1. A particle is executing simple harmonic motion with a time period T. AT time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like
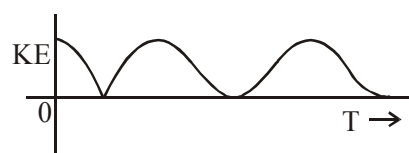
(1)

(2)

(3)

(4)

Ans. (2)

Sol.

Time taken to reach the extreme position from equilibrium position is $\dfrac{T}{4}$. Velocity is maximum at equilibrium position and zero at extreme position.

$V = A \omega \cos \omega t$

$K.E. = \dfrac{1}{2}mv^2$

(m is the mass of particle and v is the velocity of particle

$K.E. = \dfrac{1}{2}mA^2\omega^2 \cos^2 \omega t$

Hence graph of K.E. v/s time is square cos function

2. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be :-

(1) 2.5×10^{25} (2) -2.5×10^{25}
(3) -1.61×10^{23} (4) 1.38×10^{23}

Ans. (2)

Sol. Using ideal gas equation

$\qquad PV = NRT$

(N is number of moles)

$P_0V_0 = N_iR \times 290 \qquad ......(1)$

$\qquad\qquad\qquad [T_i = 273 + 17 = 290 \text{ K}]$

After heating

$P_0V_0 = N_fR \times 300 \qquad ......(2)$

$\qquad\qquad\qquad [T_f = 273 + 27 = 300 \text{ K}]$

from equation (1) and (2)

$$N_f - N_i = \frac{P_0V_0}{R \times 300} - \frac{P_0V_0}{R \times 290}$$

difference in number of moles $-\dfrac{P_0V_0}{R}\left[\dfrac{10}{290 \times 300}\right]$

Hence $n_f - n_i$ is

$$= -\frac{P_0V_0}{R} \times \left[\frac{10}{290 \times 300}\right] \times 6.023 \times 10^{23}$$

putting $P_0 = 10^5 P_A$ and $V_0 = 30$ m^3

Number of molecules $n_f - n_i = -2.5 \times 10^{25}$

3. Which of the following statements is false ?

(1) A rheostat can be used as a potential divider

(2) Kirchhoff's second law represents energy conservation

(3) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.

(4) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.
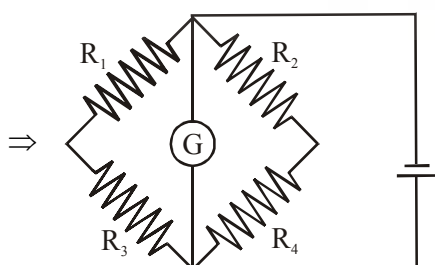
Ans. (4)

Sol. (1)
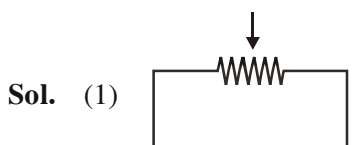
On interchanging Cell & Galvanometer.

(2)
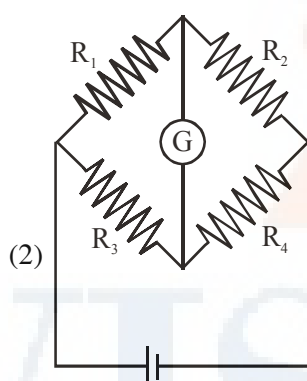
On balancing condition

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \qquad ....(1)$$

$\Rightarrow$

On balancing condition

$$\frac{R_1}{R_2} = \frac{R_3}{R} \qquad ....(2)$$

As we see both equation (1) & (2) are same. So 4th statement is false.

4. The following observations were taken for determining surface tensiton T of water by capillary method :

Diameter of capilary, D = 1.25 × 10⁻² m

rise of water, h = 1.45 × 10⁻² m

Using g = 9.80 m/s² and the simplified relation $T = \dfrac{rhg}{2} \times 10^3$ N/m, the possible error in surface tension is closest to :

(1) 2.4%   (2) 10%   (3) 0.15%   (4) 1.5%

Ans. (4)

Sol. $T = \dfrac{rhg}{2} \times 10^3$

$$\frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta h}{h} + 0$$

$$100 \times \frac{\Delta T}{T} = \left(\frac{10^{-2} \times .01}{1.25 \times 10^{-2}} + \frac{10^{-2} \times .01}{1.45 \times 10^{-2}} \right) 100$$

$$= (0.8 + 0.689)$$

$$= (1.489)$$

$$100 \times \frac{\Delta T}{T} = 1.489 \%$$

$$\simeq 1.5\%$$

5. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth $(\Delta\omega_m)$ of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave ?

(1) $\omega_m + \omega_c$ (2) $\omega_c - \omega_m$

(3) ω_m (4) ω_c

Ans. (3)

**Sol.** Refer NCERT Page No. 526

Three frequencies are contained

$\omega_m + \omega_c, \omega_c - \omega_m$ & ω_c

6. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is :

(1) real and at a distance of 40 cm from the divergent lens

(2) real and at a distance of 6 cm from the convergent lens

(3) real and at a distance of 40 cm from convergent lens

(4) virtual and at a distance of 40 cm from convergent lens.

Ans. (3)

Sol. As parallel beam incident on diverging lens if forms virtual image at $v_1 = -25$ cm from the diverging lense which works as a object for the converging lense (f = 20 cm)

f = –25 cm

←15 cm
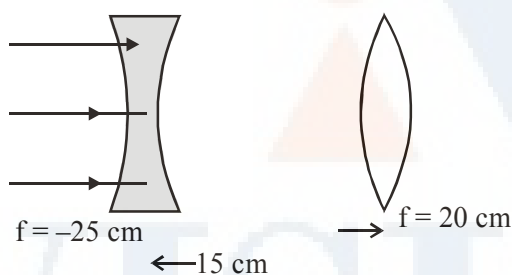
f = 20 cm

So for converging lens u = –40 cm, f = 20 cm

∴ Final image

$$\frac{1}{V} - \frac{1}{-40} = \frac{1}{20}$$

V = 40 cm from converging lenses.

7. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum ?

(1) 1   (2) $\dfrac{3}{\sqrt{2}}$ (3) $\sqrt{\dfrac{3}{2}}$ (4) $\dfrac{\sqrt{3}}{2}$

Ans. (3)

Sol. $I = \dfrac{m\ell^2}{12} + \dfrac{mR^2}{4}$

or $I = \dfrac{m}{4}\left(\dfrac{\ell^2}{3} + R^2\right)$ ....(1)]

Also $m = \pi R^2 \ell \rho$

⇒ $R^2 = \dfrac{m}{\pi \ell \rho}$ Put in equation (1)

$$I = \frac{m}{4}\left(\frac{\ell^2}{3} + \frac{m}{\pi\ell\rho}\right)$$

For maxima & minima

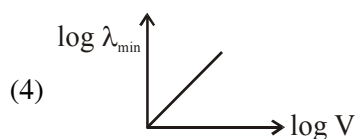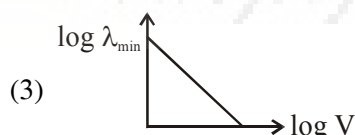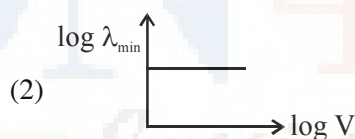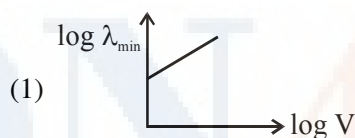$$\frac{dI}{d\ell} = \frac{m}{4}\left(\frac{2\ell}{3} - \frac{m}{\pi\ell^2\rho}\right) = 0$$

⇒ $\dfrac{2\ell}{3} = \dfrac{m}{\pi\ell^2\rho}$ ⇒ $\dfrac{2\ell}{3} = \dfrac{\pi R^2 \ell \rho}{\pi\ell^2\rho}$

or $\dfrac{2\ell}{3} = \dfrac{R^2}{\ell}$

⇒ $\dfrac{\ell^2}{R^2} = \dfrac{3}{2}$

or $\dfrac{\ell^2}{R} = \sqrt{\dfrac{3}{2}}$

8. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of log λ_{min} with log V is correctly represented in :

(1)

(2)

(3)

(4)

Ans. (3)

Sol. $\dfrac{hc}{\lambda_{min}} = eV$

$\dfrac{1}{\lambda_{min}} = \dfrac{eV}{hc}$

3

$\ell n\left(\dfrac{1}{\lambda_{min}}\right) = \ell nV + \ell n\dfrac{e}{hc}$

$-\ell n(\lambda_{min}) = \ell nV + \ell n\dfrac{e}{hc}$

$\ell n(\lambda_{min}) = -\ell nV - \ell n\left(\dfrac{e}{hc}\right)$

It is a straight line with –ve slope.

9. A radioactive nucleus A with a half life T, decays into a nucleus B. At t = 0, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by :

(1) t = T log (1.3)　　　(2) $t = \dfrac{T}{\log(1.3)}$

(3) $t = \dfrac{T}{2}\dfrac{\log 2}{\log 1.3}$　　　(4) $t = T\dfrac{\log 1.3}{\log 2}$

Ans. (4)

Sol. At time t

$\dfrac{N_B}{N_A} = .3 \Rightarrow N_B = .3\,N_A$

also let initially there are total N_0 number of nuclei
$N_A + N_B = N_0$

$N_A = \dfrac{N_0}{1.3}$

Also as we know
$N_A = N_0 e^{-\lambda t}$

$\dfrac{N_0}{1.3} = N_0 e^{-\lambda t}$

$\dfrac{1}{1.3} = e^{-\lambda t} \Rightarrow \ell n(1.3) = \lambda t$ or $t = \dfrac{\ell n(1.3)}{\lambda}$
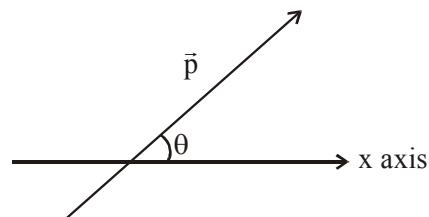
$t = \dfrac{\ell n(1.3)}{\dfrac{\ell n(2)}{T}} = \dfrac{\ell n(1.3)}{\ell n(2)}T$

10. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is :

(1) 60°　　(2) 90°　　(3) 30°　　(4) 45°

Ans. (1)

Sol. So from　　$\vec{\tau} = \vec{p} \times \vec{E}$

$\tau\hat{k} - \tau\hat{k} = (p_x\hat{i} + p_y\hat{j}) \times (E\hat{i} + \sqrt{3}E\hat{j})$

$= p_x \times \sqrt{3}E\,\hat{k} + p_y E(-\hat{k})$

$0 = E\hat{k}(\sqrt{3}p_x - p_y)$

$\dfrac{p_y}{p_x} = \sqrt{3}$

∴　　$\tan\theta = \sqrt{3}$

　　$\theta = 60°$

11. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be :

(1) 135°　(2) 180°　(3) 45°　(4) 90°

Ans. (2)

Sol. In common emitter amplifier circuit input and out put voltage are out of phase. When input voltage is increased then i_b is increased, i_c also increases so voltage drop across R_c is increased. However increase in voltage across R_C is in opposite sense.

12. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$C_p - C_v = a$ for hydrogen gas
$C_p - C_v = b$ for nitrogen gas

The correct relation between a and b is :

(1) a = 14 b　　　(2) a = 28 b

(3) $a = \dfrac{1}{14}b$　　　(4) a = b

Ans. (1)

Sol. $C_P - C_V = R$

where C_P and C_V are molar specific heat capacities
As per the question

$a = \dfrac{R}{2}$　　　$b = \dfrac{R}{28}$

a = 14 b

13. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by : (Given : room temperature = 30° C, specific heat of copper = 0.1 cal/gm°C

(1) 1250°C   (2) 825°C

(3) 800°C   (4) 885° C

Ans. (4)

Sol. Heat given = Heat taken

$(100)(0.1)(T-75) = (100)(0.1)(45) + (170)(1)(45)$

$10(T - 75) = 450 + 7650 = 8100$

$T - 75 = 810$

$T = 885\ °C$

14. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its intial speed is $v_0 = 10$ ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be:-

(1) 10^{-4} kg m^{-1} (2) 10^{-1} kg m^{-1} s^{-1}

(3) 10^{-3} kg m^{-1} (4) 10^{-3} kg s^{-1}

Ans. (1)

Sol. $\frac{1}{2}mv_f^2 = \frac{1}{8}mv_0^2$

$v_f = \frac{v_0}{2} = 5$ m/s

$(10^{-2})\frac{dv}{dt} = -kv^2$

$\int_{10}^{5}\frac{dv}{v^2} = -100\,k\int_{0}^{10}dt$

$\frac{1}{5} - \frac{1}{10} = 100\,k\,(10)$

$k = 10^{-4}$

15. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into to voltmeter of range 0 – 10 V is:

(1) $2.535 × 10^3$ Ω   (2) $4.005 × 10^3$ Ω

(3) $1.985 × 10^3$ Ω   (4) $2.045 × 10^3$ Ω

Ans. (3)

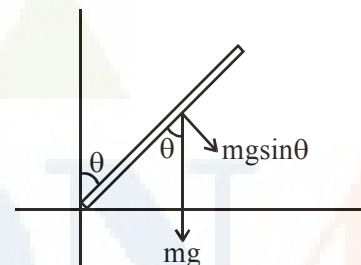**Sol.** $10 = (5 × 10^{-3})(15 + R)$

$r = 1985\ Ω$

16. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is :
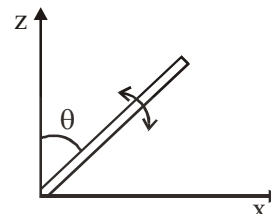
(1) $\frac{3g}{2\ell}\cos\theta$ (2) $\frac{2g}{3\ell}\cos\theta$

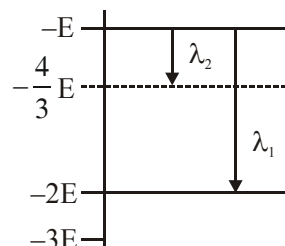(3) $\frac{3g}{2\ell}\sin\theta$ (4) $\frac{2g}{3\ell}\sin\theta$

Ans. (3)

Sol.

Taking torque about pivot $\tau = I\alpha$

$mg\sin\theta\,\frac{\ell}{2} = \frac{m\ell^2}{3}\alpha$

$\alpha = \frac{3g}{2\ell}\sin\theta$

17. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1/\lambda_2$, is given by :

(1) $r = \frac{3}{4}$ (2) $r = \frac{1}{3}$

(3) $r = \frac{4}{3}$ (4) $r = \frac{2}{3}$

5

Ans. (2)

Sol. using $\Delta E = \dfrac{hC}{\lambda}$

for λ_1 $\quad -E - (-2E) = \dfrac{hC}{\lambda_1}$

$\lambda_1 = \dfrac{hC}{E}$

for λ_2 $\quad -E - \left(-\dfrac{4E}{3}\right) = \dfrac{hC}{\lambda_2}$

$\lambda_2 = \dfrac{3hC}{E}$

$\dfrac{\lambda_1}{\lambda_2} = r = \dfrac{1}{3}$

18. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of :

(1) 81 $\qquad$ (2) $\dfrac{1}{81}$ $\qquad$ (3) 9 $\qquad$ (4) $\dfrac{1}{9}$
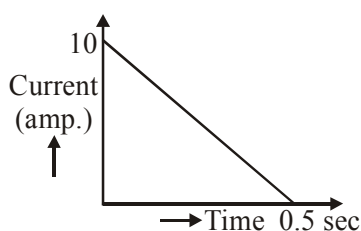
Ans. (3)

Sol. $\text{Stress} = \dfrac{\text{Force}}{\text{area}} = \dfrac{mg}{A} = \dfrac{\text{volume} \times \text{density} \times g}{\text{Area}}$
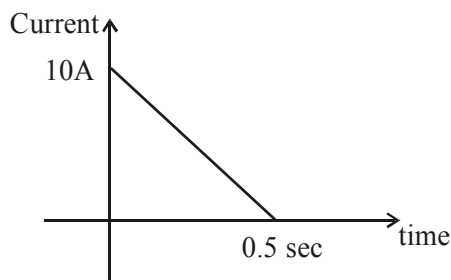
$\text{Stress} = \dfrac{L^3 \rho g}{L^2}$

$\text{Stress} \propto L$

19. In a coil of resistance $100\,\Omega$, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is :

(1) 250 Wb $\qquad$ (2) 275 Wb

(3) 200 Wb $\qquad$ (4) 225 Wb

Ans. (1)

Sol.

$q = \dfrac{\Delta\phi}{R}$

$\Delta\phi = $ change in flux

$q = \int I\,dt$

$= $ Area of current-time graph

$= \dfrac{1}{2} \times 10 \times 0.5 = 2.5$ coloumb

$q = \dfrac{\Delta\phi}{R}$

$\Delta\phi = 2.5 \times 100 = 250$ wb

20. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is :

(1) 9.75 mm $\qquad$ (2) 15.6 mm

(3) 1.56 mm $\qquad$ (4) 7.8 mm

Ans. (4)

Sol. For common maxima

$n_1\lambda_1 = n_2\lambda_2$

$n_1 \times 650 = n_2 \times 520$

$\dfrac{n_1}{n_2} = \dfrac{4}{5}$

$\dfrac{yd}{D} = n\lambda$

$y = \dfrac{4 \times 650 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}}$

$y = 7.8$ mm

21. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is :

(1) 6.98 s (2) 8.76 s

(3) 6.65 s (4) 8.89 s

Ans. (3)

Sol. $T = 2\pi\sqrt{\dfrac{I}{MB}}$

$I = 7.5 \times 10^{-6}$ kg $- $ m^2

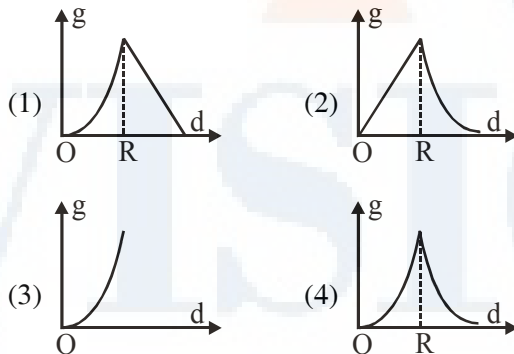$M = 6.7 \times 10^{-2}$ Am^2

By substituting value in the formula

T = .665 sec

for 10 oscillation, time taken will be

Time = 10 T = 6.65 sec

Answer option 3

22. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius):
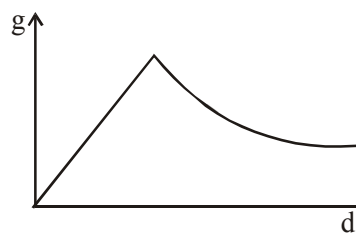

Ans. (2)

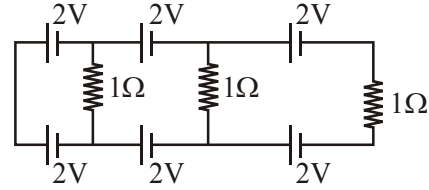**Sol.** $g = \dfrac{GMx}{R^3}$ inside the Earth (straight line)

$g = \dfrac{GM}{r^2}$ outside the Earth
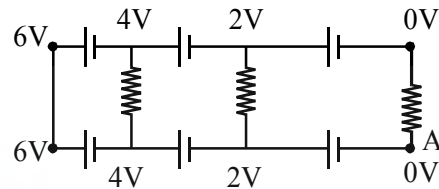
where M is Mass of Earth

option (2)

23. In the above circuit the current in each resistance is :

(1) 0.5 A (2) 0 A (3) 1 A (4) 0.25 A

Ans. (2)

Sol.

Taking voltage of point A as = 0

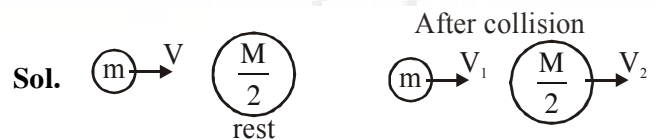Then voltage at other points can be written as shown in figure

Hence voltage across all resistance is zero.

Hence current = 0

24. A particle A of mass m and initial velocity v collides with a particle B of mass $\dfrac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de–Broglie wavelengths λ_A to λ_B after the collision is :

(1) $\dfrac{\lambda_A}{\lambda_B} = \dfrac{2}{3}$ (2) $\dfrac{\lambda_A}{\lambda_B} = \dfrac{1}{2}$

(3) $\dfrac{\lambda_A}{\lambda_B} = \dfrac{1}{3}$ (4) $\dfrac{\lambda_A}{\lambda_B} = 2$

Ans. (4)

Sol.

By conservation of linear momentum

$mv = mv_1 + \dfrac{m}{2}v_2$

$2v = 2v_1 + v_2$ ....(1)

by law of collision

$e = \dfrac{v_2 - v_1}{u_1 - u_2}$

$u = v_2 - v_1$ ....(2)

By equation (1) and (2)

$$v_1 = \frac{v}{3} \quad ; \qquad v_2 = \frac{4v}{3}$$

$$\lambda_1 = \frac{h}{p_1} \quad ; \qquad \lambda_2 = \frac{h}{o_2}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{2}{1}$$

Option (4)

25. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by :

(1) $\dfrac{3\alpha}{PK}$

(2) $3PK\alpha$

(3) $\dfrac{P}{3\alpha K}$

(4) $\dfrac{P}{\alpha K}$

Ans. (3)

Sol. Due to thermal expansion

$$\frac{\Delta v}{v} = 3\alpha\Delta T$$

Due to External pressure

$$\frac{\Delta v}{v} = \frac{P}{K}$$

Equating both w_1 get

$$3\alpha\Delta T = \frac{P}{K}$$

$$\Delta T = \frac{P}{3\alpha K}$$

26. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be :

(1) 9 J   (2) 18 J   (3) 4.5 J   (4) 22 J

Ans. (3)

Sol. F = 6t = ma

$$\Rightarrow a = 6t$$

$$\Rightarrow \frac{dv}{dt} = 6t$$

$$\int_0^v dv = \int_0^1 6t \, dt$$

$$v = (3t^2)_0^1 = 3 \text{ m/s}$$

From work energy theorem

$$W_F = \Delta K.E = \frac{1}{2}m\left(v^2 - u^2\right)$$

$$= \frac{1}{2}(1)(9 - 0) = 4.5 \text{ J}$$

27. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = 3×10^8 ms^{-1})

(1) 17.3 GHz   (2) 15.3 GHz

(3) 10.1 GHz   (4) 12.1 GHz
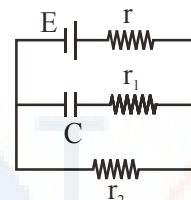
Ans. (1)

Sol. Doppler effect in light (speed of observer is not very small compare to speed of light)

$$f^1 = \sqrt{\frac{1 + V/C}{1 - V/C}} \; f_{source} = \sqrt{\frac{1 + 1/2}{1 - 1/2}}(10 \text{ GHz})$$
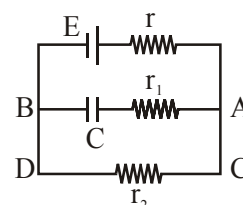
$$= 17.3 \text{ GHz}$$

28. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be :

(1) $CE\dfrac{r_2}{(r + r_2)}$

(2) $CE\dfrac{r_1}{(r_1 + r)}$

(3) CE

(4) $CE\dfrac{r_1}{(r_2 + r)}$

Ans. (1)

Sol. It steady state, current through AB = 0

$$\Rightarrow V_{AB} = V_{CD}$$

$$\Rightarrow V_{AB} = \left(\frac{\varepsilon}{r + r_2} \times r_2\right) - V_{CD}$$

$$\Rightarrow Q_C = CV_{AB}$$

$$= CE\left(\frac{r_2}{r + r_2}\right)$$

VISION40

29. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V.

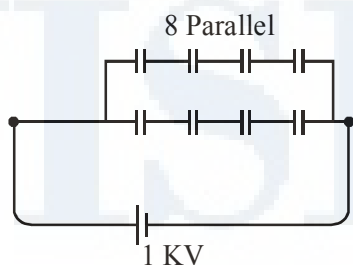The minimum number of capacitors required to achieve this is :

(1) 24   (2) 32

(3) 2   (4) 16

Ans. (2)

Sol. To hold 1 KV potential difference
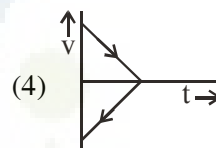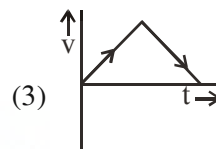
minimum four capacitors are required in series
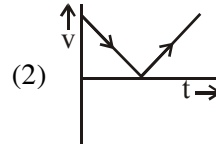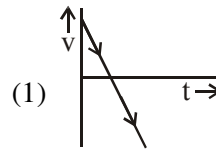
$$\Rightarrow \quad C_1 = \frac{1}{4} \text{ for one series.}$$

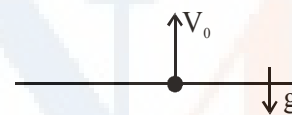So for C_{eq} to be 2μF, 8 parallel combinations are required.

8 Parallel

1 KV

$\Rightarrow$ Minimum no. of capacitors = $8 \times 4 = 32$

30. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?

(1)

(2)

(3)

(4)

Ans. (1)

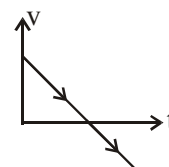**Sol.** Velocity at any time t is

given by

$v = u + at$

$v = v_0 + (-g)t$

$v = v_0 - gt$

$\Rightarrow$ straight line with negative slope

PART B – MATHEMATICS

31. Let k be an integer such that triangle with vertices (k, −3k), (5, k) and (−k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point :

(1) $\left(2,\dfrac{1}{2}\right)$ (2) $\left(2,-\dfrac{1}{2}\right)$

(3) $\left(1,\dfrac{3}{4}\right)$ (4) $\left(1,-\dfrac{3}{4}\right)$

Ans. (1)

Sol. We have

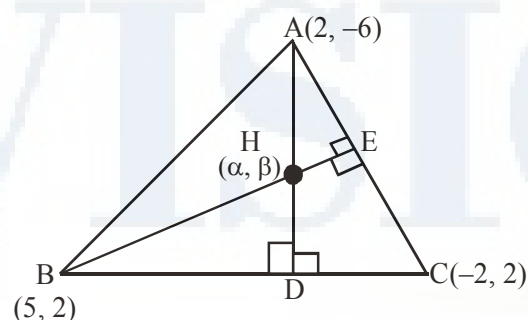$$\frac{1}{2}\left\|\begin{matrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{matrix}\right\|=28$$

$\Rightarrow$ $5k^2 + 13k − 46 = 0$
or
$5k^2 + 13k + 66 = 0$ (no real solution exist)

$\therefore$ $k = \dfrac{-23}{5}$ or $k = 2$

As k is an integer, so k = 2

$\Rightarrow$ orthocentre is $\left(2,\dfrac{1}{2}\right)$

32. If, for a positive integer n, the quadratic equation,

$x(x + 1) + (x + 1) (x + 2) + ...$

$+ (x + \overline{n-1}) (x + n) = 10n$

has two consecutive integral solutions, then n is equal to :

(1) 11   (2) 12

(3) 9   (4) 10

Ans. (1)

Sol. We have

$$\sum_{r=1}^{n}(x+r-1)(x+r)=10n$$

$$\Rightarrow \sum_{r=1}^{n}(x^2+(2r-1)x+(r^2-r))=10n$$

$\therefore$ On solving, we get

$$\underbrace{x^2 + nx + \left(\frac{n^2-31}{3}\right)}_{\alpha \quad \alpha+1}=0$$

$\therefore$ $(2\alpha + 1) = -n \Rightarrow \alpha=\dfrac{-(n+1)}{2}$ ...(1)

and $\alpha(\alpha+1)=\dfrac{n^2-31}{3}$ ...(2)

$\Rightarrow$ $n^2 = 121$ (using (1) in (2))
or $n = 11$

33. The function $f : R \to \left[-\dfrac{1}{2},\dfrac{1}{2}\right]$ defined as $f(x) = \dfrac{x}{1+x^2}$, is :

(1) neither injective nor surjective.
(2) invertible.
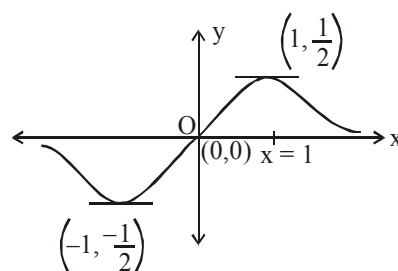(3) injective but not surjective.
(4) surjective but not injective

Ans. (4)

Sol. $f : R \to \left[-\dfrac{1}{2},\dfrac{1}{2}\right]$,

$f(x) = \dfrac{x}{1+x^2} \forall x \in R$

$\Rightarrow$ $f'(x)=\dfrac{(1+x^2)\cdot 1 - x\cdot 2x}{(1+x^2)^2}=\dfrac{-(x+1)(x-1)}{(1+x^2)^2}$

$\therefore$ From above diagram of f(x), f(x) is surjective but not injective.

34. The following statement

$(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ is :

(1) a fallacy

(2) a tautology

(3) equivalent to $\sim p \rightarrow q$

(4) equivalent to $p \rightarrow \sim q$

Ans. (2)

Sol. $(p \rightarrow q) [(\sim p \rightarrow q) \rightarrow q]$

p	q	~ p	p→q	~ p→q	(~ p→q)→q	(p→q)→((~ p→q)→q)
T	F	F	F	T	F	T
T	T	F	T	T	T	T
F	F	T	T	F	T	T
F	T	T	T	T	T	T

∴ It is tautology

35. If S is the set of distinct values of 'b' for which the following system of linear equations

$x + y + z = 1$

$x + ay + z = 1$

$ax + by + z = 0$

has no solution, then S is :

(1) a singleton

(2) an empty set

(3) an infinite set

(4) a finite set containing two or more elements

Ans. (1)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0 \Rightarrow a = 1$

and at $a = 1$ $\qquad D_1 = D_2 = D_3 = 0$

but at $a = 1$ and $b = 1$

First two equations are $\quad x+y+z=1$
and third equation is $\quad x+y+z=0$ $\left.\right] \Rightarrow$ There is no solution.

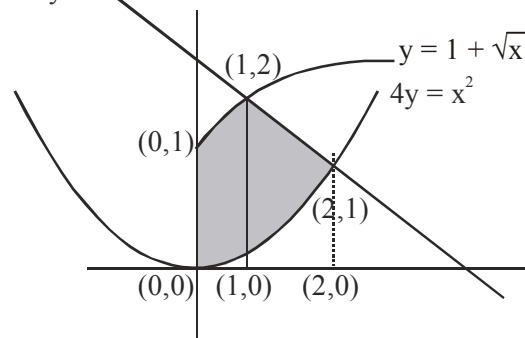∴ $b = \{1\} \Rightarrow$ it is a singleton set

36. The area (in sq. units) of the region

$\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is :

(1) $\dfrac{5}{2}$ (2) $\dfrac{59}{12}$ (3) $\dfrac{3}{2}$ (4) $\dfrac{7}{3}$

Ans. (1)

Sol.

Area $= \displaystyle\int_0^1 (1 + \sqrt{x})\,dx + \int_1^2 (3 - x)\,dx - \int_0^2 \dfrac{x^2}{4}\,dx$

$= \dfrac{5}{2}$

37. For any three positive real numbers a, b and c,

$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$.

Then :

(1) a, b and c are in G.P.

(2) b, c and a are in G.P.

(3) b, c and a are in A.P.

(4) a, b and c are in A.P.

Ans. (3)

Sol. $(15a)^2 + (3b)^2 + (5c)^2 - (15a)(5c) - (15a)(3b) - (3b)(5c) = 0$

$\dfrac{1}{2}[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2] = 0$

it is possible when $15a = 3b = 5c$

∴ $b = \dfrac{5c}{3}$, $a = \dfrac{c}{3}$

$a + b = 2c$

$\Rightarrow$ b, c, a in A.P.

38. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is :

(1) 484   (2) 485   (3) 468   (4) 469

Ans. (2)

Sol.

X ⎰→ 4 Ladies
  ⎱→ 3 Men

y ⎰→ 3 Ladies
  ⎱→ 4 Men
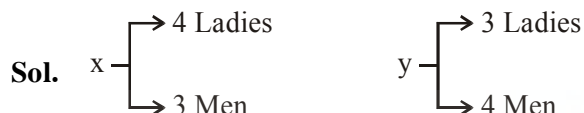
Total number of ways
$^4C_0 \cdot {}^3C_3 \cdot {}^3C_3 \cdot {}^4C_0 + {}^4C_1 \cdot {}^3C_2 \cdot {}^3C_2 \cdot {}^4C_1$
$+ {}^4C_2 \cdot {}^3C_1 \cdot {}^3C_1 \cdot {}^4C_2 + {}^4C_3 \cdot {}^3C_0 \cdot {}^3C_0 \cdot {}^4C_3$
= 485

39. The normal to the curve $y(x - 2)(x - 3) = x + 6$ at the point where the curve intersects the y-axis passes through the point :

(1) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (2) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

(3) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (4) $\left(\frac{1}{2}, -\frac{1}{3}\right)$

Ans. (3)

Sol. $y = \frac{x+6}{(x-2)(x-3)}$

Point of intersection with y–axis (0, 1)

$y' = \frac{(x^2 - 5x + 6)(1) - (x+6)(2x-5)}{(x^2 - 5x + 6)^2}$

y′= 1 at point (0, 1)

∴ Slope of normal is –1

Hence equation of normal is x + y = 1

∴ $\left(\frac{1}{2}, \frac{1}{2}\right)$ satisfy it.

40. A hyperbola passes through the point $P\left(\sqrt{2}, \sqrt{3}\right)$ and has foci at (± 2, 0). Then the tangent to this hyperbola at P also passes through the point :

(1) $\left(-\sqrt{2}, -\sqrt{3}\right)$ (2) $\left(3\sqrt{2}, 2\sqrt{3}\right)$

(3) $\left(2\sqrt{2}, 3\sqrt{3}\right)$ (4) $\left(\sqrt{3}, \sqrt{2}\right)$

Ans. (3)

Sol. Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

foci is (±2, 0) hence ae = 2, $\Rightarrow a^2e^2 = 4$
$b^2 = a^2(e^2 - 1)$
∴ $a^2 + b^2 = 4$ ...(1)

Hyperbola passes through $\left(\sqrt{2}, \sqrt{3}\right)$

∴ $\frac{2}{a^2} - \frac{3}{b^2} = 1$ ...(2)

On solving (1) and (2)
$a^2 = 8$ (is rejected) and $a^2 = 1$ and $b^2 = 3$

∴ $\frac{x^2}{1} - \frac{y^2}{3} = 1$

Equation of tangent is $\frac{\sqrt{2}x}{1} - \frac{\sqrt{3}y}{3} = 1$

Hence $\left(2\sqrt{2}, 3\sqrt{3}\right)$ satisfy it.

41. Let a, b, c ∈ R. If $f(x) = ax^2 + bx + c$ is such that a + b + c = 3 and $f(x + y) = f(x) + f(y) + xy$, ∀ x, y ∈ R, then $\sum_{n=1}^{10} f(n)$ is equal to :

(1) 255   (2) 330   (3) 165   (4) 190

Ans. (2)

Sol. $f(x) = ax^2 + bx + c$
$f(1) = a + b + c = 3$
Now $f(x + y) = f(x) + f(y) + xy$
put y = 1
    $f(x + 1) = f(x) + f(1) + x$
    $f(x + 1) = f(x) + x + 3$
Now
    f(2) = 7
    f(3) = 12
Now
    $S_n = 3 + 7 + 12 + ... t_n$ ...(1)
    $S_n = \quad 3 + 7 + ...t_{n-1} + t_n$ ...(2)
    On subtracting (2) from (1)
    $t_n = 3 + 4 + 5 + ...$ upto n terms

    $t_n = \frac{(n^2 + 5n)}{2}$

    $S_n = \Sigma t_n = \sum \frac{(n^2 + 5n)}{2}$

    $S_n = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{5n(n+1)}{2}\right]$

    $S_{10} = 330$

42. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $\left|(\vec{a} \times \vec{b}) \times \vec{c}\right| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30º. Then $\vec{a} \cdot \vec{c}$ is equal to :

(1) $\dfrac{1}{8}$ (2) $\dfrac{25}{8}$ (3) 2   (4) 5

Ans. (3)

Sol. $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$ and $|\vec{a}| = 3$

$\therefore$ $\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = 3$

Now : $(\vec{a} \times \vec{b}) \times \vec{c} = |\vec{a} \times \vec{b}| \, |\vec{c}| \sin 30 \; \hat{n}$

$\left|(\vec{a} \times \vec{b}) \times \vec{c}\right| = 3 \cdot |\vec{c}| \cdot \dfrac{1}{2}$

$3 = 3|\vec{c}| \cdot \dfrac{1}{2}$

$\therefore |\vec{c}| = 2$

Now : $|\vec{c} - \vec{a}| = 3$

$c^2 + a^2 - 2\vec{c} \cdot \vec{a} = 9$

$4 + 9 - 2\vec{a} \cdot \vec{c} = 9$

$\vec{a} \cdot \vec{c} = 2$

43. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to :-
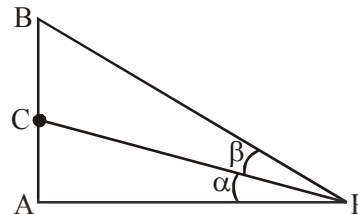
(1) $\dfrac{4}{9}$ (2) $\dfrac{6}{7}$ (3) $\dfrac{1}{4}$ (4) $\dfrac{2}{9}$

Ans. (4)

Sol. $\dfrac{AB}{AP} = \dfrac{1}{2}$

Let $\angle APC = \alpha$

$\tan\alpha = \dfrac{AC}{AP} = \dfrac{1}{2}\dfrac{AB}{AP} = \dfrac{1}{4}$ $\left(AC = \dfrac{1}{2}AB\right)$

Now

$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$\dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta} = \dfrac{1}{2}$ $\left[\begin{array}{l} \tan(\alpha + \beta) = \dfrac{AB}{AP} \\ \tan(\alpha + \beta) = \dfrac{1}{2} \end{array}\right]$

on solving $\tan\beta = \dfrac{2}{9}$

44. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is :-

(1) 30   (2) 12.5   (3) 10   (4) 25

Ans. (4)
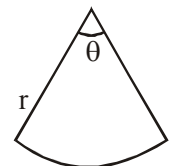
Sol. Total length = r + r + rθ = 20

$\theta = \dfrac{20 - 2r}{r}$

Area $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}r^2\left(\dfrac{20 - 2r}{r}\right)$

$A = 10r - r^2$

$\dfrac{dA}{dr} = 0$ $10 - 2r = 0$, r = 5

$A = 50 - 25 = 25$

45. The integral $\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \dfrac{dx}{1+\cos x}$ is equal to :-

(1) -1 (2) -2 (3) 2 (4) 4

Ans. (3)

Sol. $I = \int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \dfrac{dx}{1+\cos x}$ ...(1)

$I = \int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \dfrac{dx}{1-\cos x}$ ...(2)

Adding (1) and (2)

$2I = \int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \dfrac{2}{\sin^2 x}dx$

$I = \int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \operatorname{cosec}^2 x\, dx$

$I = -(\cot x)_{\pi/4}^{3\pi/4} = 2$

46. If $(2+\sin x)\dfrac{dy}{dx}+(y+1)\cos x=0$ and $y(0) = 1$, then $y\left(\dfrac{\pi}{2}\right)$ is equal to :-

(1) $\dfrac{4}{3}$ (2) $\dfrac{1}{3}$ (3) $-\dfrac{2}{3}$ (4) $-\dfrac{1}{3}$

Ans. (2)

Sol. $(2 + \sin x)\dfrac{dy}{dx}+(y+1)\cos x=0$

$\dfrac{d}{dx}(2+\sin x)(y+1)=0$

$(2 + \sin x)(y + 1) = c$

$x = 0,\ y = 1 \Rightarrow c = 4$

$y+1=\dfrac{4}{2+\sin x}$

$y\left(\dfrac{\pi}{2}\right)=\dfrac{4}{3}-1=\dfrac{1}{3}$

47. Let $I_n = \int \tan^n x\, dx, (n>1)$. $I_4 + I_6 = a\tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to :-

(1) $\left(-\dfrac{1}{5},0\right)$ (2) $\left(-\dfrac{1}{5},1\right)$ (3) $\left(\dfrac{1}{5},0\right)$ (4) $\left(\dfrac{1}{5},-1\right)$

Ans. (3)

Sol. $I_4 + I_6 = \int (\tan^4 x + \tan^6 x)dx = \int \tan^4 x \sec^2 x\, dx$

$= \dfrac{1}{5}\tan^5 x + c \Rightarrow a = \dfrac{1}{5},\ b = 0$

48. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix}=3k,$

then k is equal to :-

(1) 1 (2) $-z$ (3) z (4) -1

Ans. (2)

Sol. Here ω is complex cube root of unity

$R_1 \to R_1 + R_2 + R_3$

$= \begin{vmatrix} 3 & 0 & 0 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$

$= 3(-1 - \omega - \omega) = -3z \Rightarrow k = -z$

49. The value of
$(^{21}C_1 - {}^{10}C_1) + (^{21}C_2 - {}^{10}C_2) + (^{21}C_3 - {}^{10}C_3) + (^{21}C_4 - {}^{10}C_4) + ... + (^{21}C_{10} - {}^{10}C_{10})$ is :-

(1) $2^{20} - 2^{10}$ (2) $2^{21} - 2^{11}$

(3) $2^{21} - 2^{10}$ (4) $2^{20} - 2^9$

Ans. (1)

Sol. $(^{21}C_1 + {}^{21}C_2 ....+ {}^{21}C_{10})$
$- ({}^{10}C_1 + {}^{10}C_2 .... {}^{10}C_{10})$

$= \dfrac{1}{2}[({}^{21}C_1 + ...+ {}^{21}C_{10}) + ({}^{21}C_{11} +.....{}^{21}C_{20})]$
$- (2^{10} - 1)$

$= \dfrac{1}{2}[2^{21} - 2] - (2^{10} - 1)$

$= (2^{20} - 1) - (2^{10} - 1) = 2^{20} - 2^{10}$

14

50. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cot x - \cos x}{(\pi - 2x)^3}$ equals :-

(1) $\dfrac{1}{4}$ (2) $\dfrac{1}{24}$ (3) $\dfrac{1}{16}$ (4) $\dfrac{1}{8}$

Ans. (3)

Sol. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cot x(1 - \sin x)}{-8\left(x - \dfrac{\pi}{2}\right)^3}$

$= \lim\limits_{x \to \frac{\pi}{2}} \dfrac{\tan\left(\dfrac{\pi}{2} - x\right)}{8\left(\dfrac{\pi}{2} - x\right)} \cdot \dfrac{\left(1 - \cos\left(\dfrac{\pi}{2} - x\right)\right)}{\left(\dfrac{\pi}{2} - x\right)^2}$

$= \dfrac{1}{8} \cdot 1 \cdot \dfrac{1}{2} = \dfrac{1}{16}$

51. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is :-

(1) $-\dfrac{7}{9}$ (2) $-\dfrac{3}{5}$ (3) $\dfrac{1}{3}$ (4) $\dfrac{2}{9}$

Ans. (1)

Sol. $5\left[\dfrac{1-t}{t} - t\right] = 2(2t - 1) + 9$ {Let $\cos^2 x = t$}

$\Rightarrow 5(1 - t - t^2) = t(4t + 7)$

$\Rightarrow 9t^2 + 12t - 5 = 0$

$\Rightarrow 9t^2 + 15t - 3t - 5 = 0$

$\Rightarrow (3t - 1)(3t + 5) = 0$

$\Rightarrow t = \dfrac{1}{3}$ as $t \neq -\dfrac{5}{3}$.

$\cos 2x = 2\left(\dfrac{1}{3}\right) - 1 = -\dfrac{1}{3}$

$\cos 4x = 2\left(-\dfrac{1}{3}\right)^2 - 1 = -\dfrac{7}{9}$

52. If the image of the point P(1, –2, 3) in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to line, $\dfrac{x}{1} = \dfrac{y}{4} = \dfrac{z}{5}$ is Q, then PQ is equal to :-

(1) $6\sqrt{5}$ (2) $3\sqrt{5}$

(3) $2\sqrt{42}$ (4) $\sqrt{42}$

Ans. (3)

Sol. Line PQ ; $\dfrac{x-1}{1} = \dfrac{y+2}{4} = \dfrac{z-3}{5}$

Let F($\lambda + 1$, $4\lambda - 2$, $5\lambda + 3$)
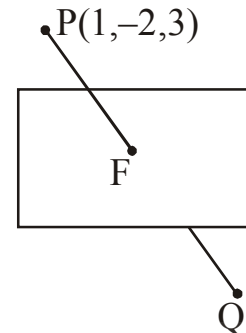
F lies on the plane

$2(\lambda + 1) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$

$\Rightarrow -6\lambda + 6 = 0 \Rightarrow \lambda = 1$

F (2, 2, 8)

PQ = 2 PF = $2\sqrt{42}$.

53. The distantce of the point (1, 3, –7) from the plane passing through the point (1, –1, –1), having normal perpendicular to both the lines

$\dfrac{x-1}{1} = \dfrac{y+2}{-2} = \dfrac{z-4}{3}$ and $\dfrac{x-2}{2} = \dfrac{y+1}{-1} = \dfrac{z+7}{-1}$, is :-

(1) $\dfrac{10}{\sqrt{74}}$ (2) $\dfrac{20}{\sqrt{74}}$

(3) $\dfrac{10}{\sqrt{83}}$ (4) $\dfrac{5}{\sqrt{83}}$

Ans. (3)

Sol. Normal vector

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i} + 7\hat{j} + 3\hat{k}$

So plane is $5(x - 1) + 7(y + 1) + 3(z + 1) = 0$

$\Rightarrow 5x + 7y + 3z + 5 = 0$

Distance $\dfrac{5 + 21 - 21 + 5}{\sqrt{25 + 49 + 9}} = \dfrac{10}{\sqrt{83}}$

54. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then g(x) equals :-

(1) $\frac{3}{1+9x^3}$ (2) $\frac{9}{1+9x^3}$ (3) $\frac{3x\sqrt{x}}{1-9x^3}$ (4) $\frac{3x}{1-9x^3}$

Ans. (2)

Sol. Let $y = \tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ where $x \in \left(0, \frac{1}{4}\right)$

$= \tan^{-1}\left(\frac{2.(3x^{3/2})}{1-(3x^{3/2})}\right) = 2\tan^{-1}(3x^{3/2})$

As $3x^{3/2} \in \left(0, \frac{3}{8}\right)$

$\therefore \frac{dy}{dx} = 2 \times \frac{1}{1+9x^3} \times 3 \times \frac{3}{2} \times x^{1/2}$

$= \frac{9}{1+9x^3}\sqrt{x}$

$\therefore g(x) = \frac{9}{1+9x^3}$

55. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is :-

(1) $4(\sqrt{2}+1)$ (2) $2(\sqrt{2}+1)$

(3) $2(\sqrt{2}-1)$ (4) $4(\sqrt{2}-1)$

Ans. (Bonus or 4)

Sol.
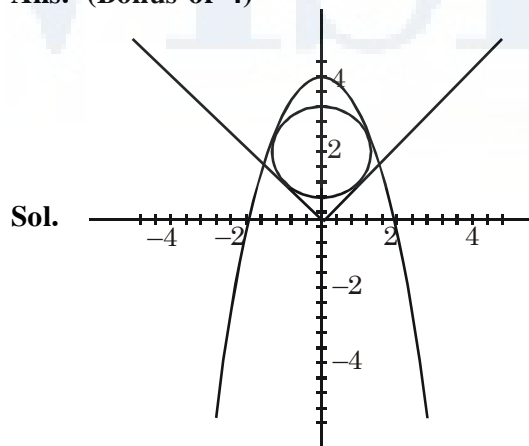
$x^2 + (y-\beta)^2 = r^2$
$x - y = 0$
$\left|\frac{0-\beta}{\sqrt{2}}\right| = r \Rightarrow \beta = r\sqrt{2}$

$x^2 + (y-\beta)^2 = \frac{\beta^2}{2}$

$\Rightarrow 4 - y + (y-\beta)^2 = \frac{\beta^2}{2}$

$\Rightarrow y^2 - y(2\beta+1) + \frac{\beta^2}{2} + 4 = 0$

$\Rightarrow (2\beta+1)^2 - 4\left(\frac{\beta^2}{2}+4\right) = 0$

$4\beta^2 + 4\beta + 1 - 2\beta^2 - 16 = 0$

$\Rightarrow 2\beta^2 + 4\beta - 15 = 0$

$\beta = \frac{-4 \pm \sqrt{16+120}}{4} = \frac{-4 \pm 2\sqrt{34}}{4}$

$= \frac{-2 \pm \sqrt{34}}{2} \Rightarrow \frac{\sqrt{34}-2}{2}$

$r = \frac{\sqrt{34}-2}{2\sqrt{2}}$

which is not in options therefore it must be bonus. But according to history of JEE-Mains it seems they had following line of thinking.
Given curves are $y = 4 - x^2$ and $y = |x|$
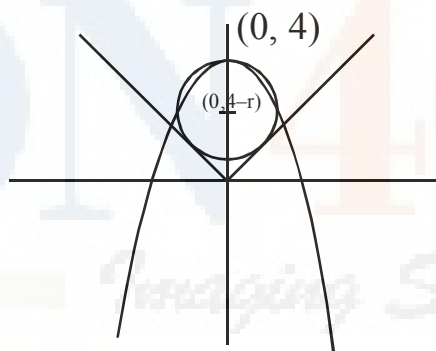
There are two circles satisfying the given conditions. The circle shown is of least area. Let radius of circle is 'r'

$\therefore$ co-ordinates of centre $= (0, 4 - r)$

$\because$ circle touches the line $y = x$ in first quadrant

$\therefore \left|\frac{0-(4-r)}{\sqrt{2}}\right| = r \Rightarrow r - 4 = \pm r\sqrt{2}$

$\therefore r = \frac{4}{\sqrt{2}+1} = 4(\sqrt{2}-1)$

which is given in option 4.

56. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one–by–one, with replacement, then the variance of the number of green balls drawn is :-

(1) $\dfrac{6}{25}$　(2) $\dfrac{12}{5}$　(3) 6　(4) 4

Ans. (2)

Sol. We can apply binomial probability distribution

Variance = npq
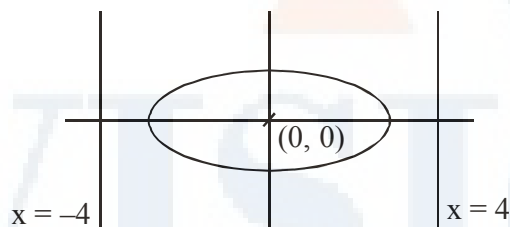
$= 10 \times \dfrac{3}{5} \times \dfrac{2}{5} = \dfrac{12}{5}$

57. The eccentricity of an ellipse whose centre is at the origin is $\dfrac{1}{2}$. If one of its directices is x= –4, then the equation of the normal to it at $\left(1, \dfrac{3}{2}\right)$ is :-

(1) x + 2y = 4　　(2) 2y – x = 2
(3) 4x – 2y = 1　　(4) 4x + 2y = 7

Ans. (3)

Sol. Eccentricity of ellipse $= \dfrac{1}{2}$

$x = -4$　$x = 4$

Now, $-\dfrac{a}{e} = -4 \implies a = 4 \times \dfrac{1}{2} = 2$

$\therefore b^2 = a^2(1 - e^2) = a^2\left(1 - \dfrac{1}{4}\right) = 3$

$\therefore$ Equation of ellipse

$\dfrac{x^2}{4} + \dfrac{y^2}{3} = 1$

$\implies \dfrac{x}{2} + \dfrac{2y}{3} \times y' = 0 \implies y' = -\dfrac{3x}{4y}$

$y'|_{(1, 3/2)} = -\dfrac{3}{4} \times \dfrac{2}{3} = -\dfrac{1}{2}$

$\therefore$ Equation of normal at $\left(1, \dfrac{3}{2}\right)$

$y - \dfrac{3}{2} = 2(x - 1) \implies 2y - 3 = 4x - 4$

$\therefore 4x - 2y = 1$

58. If two different numbers are taken from the set {0, 1, 2, 3, ..., 10), then the probability that their sum as well as absolute difference are both multiple of 4, is :-

(1) $\dfrac{7}{55}$　　　(2) $\dfrac{6}{55}$

(3) $\dfrac{12}{55}$　　　(4) $\dfrac{14}{45}$

Ans. (2)

Sol. Let $A \equiv \{0, 1, 2, 3, 4 ......, 10 \}$

$n(s) = {}^{11}C_2$ (where 'S' denotes sample space)

Let E be the given event

$\therefore$ E $\equiv$ {(0, 4), (0, 8), (2, 6), (2, 10), (4, 8), (6, 10)}

$n(E) = 6$

$\therefore P(E) = \dfrac{6}{55}$

59. For three events A, B and C,

P(Exactly one of A or B occurs)

= P(Exactly one of B or C occurs)

= P(Exactly one of C or A occurs) = $\dfrac{1}{4}$ and

P(All the three events occur simultaneously) = $\dfrac{1}{16}$.

Then the probability that at least one of the events occurs, is :-

(1) $\dfrac{3}{16}$　(2) $\dfrac{7}{32}$　(3) $\dfrac{7}{16}$　(4) $\dfrac{7}{64}$

Ans. (3)

P(exactly one of A or B occurs)

$= P(A) + P(B) - 2P(A \cap B) = \dfrac{1}{4}$

P(Exactly one of B or C occurs)

$= P(B) + P(C) - 2P(B \cap C) = \dfrac{1}{4}$

P(Exactly one of C or A occurs)

$= P(C) + P(A) - 2P(C \cap A) = \dfrac{1}{4}$

Adding all, we get

$2\Sigma P(A) - 2\Sigma P(A \cap B) = \dfrac{3}{4}$

$\therefore \ \Sigma P(A) - \Sigma P(A \cap B) = \dfrac{3}{8}$

Now, $P(A \cap B \cap C) = \dfrac{1}{16}$ (Given)

$\therefore \ P(A \cup B \cup C)$

$\quad = \Sigma P(A) - \Sigma \ P(A \cap B) + P(A \cap B \cap C)$

$\quad = \dfrac{3}{8} + \dfrac{1}{16} = \dfrac{7}{16}$

60. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal

to :-

(1) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (2) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

(3) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (4) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

Ans. (3)

Sol. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$3A^2 = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$

$12A = \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$

$\therefore \ 3A^2 + 12A = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

adj $(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

PART C – CHEMISTRY

61. Which of the following compounds will significant amont of meta product during mono-nitration reaction ?

(1) OH (phenol)

(2) OCOCH$_3$

(3) NH$_2$

(4) NHCOCH$_3$
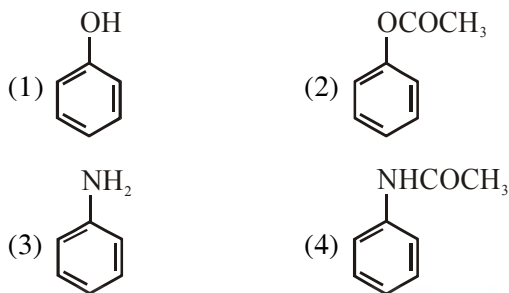
Ans. (3)

Sol. (i) Nitration is carried out in presence of concentrated HNO_3 + concentrated H_2SO_4.

(ii) Aniline acts as base. In presence of H_2SO_4 its protonation takes place and anilinium ion is formed

NH$_2$ $\xrightarrow[H_2SO_4]{H^\oplus}$ $\overset{\oplus}{N}H_3 HSO_4^\ominus$
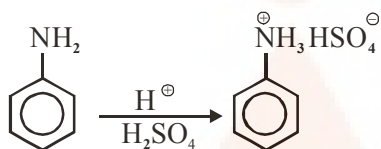
(iii) Anilinium ion is strongly deactivating group and meta directing in nature so it give meta nitration product in significant amount.

NH$_2$ $\xrightarrow[\text{+ conc. } HNO_3]{\text{conc.} H_2SO_4}$ $\overset{\oplus}{N}H_3 HSO_4^\ominus$ with NO_2
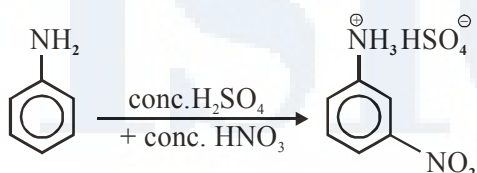
62. ΔU is equal to

(1) Isochoric work   (2) Isobaric work

(3) Adiabatic work   (4) Isothermal work

Ans. (3)

Sol. From 1st law :

ΔU = q + w

For adiabatic process :

q = 0

∴ ΔU = w

∴ Work involve in adiabatic process is at the expense of change in internal energy of the system.

63. The increasing order of the reactivity of the following halides for the S_N1 reaction is

$CH_3CHCH_2CH_3$ with Cl $CH_3CH_2CH_2Cl$

(I)   (II)

$p\text{-}H_3CO\text{–}C_6H_4\text{–}CH_2Cl$

(III)
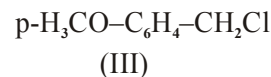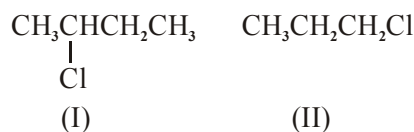
(1) (III) < (II) < (I)   (2) (II) < (I) < (III)

(3) (I) < (III) < (II)   (4) (II) < (III) < (I)

Ans. (2)

Sol. For any S_N1 reaction reactivity is decided by ease of dissociation of alkyl halide

$$R - X \rightleftharpoons R^\oplus + X^\ominus$$
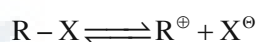
Higher the stability of $R^\oplus$ (carbocation) higher would be reactivity of S_N1 reaction.

Since stability of cation follows order.

$$CH_3 - CH_2 - \overset{\oplus}{C}H_2$$
$$< CH_3 - \overset{\oplus}{C}H - CH_2 - CH_3$$
$$< p - H_3CO - C_6H_4 - \overset{\oplus}{C}H_2$$
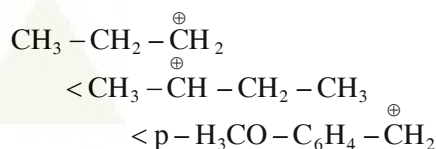
Hence correct order is

$$\boxed{II < I < III}$$
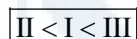
64. The radius of the second Bohr orbit for hydrogen atom is :

(Plank's const. h = 6.6262 × 10^{-34} Js ; mass of electron = 9.1091 × 10^{-31} kg ; charge of electron e = 1.60210 × 10^{-19} C ; permittivity of vaccum

$\in_0$ = 8.854185 × 10^{-12} kg^{-1} m^{-3} A^2)

(1) 1.65Å   (2) 4.76Å

(3) 0.529Å   (4) 212Å

Ans. (4)

Sol. Radius of nth Bohr orbit in H-atom

= 0.53 n^{2}Å

Radius of II Bohr orbit = 0.53 × (2)^2

= 2.12 Å

65. pK_a of a weak acid (HA) and pK_b of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is

(1) 7.2   (2) 6.9

(3) 7.0   (4) 1.0

Ans. (2)

Sol. Given

pK_a (HA) = 3.2

pK_b (BOH) = 3.4

As given salt is of weak acid and weak bas

$$\therefore \; pH = 7 + \frac{1}{2}pK_a - \frac{1}{2}pK_b$$

$$= 7 + \frac{1}{2}(3.2) - \frac{1}{2}(3.4)$$

$$= 6.9$$

66. The formation of which of the following polymers involves hydrolysis reaction ?

(1) Nylon 6   (2) Bakelite

(3) Nylon 6, 6   (4) Terylene

Ans. (1)

Sol. Formation of Nylon-6 involves hydrolysis of its monomer (caprolactum) in initial state.
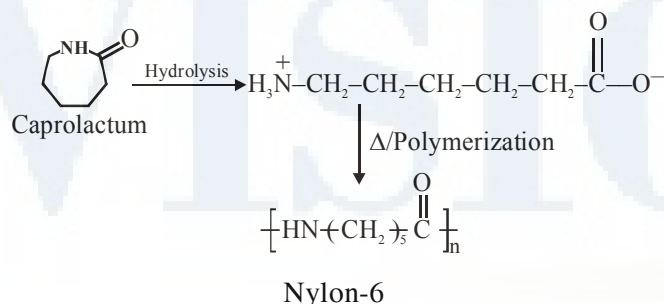
67. The most abundant elements by mass in the body of a healthy human adult are :

Oxygen (61.4%) ; Carbon (22.9%), Hydrogen (10.0%) ; and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ^{1}H atoms are replaced by ^{2}H atoms is

(1) 15 kg   (2) 37.5 kg

(3) 7.5 kg   (4) 10 kg

Ans. (3)

Sol. Mass in the body of a healthy human adult has :-

Oxygen = 61.4%, Carbon = 22.9%,

Hydrogen = 10.0% and Nitrogen = 2.6%

Total weight of person = 75 kg

Mass due to ^{1}H is $= 75 \times \dfrac{10}{100} = 7.5 \text{kg}$

^{1}H atoms are replaced by ^{2}H atoms.

So mass gain by person =7.5 kg

68. Which of the following , upon treatment with tert-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine ?
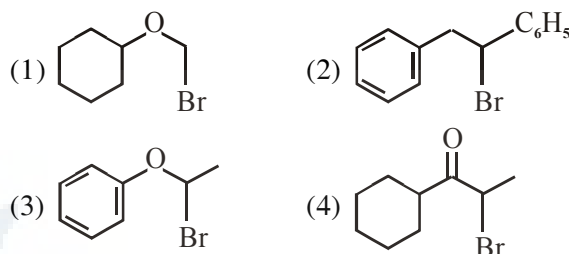
Ans. (1)

68. Ans.(1)
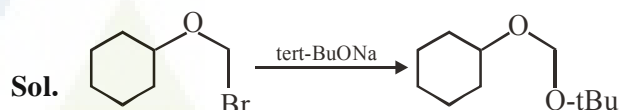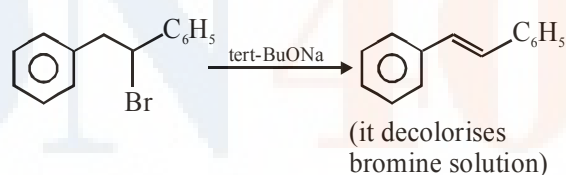
Sol.
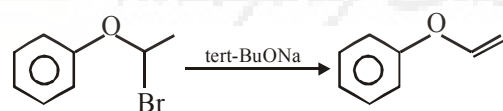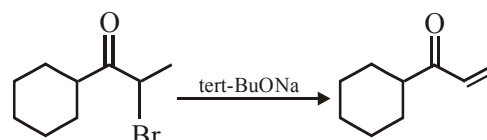
69. In the following reactions, ZnO is respectively acting as a/an :

(a) $ZnO + Na_2O \rightarrow Na_2ZnO_2$

(b) $ZnO + CO_2 \rightarrow ZnCO_3$

(1) base and acid   (2) base and base

(3) acid and acid   (4) acid and base

Ans. (4)

**Sol.** Although ZnO is an amphoteric oxide but in given reaction.

(A) $\underset{\text{acid}}{ZnO} + \underset{\text{base}}{Na_2O} \longrightarrow \underset{\text{salt}}{Na_2ZnO_2}$

(B) $\underset{\text{base}}{ZnO} + \underset{\text{acid}}{CO_2} \longrightarrow \underset{\text{salt}}{ZnCO_3}$
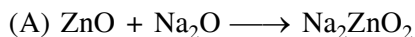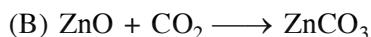
70. Both lithium and magnesium display several similar properties due to the diagonal relationship ; however, the one which is incorrect is :

(1) Both form basic carbonates

(2) Both form soluble bicarbonates

(3) Both form nitrides

(4) Nitrates of both Li and Mg yield NO_2 and O_2 on heating

Ans. (1)

Sol. Mg can form basic carbonate like

$5Mg^{+2} + 6CO_3^{2-} + 7H_2O$

$\rightarrow 4MgCO_3 . Mg(OH)_2 . 5H_2O\downarrow + 2HCO_3^-$
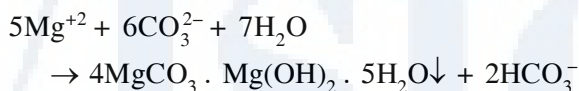
While Li can form only carbonate (Li_2CO_3) not basic carbonate.

71. 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is :-

(1) Six   (2) Zero

(3) Two   (4) Four

Ans. (4)

Sol.

3-methyl pent-2-ene   Anti markownikov product (4 stereo isomers possible due to 2 chiral centre as molecule is nonsymmetric)

(I)   (II)

(III)   (IV)
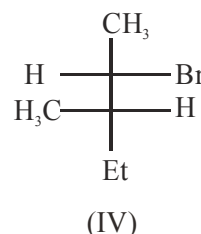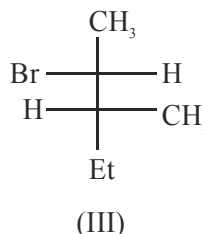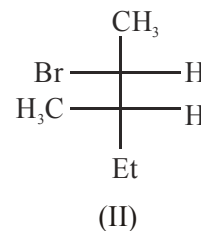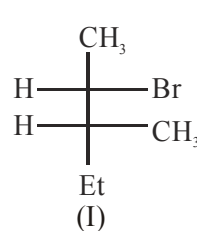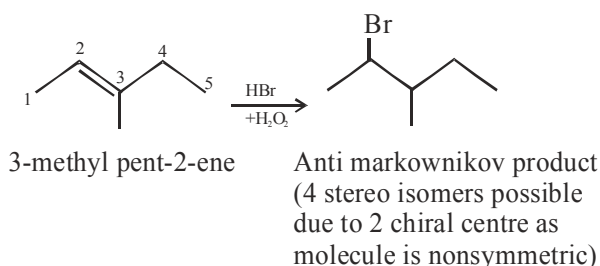
72. A metal crystallises in a face centred cubic structure. If the edge length of its unit cell is 'a', the closest approach between two atoms in metallic crystal will be :-

(1) 2a   (2) $2\sqrt{2}\,a$

(3) $\sqrt{2}\,a$ (4) $\dfrac{a}{\sqrt{2}}$

Ans. (4)

Sol. In FCC unit cell atoms are in contant along face diagonal

So, $\sqrt{2}\,a = 4R$

$\therefore$ closest distance $(2R) = \dfrac{\sqrt{2}\,a}{2} = \dfrac{a}{\sqrt{2}}$

73. Two reactions R_1 and R_2 have identical pre-exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1}. If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K, then $\ln(k_2/k_1)$ is equal to :- ($R = 8.314$ J mol^{-1}K^{-1})

(1) 8   (2) 12

(3) 6   (4) 4

Ans. (4)

Sol. From arrhenius equation,

$$K = A.e^{\frac{-Ea}{RT}}$$
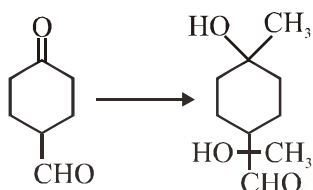
so, $K_1 = A.e^{-E_{a_1}/RT}$ .....(1)

$K_2 = A.e^{-E_{a_2}/RT}$ .....(2)

so, equation (2)/(1) $\Rightarrow \dfrac{K_2}{K_1} = e^{\frac{(E_{a_1} - E_{a_2})}{RT}}$

(As pre-exponential factors of both reactions is same)

$$\ln\left(\dfrac{K_2}{K_1}\right) = \dfrac{E_{a_1} - E_{a_2}}{RT} = \dfrac{10,000}{8.314 \times 300} = 4$$
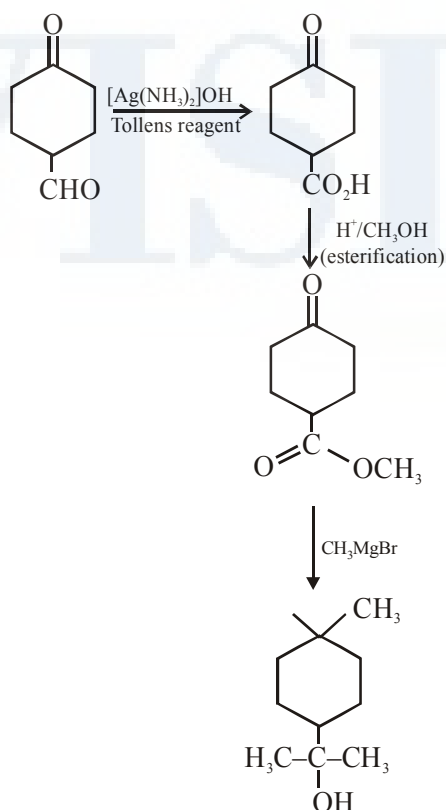
74. The correct sequence of reagents for the following conversion will be :-

(1) $[Ag(NH_3)_2]^+ OH^-$, H^+/CH_3OH, CH_3MgBr

(2) CH_3MgBr, H^+/CH_3OH, $[Ag(NH_3)_2]^+ OH^-$

(3) CH_3MgBr, $[Ag(NH_3)_2]^+ OH^-$, H^+/CH_3OH

(4) $[Ag(NH_3)_2]^+ OH^-$, CH_3MgBr, H^+/CH_3OH

Ans. (1)

Sol.

75. The Tyndall effect is observed only when following conditions are satisfied :-

(a) The diameter of the dispersed particles is much smaller than the wavelength of the ligh used.

(b) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.

(c) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude.

(d) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

(1) (a) and (d)   (2) (b) and (d)
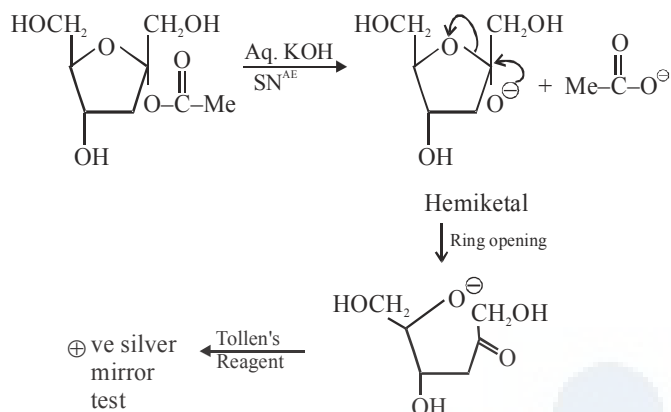
(3) (a) and (c)   (4) (b) and (c)

Ans. (2)

As per NCERT book (fact)

76. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution ?
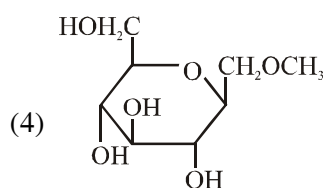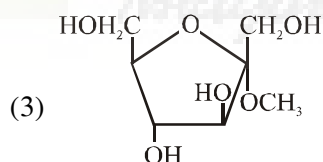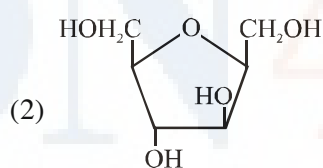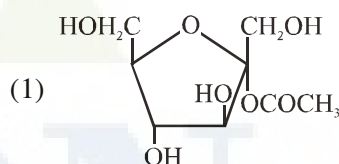
Ans. (1)

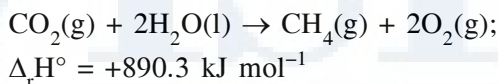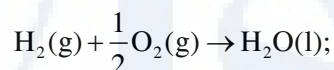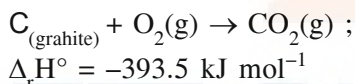**Sol.** (1) Ester in presence of Aqueous KOH solution give SN^{AE} reaction so following reaction takes place

Hemiketal

Ring opening

⊕ve silver mirror test ← Tollen's Reagent


(2) In above compound in presence of Aq. KOH (SN^{AE}) reaction takes place & ∝-Hydroxy carbonyl compound is formed which give ⊕ve Tollen's test So this compound behave as reducing sugar in an aqueous KOH solution.
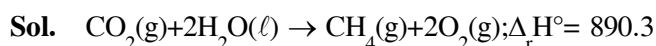
77. Given

$C_{(grahite)} + O_2(g) \rightarrow CO_2(g)$;

$\Delta_r H° = -393.5$ kJ mol^{-1}

$H_2(g) + \dfrac{1}{2}O_2(g) \rightarrow H_2O(l)$;

$\Delta_r H° = -285.8$ kJ mol^{-1}

$CO_2(g) + 2H_2O(l) \rightarrow CH_4(g) + 2O_2(g)$;

$\Delta_r H° = +890.3$ kJ mol^{-1}

Based on the above thermochemical equations, the value of $\Delta_r H°$ at 298 K for the reaction $C_{(grahite)} + 2H_2(g) \rightarrow CH_4(g)$ will be :-
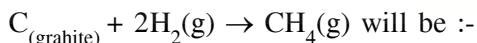
(1) +74.8 kJ mol^{-1} (2) +144.0 kJ mol^{-1}

(3) −74.8 kJ mol^{-1} (4) −144.0 kJ mol^{-1}

Ans. (3)

Sol. $CO_2(g) + 2H_2O(\ell) \rightarrow CH_4(g) + 2O_2(g)$; $\Delta_r H°$= 890.3

$\Delta_f H°$ −393.5   −285.8   ?   0

$\Delta_r H° = \sum (\Delta_f H°)_{products} - \sum (\Delta_f H°)_{Reactants}$

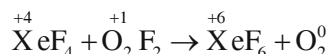$890.3 = \left[1 \times (\Delta_f H°)_{CH_4} + 2 \times 0\right] - \left[1 \times (-393.5) + 2(-285.8)\right]$

$(\Delta_f H°)_{CH_4} = 890.3 - 965.1 = -74.8$ kJ / mol

78. Which of the following reactions is an example of a redox reaction ?

(1) $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$

(2) $XeF_2 + PF_5 \rightarrow [XeF]^+PF_6^-$

(3) $XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$

(4) $XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$

Ans. (1)

Sol. In the reaction

$\overset{+4}{X}eF_4 + \overset{+1}{O_2}F_2 \rightarrow \overset{+6}{X}eF_6 + \overset{0}{O_2}$
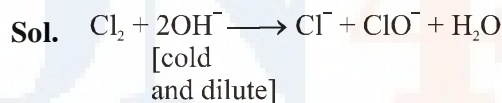
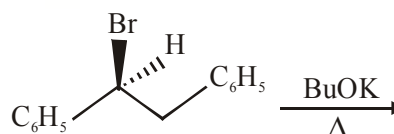Xenon undergoes oxidation while oxygen undergoes reduction.

79. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are :-

(1) ClO^- and ClO_3^-

(2) ClO_2^- and ClO_3^-

(3) Cl^- and ClO^-

(4) Cl^- and ClO_2^-

Ans. (3)

Sol. $Cl_2 + 2OH^- \longrightarrow Cl^- + ClO^- + H_2O$
            [cold and dilute]

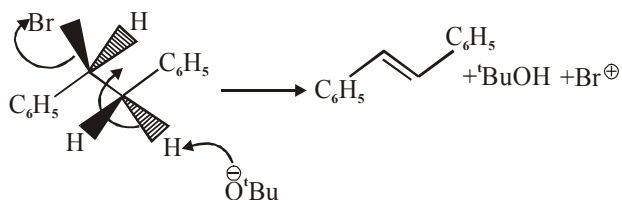**80.** The major product obtained in the following reaction is :-

(1) $(\pm)C_6H_5CH(O^tBu)CH_2CH_6H_5$

(2) $C_6H_5CH=CHC_6H_5$

(3) $(+)C_6H_5CH(O^tBu)CH_2H_5$

(4) $(-)C_6H_5CH(O^tBu)CH_2C_6H_5$

Ans. (2)

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Sol. Elimination reaction is highly favoured if

(a) Bulkier base is used

(b) Higher temperature is used

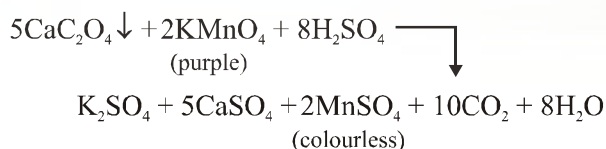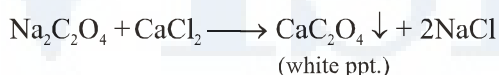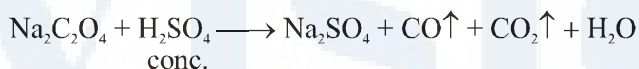Hence in given reaction biomolecular ellimination reaction provides major product.

81. Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4. 'X' reacts with the acidified aqueous $CaCl_2$ solution to give a white precipitate which decolourises acidic solution of $KMnO_4$. 'X' is :-

(1) C_6H_5COONa

(2) $HCOONa$

(3) $CH3COONa$

(4) $Na_2C_2O_4$

Ans. (4)

Sol.

$$Na_2C_2O_4 + H_2SO_4 \underset{conc.}{\longrightarrow} Na_2SO_4 + CO\uparrow + CO_2\uparrow + H_2O$$

$$Na_2C_2O_4 + CaCl_2 \longrightarrow \underset{\text{(white ppt.)}}{CaC_2O_4\downarrow} + 2NaCl$$

$$\underset{\text{(purple)}}{5CaC_2O_4\downarrow + 2KMnO_4 + 8H_2SO_4} \longrightarrow$$

$$\underset{\text{(colourless)}}{K_2SO_4 + 5CaSO_4 + 2MnSO_4 + 10CO_2 + 8H_2O}$$

82. Which of the following species is not paramagnetic :-

(1) NO   (2) CO

(3) O_2 (4) B_2

Ans. (2)

Sol. NO $\Rightarrow$ One unpaired electron is present in π^* molecular orbital.

CO $\Rightarrow$ No unpaired electron is present

$O_2 \Rightarrow$ Two unpaired electrons are present in π^* molecular orbitals.

$B_2 \Rightarrow$ Two unpaired electrons are present in π bonding molecular orbitals.

83. The freezing point of benzene decreases by 0.45°C when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be :- (K_f for benzene = 5.12 K kg mol^{-1})

(1) 64.6%   (2) 80.4%

(3) 74.6%   (4) 94.6%

Ans. (4)

Sol. In benzene

$$2CH_3COOH \rightleftharpoons (CH_3COOH)_2$$
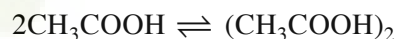
$$i = 1 + \left(\frac{1}{2} - 1\right)\alpha$$

$$i = 1 - \frac{\alpha}{2} \quad \text{Here } \alpha \text{ is degree of association}$$
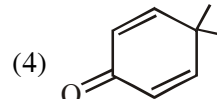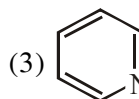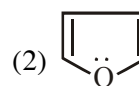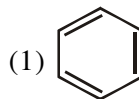
$$\Delta T_f = iK_f m$$

$$0.45 = \left(1 - \frac{\alpha}{2}\right)(5.12)\frac{\left(\frac{0.2}{60}\right)}{\frac{20}{1000}}$$
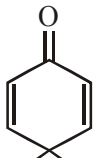
$$1 - \frac{\alpha}{2} = 0.527$$

$$\alpha = 0.945$$

% degree of association = 94.5%

84. Which of the following molecules is least resonance stabilized ?
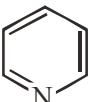
(1)   (2)

(3)   (4)

Ans. (4)

Sol. (4) is nonaromatic and hence least

reasonance stabilized

(1) Benzene is aromatic

(2) furan is aromatic

(3) pyridine is aromatic

85. On treatment of 100 mL of 0.1 M solution of $CoCl_3 \cdot 6H_2O$ with excess $AgNO_3$; 1.2×10^{22} ions are precipitated. The complex is :-

(1) $[Co(H_2O)_4 Cl_2]Cl.2H_2O$

(2) $[Co(H_2O)_3Cl_3].3H_2O$

(3) $[Co(H_2O)_6]Cl_3$

(4) $[Co(H_2O)_5Cl]Cl_2.H_2O$

Ans. (4)

Sol. Moles of complex = $\dfrac{\text{Molarity} \times \text{volume (ml)}}{1000}$

$= \dfrac{100 \times 0.1}{1000}$ = 0.01 mole

Moles of ions precipitated with excess of

$AgNO_3 = \dfrac{1.2 \times 10^{22}}{6.02 \times 10^{23}}$

= 0.02 moles

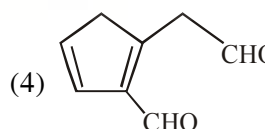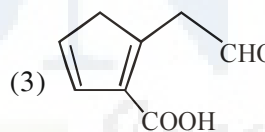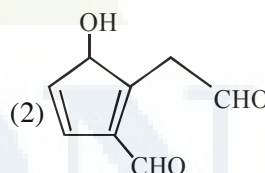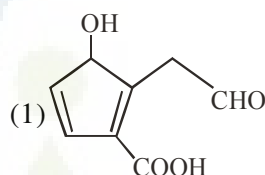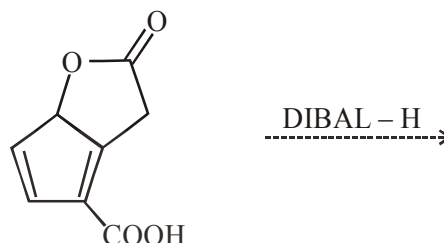Number of Cl^- present in ionization sphere =

$\dfrac{\text{Mole of ion precipitated with exess } AgNO_3}{\text{mole of complex}} = \dfrac{0.02}{0.01} = 2$

It means $2Cl^-$ ions present in ionization sphere

∴ complex is $[Co(H_2O)_5Cl]Cl_2.H_2O$

86. The major product obtained in the following reaction is :-

(1)

(2)

(3)

(4)

Ans. (2)

Sol. DIBAL – H is electrophilic reducing agent reduces cynide, esters, lactone, amide, carboxylic acid into corresponding Aldehyde (partial reduction)

87. A water sample has ppm level concentration of following anions

$$F^- = 10; SO_4^{2-} = 100; NO_3^- = 50$$

the anion/anions that make / makes the water sample unsuitable for drinking is / are :

(1) only NO_3^- (2) both SO_4^{2-} and NO_3^-

(3) only F⁻   (4) only SO_4^{2-}

Ans. (3)

Sol. NO_3^-: The maximum limit of nitrate in drinking water is 50 ppm. Excess nitrate in drinking water can cause disease. Such as methemoglobinemia.

SO_4^{2-} : above 500 ppm of SO_4^{2-} ion in drinking water causes laxative effect otherwise at moderate levels it is harmless

F^- : Above 2ppm concentration of F^- in drinking water cause brown mottling of teeth.
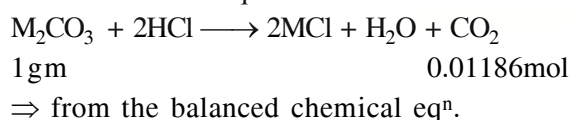
∴ The concentration given in question of SO_4^{2-} & NO_3^- in water is suitable for drinking but the concentration of F^- (i.e 10 ppm) make water unsuitable for drinking purpose :

88. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. the molar mass of M_2CO_3 in g mol⁻¹ is :-

(1) 1186   (2) 84.3

(3) 118.6   (4) 11.86

Ans. (2)

Sol. Given chemical eqⁿ
$$M_2CO_3 + 2HCl \longrightarrow 2MCl + H_2O + CO_2$$
1 gm   0.01186mol
⇒ from the balanced chemical eqⁿ.

$$\frac{1}{M} = 0.01186$$

⇒ $\boxed{M = 84.3}$ gm/mol

89. Given

$$E^o_{Cl_2/Cl^-} = 1.36\,V, E^o_{Cr^{3+}/Cr} = -0.74\,V$$

$$E^o_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33\,V, E^o_{MnO_4^-/Mn^{2+}} = 1.51\,V.$$

Among the following, the strongest reducing agent is

(1) Cr   (2) Mn^{2+}

(3) Cr^{3+} (4) Cl⁻

Ans. (1)

Sol. $E^o_{MnO_4^-/Mn^{+2}} = 1.51\,V$ ..(i)

$E^o_{Cl_2/Cl^-} = 1.36\,V$ ..(ii)

$E^o_{Cr_2O_7^{-2}/Cr^{+3}} = 1.33\,V$ ..(iii)

$E^o_{Cr^{+3}/Cr} = -0.74$ ..(iv)

Since Cr^{+3} is having least reducing potential, so Cr is the best Reducing agent.

90. The group having isoelectronic species is :-

(1) O^{2-}, F^-, Na^+, Mg^{2+}

(2) O^-, F^-, Na, Mg^+

(3) O^{2-}, F^-, Na, Mg^{2+}

(4) O^-, F^-, Na^+, Mg^{2+}

Ans. (1)

Sol.

ions	O^{-2}	F^-	Na^+	Mg^{+2}
Atomic number =	8	9	11	12
No. of e⁻ =	10	10	10	10

therefore O^{2-}, F^-, Na^+, Mg^{+2} are isoelectronic