INSTITUTE OF MATHEMATICS AND APPLICATIONS
BHUBANESWAR
ENTRANCE EXAMINATION-2015
B.Sc.(Honours) in Mathematics & Computing

Full Marks: 100 **Time: 2 Hours.**

This question booklet contains 50 questions. For each question/incomplete statement, four alternative answers are suggested and given against (A), (B), (C) and (D) of which, only one of them is correct. Choose the correct answer in each question. Each correct answer carries 2 marks. More than one answer marked against a question will be deemed as incorrect.

Multiple-Choice Questions

1. Let $U = \{(x, y) \in \mathbb{R}^2 : y = e^x\}$ and $V = \{(x, y) \in \mathbb{R}^2 : y = \log(x), x > 0\}$. Then

   (A) $U \subset V$. (B) $U \cap V = \emptyset$.

   (C) $V \subset U$ and $U - V \neq \emptyset$. (D) $U \cap V = \{0, 1\}$.

2. Of the following choices of δ, which is the largest that could be used successfully with an arbitrary ϵ in an epsilon-delta proof of $\lim_{x \to 2}(1 - 3x) = -5$?

   (A) $\delta = 3\epsilon$. (B) $\delta = \epsilon$. (C) $\delta = \epsilon/2$. (D) $\delta = \epsilon/4$.

3. If the domain of the function f given by $f(x) = \dfrac{1}{1 - x^2}$ is $\{x \in \mathbb{R} : |x| > 1\}$, then the range of f is

   (A) $\{x \in \mathbb{R} : -\infty < x < -1\}$. (B) $\{x \in \mathbb{R} : -\infty < x < 0\}$.

   (C) $\{x \in \mathbb{R} : -\infty < x < 1\}$. (D) $\{x \in \mathbb{R} : -1 < x < \infty\}$.

4. A function f from the set of natural numbers to the set of integers defined by

   $$f(n) = \begin{cases} \frac{n-1}{2}, & n \text{ is odd} \\ -\frac{n}{2}, & n \text{ is even} \end{cases}$$ is

   (A) one-one but not onto. (B) onto but not one-one.

   (C) both one-one and onto. (D) neither one-one nor onto.

5. If a complex number z satisfies $|z^2 - 1| = |z|^2 + 1$, then z lies on

   (A) the X-axis. (B) an ellipse. (C) a circle. (D) the Y-axis.

6. The number of integers lying between 100 and 500 that are divisible by 7, but not by 21 is

   (A) 57. (B) 38. (C) 19. (D) 11.

7. If $x^2 + y^2 + z^2 = 1$, then $xy + yz + xz$ lies in the interval

   (A) $[1, 2]$. (B) $[0, 1]$. (C) $[0, 1/2]$. (D) $[-1/2, 1]$.

8. If $p(x) = ax^2 + bx + c$ and $q(x) = -ax^2 + dx + c$ $(ac \neq 0)$, then the equation $p(x)q(x) = 0$ has

(A) four real roots.

(B) exactly two real roots.

(C) either two or four real roots

(D) at most two real roots.

9. The system of linear equations: $x + 2y + z = 3, 2x + 3y + z = 3, 3x + 5y + 2z = 1$ has

(A) infinite number of solutions.

(B) exactly three solutions.

(C) a unique solution.

(D) no solution.

10. Consider the set U consisting of all determinants of order 3 with entries 0 and 1 only. Let V be the subset of U consisting of all determinants with value 1 and W be the subset of U consisting of all determinants with value -1. Then

(A) $U = V \cup W$.

(B) V and W have the same number of elements.

(C) $W = \emptyset$.

(D) V has twice as many elements as W.

11. Let X and Y be two sets containing 2 and 4 elements respectively. Then the number of subsets of $X \times Y$ having 3 or more elements is

(A) 256. (B) 220. (C) 219. (D) 211.

12. The range of the function $f(x) = [|\sin x| + |\cos x|]$ $(x \in \mathbb{R})$, where $[\cdot]$ denotes the greatest integer function, is

(A) $\{0\}$. (B) $\{0, 1\}$. (C) $\{1\}$. (D) $\{0, 1, 2\}$.

13. If $\lim_{x \to 0} \dfrac{\log(a + x) - \log a}{x} + \alpha \lim_{x \to 0} \dfrac{\log x - 1}{x - e} = 1$, then the value of α is

(A) $e\left(1 - \frac{1}{a}\right)$. (B) $e(1 + a)$. (C) $e(2 - a)$. (D) None of these.

14. The function $f(x) = \begin{cases} e^x, & x \leq 0 \\ |1 - x|, & x > 0 \end{cases}$ is

(A) not continuous at $x = 0, 1$.

(B) differentiable at $x = 1$.

(C) differentiable at $x = 0$.

(D) continuous at $x = 0, 1$, but not differentiable at these points.

15. A particle moves along the X-axis so that at any time t its position is given by $x(t) = te^{-2t}$. For what value of t is the particle at rest ? .

(A) No values. (B) 0 only. (C) 1/2 only. (D) 1 only.

16. A curve in the plane is defined parametrically by the equations $x = t^3 + t$ and $y = t^4 + 2t^2$. An equation of the line tangent to the curve at $t = 1$ is

(A) $y = 2x$. (B) $y = 8x$. (C) $y = 2x - 1$. (D) $y = 4x - 5$.

17. A region in the plane is bounded by the graph of $y = 1/x$, the X-axis, the line $x = m$, and the line $x = 2m$ $(m > 0)$. The area of the region

(A) is independent of m.

(B) increases as m increases.

(C) increases as m increases when $m < 1/2$; decreases as m increases when $m > 1/2$.

(D) decreases as m increases when $m < 1/2$; increases as m increases when $m > 1/2$.

18. If M is a 3×3 matrix such that $M^2 = 0$, then the determinant of the matrix $(I + M)^{50} - 50M$ is equal to

(A) 1. (B) 2. (C) 3. (D) 50.

19. The contrapositive of the statement "I go to school, if it does not rain" is

(A) If it rains, I go to school.

(B) If it rains, I do not go to school.

(C) If I go to school, it rains.

(D) If I do not go to school, it rains.

20. If the Rolle's theorem holds for the function $f(x) = 2x^3 + ax^2 + bx$ in the interval $[-1, 1]$ for the point $c = 1/2$, then the value of $2a + b$ is equal to

(A) -2. (B) -1. (C) 1. (D) 2.

21. If $|\vec{a}| = 2, |\vec{b}| = 3$ and $|2\vec{a} - \vec{b}| = 5$, then the value of $|2\vec{a} + \vec{b}|$ is equal to

(A) 17. (B) 7. (C) 5. (D) 1.

22. If X and Y are two events such that $P(X \cup Y) = P(X \cap Y)$, then the **incorrect** statement amongst the following is

(A) $P(X) + P(Y) = 1$.

(B) $P(X \cap Y') = 0$.

(C) $P(X' \cap Y) = 0$.

(D) X and Y are equally likely.

23. Let R be the relation defined on the set of real numbers such that $R = \{(x, y) : \sec^2 x - \tan^2 y = 1\}$. Then R is

(A) reflexive and symmetric, but not transitive.

(B) symmetric and transitive, but not reflexive.

(C) reflexive and transitive, but not symmetric.

(D) an equivalence relation.

24. One ticket is selected at random from 50 tickets numbered $00, 01, 02, \cdots, 49$. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of the digits on the ticket is zero, is

(A) 1/14. (B) 1/7. (C) 5/14. (D) 1/50.

25. $\int \frac{dx}{\sqrt{x}(x+9)} =$

(A) $\frac{2}{3}\tan^{-1}\sqrt{x}$ + Constant.

(B) $\frac{2}{3}\tan^{-1}\left(\frac{\sqrt{x}}{3}\right)$ + Constant.

(C) $\tan^{-1}\sqrt{x}$ + Constant.

(D) $\tan^{-1}\left(\frac{\sqrt{x}}{3}\right)$ + Constant.

26. A circle with centre $(2, 4)$ is such that the line $x + y + 2 = 0$ cuts a chord of length 6 units. Then the radius of the circle is

(A) $\sqrt{41}$ units. (B) $\sqrt{11}$ units. (C) $\sqrt{21}$ units. (D) $\sqrt{31}$ units.

27. If $\cos^{-1}x = \cos^{-1}(5/13) + \cos^{-1}(3/5)$, then $x =$

(A) 3/65. (B) −36/65. (C) −33/65. (D) −1.

28. If the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and the hyperbola $\frac{x^2}{4} - \frac{y^2}{\alpha^2} = 1$ coincide, then $\alpha^2 =$

(A) 4. (B) 5. (C) 8. (D) 9.

29. If $\cos\theta + \sin\theta = p$ and $\cos^3\theta + \sin^3\theta = q$, then $p(p^2 - 3) =$

(A) q. (B) $-2q$. (C) $2q$. (D) $2q^2$.

30. Consider the following two statements for the parabola $y^2 + 6y - 2x + 5 = 0$.

S_1 : The vertex is $(-2, -3)$. S_2 : The directrix is $y + 3 = 0$.

Which one of the following is true ?

(A) Both S_1 and S_2 are true.

(B) S_1 is true, but S_2 is false.

(C) S_1 is false, but S_2 is true.

(D) Both S_1 and S_2 are false.

31. The value of $\lim_{x\to 0} \frac{e^x - e^{\sin x}}{2(x - \sin x)}$ equals to

(A) −1/2. (B) 1/2. (C) 1. (D) 3/2.

32. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10^7 + \ldots + 10(11)^9 = \alpha(10)^9$, then the value of α is equal to

(A) 121/10. (B) 441/100. (C) 110. (D) 100.

33. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z = -3$ is the line

(A) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$.

(B) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$.

(C) $\frac{x-3}{3} = \frac{y-+5}{1} = \frac{z-2}{-5}$

(D) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$.

34. If $[\vec{a}\times\vec{b}\,\vec{b}\times\vec{c}\,\vec{c}\times\vec{a}] = \lambda[\vec{a}\,\vec{b}\,\vec{c}]$, then $\lambda =$

(A) 0. (B) 1. (C) 2. (D) 3.

35. A multiple choice examination has 5 questions and each question has 3 alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is

(A) $\frac{17}{3^5}$. (B) $\frac{13}{3^5}$. (C) $\frac{11}{3^5}$. (D) $\frac{10}{3^5}$.

36. The coefficient of x^{50} in the expansion of $(1+x)^{41}(1-x+x^2)^{40}$ is

(A) 3. (B) 2. (C) 1. (D) 0.

37. If the points $(1,2,3)$ and $(2,-1,0)$ on the opposite side of the plane $2x+3y-2z-\alpha=0$, then the value of α is

(A) $1<\alpha<2$. (B) $\alpha<1$. (C) $\alpha>2$. (D) $\alpha=1$.

38. Which one of the following is correct ?

(A) $\sin 1 > \sin 1^\circ$. (B) $\sin 1 < \sin 1^\circ$. (C) $\sin 1 = \sin 1^\circ$. (D) $\sin 1^\circ = \frac{\pi}{18^\circ}\sin 1$.

39. If the function $f(x)=|x-1|+|x-3|+|5-x|$ $(x\in\mathbb{R})$ decreases, then x lies in the interval

(A) $(1,\infty)$. (B) $(-\infty,3)$. (C) $(3,5)$. (D) $(-\infty,5)$.

40. The area of the region $\{(x,y)\in\mathbb{R}^2 : x^2+y^2\le 1\le x+y\}$ is

(A) $\frac{\pi}{4}-\frac{1}{2}$. (B) $\frac{\pi^2}{2}$. (C) $\frac{\pi^2}{3}$. (D) $\frac{\pi^2}{5}$.

41. If $I_1=\int_0^1 \frac{\sin x}{\sqrt{x}}dx$ and $I_2=\int_0^1 \frac{\cos x}{\sqrt{x}}dx$, then which one of the following is true ?

(A) $I_1>\frac{2}{3}$ and $I_2<2$. (B) $I_1>\frac{2}{3}$ and $I_2>2$.

(C) $I_1<\frac{2}{3}$ and $I_2<2$. (D) $I_1<\frac{2}{3}$ and $I_2>2$.

42. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ is

(A) $2\sqrt{5}$ sq.units. (B) 10 sq. units. (C) 20 sq. units. (D) 40 sq. units.

43. If $y=(1-x)(1+x^2)(1+x^4)\cdots(1+x^{100})$, then $\frac{dy}{dx}$ at $x=0$ is

(A) -1. (B) 0. (C) 99. (D) 100.

44. If f is an odd differentiable function defined on $\mathbb{R}$ and $f'(3)=-2$, then

(A) $f'(-3)=-2$. (B) $f'(-3)=0$. (C) $f'(-3)=2$. (D) $f'(-3)=4$.

45. For what value of α, the function $f(x) = \begin{cases} \dfrac{\sqrt{1+\alpha x} - \sqrt{1-\alpha x}}{x}, & x \neq 0 \\ \dfrac{2x+1}{x-2}, & x = 0 \end{cases}$ is continuous at $x = 0$?

(A) $-\frac{3}{2}$. (B) $-\frac{1}{2}$. (C) $\frac{1}{2}$. (D) 1.

46. If $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = \min\{1+x, 1+|x|\}$, then which one of the following is true ?

(A) $f(x) \geq 1$ for all $x \in \mathbb{R}$.

(B) f is not differentiable at any point of $\mathbb{R}$.

(C) f is differentiable at all the points of $\mathbb{R}$.

(D) f is not differentiable only at $x = 1$.

47. The number of groups that can be formed from 5 different green balls, 4 different blue balls, and three different red bulbs, if at least 1 green and 1 blue ball is to be included is

(A) 3700. (B) 3720. (C) 3740. (D) 3760.

48. If $\sim (p \wedge q)$ is false, then the corresponding values of p and q are, respectively

(A) T, T. (B) T, F. (C) F, T. (D) None of these.

49. If $\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d and e are constants, then $e =$

(A) -1. (B) 0. (C) 1. (D) 2.

50. If two sets X and Y have 80 elements in common, then the number of elements common to the sets $X \times Y$ and $Y \times X$ are

(A) 2^{80}. (B) 80. (C) 80^2. (D) 2^{40}.