Institute of Mathematics and Applications, Bhubaneswar
Entrance Test-2012
B.Sc (Hons): Mathematics and Computing

Max Marks: 100 **Max Time: Two Hours**

All questions are compulsory. Each question has 4 choices A, B, C and D, out of which *only one* is correct. Choose the correct answer. Each question carries +4 marks for the correct answer and -1 mark for a wrong answer.

1. If $\mathbb{N}$ is the set of all natural numbers. Let mRn, if n is divisible by m. The relation R is
   A. reflexive and symmetric
   B. transitive and antisymmetric
   C. symmetric but not transitive
   D. none of the above

2. Only one of the following is a function which one is it?
   A. $\{(x^2, x) : x \in \mathbb{R}\}$
   B. $\{(x, y) : x^2 + y^2 = 25, x, y \in \mathbb{R}\}$
   C. $\{(x, \cos x) : x \in \mathbb{R}\}$
   D. $\{(x, y) : x^3 + y^3 - 3xy = 0, x, y \in \mathbb{R}\}$

3. Let ABC be an equilateral triangle and P is a point within it satisfying $AP^2 = BP^2 + CP^2$. The locus of P is
   A. a straight line
   B. a parabola
   C. a circle
   D. an ellipse

4. The last two digits of 19^{39} are
   A. 10
   B. 03
   C. 59
   D. 79

5. Let (x_0, y_0) be the solution of following equation

$$(2x)^{\ln 2} = (3y)^{\ln 3}, (3)^{\ln x} = (2)^{\ln y},$$

then x_0 is

A. 1/6

B. 1/3

C. 1/2

D. 6

6. In how many ways 5 sweets can be distributed among 3 children so that every one gets at least one?

A. 10

B. 20

C. 6

D. 4

7. $1 - x - e^{-x} > 0$ for

A. all $x \in \mathbb{R}$

B. no $x \in \mathbb{R}$

C. $x > 0$

D. $x < 0$

8. Let $A = \{\sin x | 0 < x < \pi\}$. What does it mean if we say y is an element of A?

A. $\sin y$ is between 0 and π

B. y is between $\sin(0)$ and $\sin(\pi)$

C. y is between 0 and π

D. $y = \sin x$ for some $0 < x < \pi$

9. The number of points where the graph of the function $f(x) = x^3 + 2x^2 + 2x + 1$ cuts the abscissa is

A. 1

B. 2

C. 3

D. 0

10. If one is solving three linear equations involving two unknowns, what happens?

    A. usually there will be one solution, but occasionally there will be no solution or infinitely many solutions.

    B. anything can happen.

    C. usually there will never be a solution.

    D. there will always be a solution.

11. The number of solutions of the following system

$$\begin{aligned} x + y + z &= 3, \\ 2x + 3y + 4z &= 9, \\ 4x + 5y + 6z &= 10, \end{aligned}$$

    is

    A. 0

    B. 1

    C. 2

    D. infinitely many

12. If $a_1, a_2, ..., a_n$ are positive real numbers then $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \cdots + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1}$ is always

    A. $\geq n$

    B. $\leq n$

    C. $\leq n^{1/n}$

    D. $\geq n^{1/n}$

13. The coefficient of t^3 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is

    A. 10

    B. 12

    C. 8

    D. 9

14. $(\sqrt{5}+2)^{10}+(\sqrt{5}-2)^{10}$ is equal to

   A. $[(\sqrt{5}+2)^{10}]+1$

   B. 4149

   C. 10249

   D. none of the above

15. Sum of $\binom{n}{0}-\binom{n}{2}+\binom{n}{4}-\binom{n}{6}$..... equal to

   A. 2^n

   B. 0

   C. $2^{(n+2)/2}\cos(n\pi)/4$

   D. $2^{(n+1)/2}\sin(n\pi)/4$

16. The set of complex numbers z satisfying the equation $(3+7i)z+(10-2i)\bar{z}+100=0$ represents in the complex plane

   A. a point

   B. a straight line

   C. a pair of intersecting straight lines

   D. a pair of distinct parallel lines

17. Let $\mathbb{Z}_3=\{0,1,2\}$. The number of 2×2 matrices with entries from the set $\mathbb{Z}_3$ with determinant 1 is

   A. 24

   B. 60

   C. 20

   D. 30

18. Let A be 4×4 matrix with determinant 3. Let B be the matrix formed by subtracting two copies of the third row from first. What is $\det(B)$?

   A. -6

   B. 6

   C. 3

   D. 0

19. In the Taylor expansion of the function $f(x) = e^{x/2}$ about $x = 3$, the coefficient of $(x-3)^5$ is

   A. $e^{3/2}\frac{1}{5!}$

   B. $e^{3/2}\frac{1}{2^5 5!}$

   C. $e^{-3/2}\frac{1}{2^5 5!}$

   D. $e^{-3/2}\frac{1}{5!}$

20. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from $(0, 0)$ to (x, y) in $1 : 3$. Then locus of P is

   A. $x^2 = y$

   B. $y^2 = 2x$

   C. $y^2 = x$

   D. $x^2 = 2y$

21. $\lim_{x\to 0} \dfrac{(1+x)^{1/x} - e}{x}$ is

   A. 0

   B. $\dfrac{-e}{2}$

   C. $\dfrac{5e}{2}$

   D. doesn't exit.

22. If $f(x) = \begin{cases} \sin[x] & , x \neq 0, \text{where } [x] \text{ is a greatest integer function.} \\ -x & , x = 0. \end{cases}$
   Then $\lim_{x\to 0} f(x)$ is

   A. 0

   B. 1

   C. -1

   D. doesn't exit.

23. Let $f(x) = \min\{x, x^2, x^3\}$. The number of points where f is not differentiable but continuous is

   A. 1

   B. 2

   C. 3

D. none of the above

24. Let $f(x)$ be a polynomial of degree 23 and $f(-x) = -f(x)$ for $|x| \geq 10$. If $\int_{-1}^{1}(f(x) + c)\, dx = 4$, then c is equal to

A. 0

B. 1

C. 2

D. 10

25. $\int_0^{\pi} \frac{1}{1+\sin x} dx$ is equal to

A. 0

B. 1

C. 2

D. 5