INSTITUTE OF MATHEMATICS AND APPLICATIONS, BHUBANESWAR

ENTRANCE EXAMINATION-2014

B.Sc.(Hons.) in Mathematics & Computing

Full Marks: 100 **Time: 2 Hours**

This question booklet contains 50 questions. For each question, four answers are suggested and given against (A), (B), (C) and (D) of which, only one will be the correct answer. Choose the correct answer in each question. Each correct answer will be awarded 2 marks. More than one answer marked against a question will be deemed as incorrect.

1. Let A and B be two sets with $n(A) = 43, n(B) = 51$ and $n(A \cup B) = 75$. Then $n((A - B) \cup (B - A)) =$

   (A) 45. (B) 53. (C) 56. (D) 66.

2. If A and B are non-empty sets such that $B \subset A$, then

   (A) $B' - A' = A - B$. (B) $B' - A' = B - A$.

   (C) $A' \cap B' = B - A$. (D) $A' \cup B' = A' - B'$.

3. Let A and B be two sets containing 2 and 4 elements respectively. The number of subsets of $A \times B$ having at least 3 elements is

   (A) 211. (B) 219. (C) 221. (D) 256.

4. On a set with 4 elements, the number of symmetric relations is

   (A) 2^4. (B) 2^5. (C) $2^{10} - 1$. (D) 2^{10}.

5. The domain of the real-valued function $f(x) = \sqrt{5 - 4x - x^2} + x^2 \log(x + 4)$ is

   (A) $-5 \leq x \leq 1$. (B) $-5 \leq x$ and $x \geq 1$.

   (C) $-4 < x \leq 1$. (D) $0 \leq x \leq 1$.

6. The number of onto functions that can be defined from the set $A = \{1, 2, 3, 4\}$ to the set $B = \{5, 6, 7\}$ is

   (A) 12 (B) 24. (C) 30 (D) 36.

7. The set of solutions satisfying both the inequalities $x^2 + 6x - 27 \geq 0$ and $-x^2 + 3x + 4 < 0$ is

(A) $(3, 4)$. (B) $[3, 4]$. (C) $(-\infty, 3] \cup [4, \infty)$. (D) $[3, 4)$.

8. Let $\mathbb{R}$ denote the set of real numbers, and $\mathbb{R}^+$ be the set of positive real numbers. For the subsets A and B of $\mathbb{R}$, define the function $f : A \longrightarrow B$ by $f(x) = x^2$ for $x \in A$ and consider the two list given below:

List-I	List-II
(i) f is one-one and onto, if	(a) $A = \mathbb{R}^+, B = \mathbb{R}$
(ii) f is one-one and but onto, if	(b) $A = B = \mathbb{R}$
(iii) f is not one-one and but onto, if	(c) $A = \mathbb{R}, B = \mathbb{R}^+$
(iv) f is neither one-one nor onto, if	(d) $A = B = \mathbb{R}^+$

The correct matching of List-I to List-II is

	(i)	(ii)	(iii)	(iv)
(A)	(a)	(b)	(c)	(d)
(B)	(d)	(b)	(a)	(c)
(C)	(d)	(a)	(c)	(b)
(D)	(d)	(b)	(c)	(a)

9. If $\dfrac{3x^2 + x + 1}{(x-1)^4} = \dfrac{a}{(x-1)} + \dfrac{b}{(x-1)^2} + \dfrac{c}{(x-1)^3} + \dfrac{d}{(x-1)^4}$, then $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

(A) $\begin{bmatrix} 3 & 7 \\ 5 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 3 \\ 7 & 5 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 7 \\ 3 & 5 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 5 \\ 7 & 0 \end{bmatrix}$.

10. If $(p \wedge \sim r) \longrightarrow (\sim p \vee q)$ is false, then the truth values of p, q and r are respectively

(A) T, F and T. (B) F, T and T.

(C) F, F and T. (D) T, F and F.

11. A teacher is making a multiple choice quiz. The teacher wants to give each student the same questions, but have each student's questions appear in a different order. If there are 27 students in the class, then what is the least number of questions the quiz must
contain ?

(A) 3. (B) 4. (C) 5. (D) 6.

12. A bag contains n white balls and n black balls. Pairs of balls are drawn at random without replacement, successively until the bag is empty. If the number of ways of drawing in which each pair consists of one white and one black ball is $14,400$, then the value of n is

(A) 3. (B) 4. (c) 5. (D) 6.

13. If X and Y are two events such that $P(X \cup Y) = 5/6$, $P(X \cap Y) = 1/3$, $P(X) = 2/3$, then X and Y are

(A) dependent events.

(B) independent events.

(C) mutually exclusive events.

(D) mutually exclusive and independent events.

14. In an entrance test, there are multiple choice questions. There are four possible answers in each question, out of which one of them is correct. The probability that a student knows the answer of a question is 9/10. If he/she gets the correct answer to a question, then the probability that he/she was guessing is

(A) $\frac{1}{37}$. (B) $\frac{1}{9}$. (C) $\frac{37}{40}$. (D) $\frac{36}{37}$.

15. The probability of choosing an integer k from the set $\{1, 2, 3, \ldots, 9\}$ such that the equation $x^2 + 5x + k = 0$ has real roots is

(A) $\frac{2}{3}$. (B) $\frac{1}{9}$. (C) $\frac{4}{9}$. (D) $\frac{7}{9}$.

16. If $\alpha > 0$ and $\beta^2 - 4\alpha\gamma = 0$, then the curve $y = \alpha x^2 + \beta x + \gamma$

(A) cuts the X-axis.

(B) touches the X-axis and lies below it.

(C) lies entirely above or below the X-axis.

(D) touches the X-axis and lies above it.

17. If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then the value of x is

(A) -1. (B) 0. (C) $\pi\sqrt{5/8}$. (D) $\frac{3}{2}$.

18. If ω is a complex cube root of unity, then $(z+1)(z+\omega)(z-\omega-1) =$

(A) $z^3 - 1$. (B) $z^3 + 1$. (C) $z^3 + 2$. (D) $z^3 - 2$.

19. If a non-zero complex number z satisfies $z + z^{-1} = 1$, then $z^{100} + z^{-100}$ is equal to

(A) -1. (B) $-i$. (C) 1. (D) i.

20. The locus of the complex number z satisfying $|z^2 - 1| = |z|^2 + 1$ is

(A) the real axis. (B) the imaginary axis. (C) the straight line $y = x$. (D) a circle.

21. the sum of the infinite series $\log_4 2 - \log_8 2 + \log_{16} 2 - \cdots$ is

(A) e^2. (B) $\log_e 2$. (C) $1 + \log_e 3$. (D) $1 - \log_e 2$.

22. Let $ab \neq 0$. If the sum of the coefficient of x^4 and x^7 in the binomial expansion of $\left(\frac{x^2}{a} - \frac{b}{x}\right)^{11}$ in powers of x is zero, then

(A) $a = b$. (B) $a + b = 0$. (C) $ab = -1$. (D) $ab = 1$.

23. The length of the common cord of the circles of radii 15 and 20 units whose centers are 25 units apart, is

(A) 12. units. (B) 15. units. (C) 24. units. (D) 26. units.

24. The equation of the circle concentric with the circle $x^2+y^2-6x+12y+15=0$ and of double its area is

(A) $x^2 + y^2 - 6x + 12y - 15 = 0$.
(B) $x^2 + y^2 + 6x - 12y + 25 = 0$.
(C) $x^2 + y^2 - 6x + 12y + 30 = 0$.
(D) $x^2 + y^2 - 10x + 16y + 110 = 0$.

25. $\lim\limits_{x \to 8} \dfrac{\sqrt{1 + \sqrt{x+1}} - 2}{x - 8} =$

(A) $\frac{3}{2}$. (B) $-\frac{1}{4}$. (C) $\frac{1}{4}$. (D) $\frac{1}{24}$.

26. If the function $f(x) = \begin{cases} (x-1)^{1/(2-x)}, & x \neq 2 \\ \alpha, & x = 2 \end{cases}$ is continuous at $x = 2$, then α is equal to

(A) $\frac{1}{e^2}$. (B) $\frac{1}{e}$. (C) e. (D) 1.

27. If $f(x) = |\sin x| + |x|$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then its left derivative at $x = 0$ is

(A) -3. (B) -2. (C) -1. (D) 0.

28. Let the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined by $f(x) = \min\{x+1, |x|+1\}$ for $x \in \mathbb{R}$. Then

(A) $f(x) \geq 1$ for all $x \in \mathbb{R}$.

(B) f is not differentiable at $x = 1$.

(C) f is differentiable at each point of $\mathbb{R}$.

(D) f is not differentiable at $x = 0$.

29. If $I_1 = \int_0^{\pi/2} x \sin x \, dx$ and $I_2 = \int_0^{\pi/2} x \cos x \, dx$, then

(A) $I_1 - I_2 = 0$. (B) $I_1 + I_2 = 0$. (C) $I_1 = \frac{\pi}{2} I_2$. (D) $I_1 + I_2 = \frac{\pi}{2}$.

30. If a circle passes through the point $(1, -2)$ and touches the X-axis at the point $(3, 0)$, then it also passes through the point

(A) $(-5, 2)$. (B) $(2, -5)$. (C) $(5, -2)$. (D) $(-2, 5)$.

31. If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{a^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = 1$ concide, then the value of a^2 is

(A) 1. (B) 5. (C) 7. (D) 9.

32. Let the points S, T represent the foci of an ellipse, and B represents one end of the minor axis. If the $\triangle STB$ is equilateral, then the value of the eccentricity e is

(A) $\frac{1}{2}$. (B) $\frac{1}{3}$. (C) $\frac{1}{4}$. (D) $\frac{2}{3}$.

33. If the line $x + 3y = 0$ is the tangent to at $(0, 0)$ to a circle of radius 1 unit, then the center of one such circle is

(A) $(3, 0)$. (B) $\left(-\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$. (C) $\left(\frac{3}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\right)$. (D) $\left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$.

34. The area (in square units) of the region enclosed by $|x| + |y| = 2$ is

(A) 1 sq. units. (B) 4 sq. units. (C) 8 sq. units. (D) 16 sq. units.

35. If a function f is defined, continuous on $[3, 5]$ and differentiable at $x = 4$ with $f'(x) = 6$, then $\lim_{x\to 4} \dfrac{f(x+4) - f(x-4)}{4x} =$

(A) 0. (B) 2. (C) 3. (D) 4.

36. For a 2×2 matrix A, consider the following statements:

$S_1 :\ \mathrm{adj}(\mathrm{adj}(A)) = A,$

$S_2 :\ \mathrm{adj}(A) = A.$

Then

(A) S_1 is true, S_2 is true, and S_2 is the correct justification for S_1

(B) both S_1 and S_2 are true, and S_2 is not a correct justification for S_1.

(B) S_1 is true, but S_2 is false.

(D) S_1 is false, but S_2 is true.

37. In a $\triangle ABC$, if $\dfrac{a}{\cos A} = \dfrac{b}{\cos B} = \dfrac{c}{\cos C}$ and side $a = 2$ units, then the area of the triangle is

(A) $\dfrac{2}{\sqrt{3}}$ sq. units. (B) $\dfrac{1}{\sqrt{3}}$ sq. units. (C) $2\sqrt{3}$ sq. units. (D) $\sqrt{3}$ sq. units.

38. $\lim_{x\to 0} \dfrac{\sin x - x + \dfrac{x^3}{6}}{x^5} =$

A) $\dfrac{1}{90}$. (B) $\dfrac{1}{100}$. (C) $\dfrac{1}{110}$. (D) $\dfrac{1}{120}$.

39. The differential equation of the family of circles passing through the origin and having their centers on the X-axis is

(A) $x^2 = y^2 + 3xy\dfrac{dy}{dx}$. (B) $y^2 = x^2 + 2xy\dfrac{dy}{dx}$.

(C) $x^2 = y^2 + xy\dfrac{dy}{dx}$. (D) $y^2 = x^2 - 2xy\dfrac{dy}{dx}$.

40. The differential equation $y\dfrac{dy}{dx} + x = c$(constant) represents

(A) a family of hyperbolas.

(B) a family of circles whose centers are on the Y-axis.

(C) a family of parabolas.

(D) a family of circles whose centers are on the X-axis.

41. If $\vec{a} = \hat{i} + \hat{j}, \vec{b} = -\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$, then the unit vector in the direction of the vector $3\vec{a} + \vec{b} - 2\vec{c}$ is

(A) $\hat{i} + 2\hat{j} + 2\hat{k}$. (B) $\frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k})$. (C) $\frac{1}{3}(\hat{i} - 2\hat{j} - 2\hat{k})$. (D) $\frac{1}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$.

42. A stone is thrown vertically upward from the top of a tower 64 meters high according to the law $s = 48t - 16t^2$. The greatest height attained by the stone from the ground level is

A) 100 meters. (B) 132 meters. (C) 148 meters. (D) 164 meters.

43. For the matrix $A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$, the least value of n such that $A^n =$ O(zero matrix) is

(A) 1. (B) 2. (C) 3. (D) 4.

44. If $A = \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}$, then the value of $\det\left(A^{2013} - 3A^{2012}\right) =$

(A) -8. (B) -7. (C) 7. (D) 8.

45. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & (x-1)x & x(x+1) \\ 3(x-1)x & (x-2)(x-1)x & (x-1)x(x+1) \end{vmatrix}$, then the value of $f(2014)$ is

(A) 0. (B) 1. (C) -500. (D) 2012.

46. If $B = \begin{bmatrix} 1 & a & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A, and $\det(A) = 4$, then the value of a is

(A) 0. (B) 7. (C) 10. (D) 11.

47. The values of λ and μ for which the system of equations: $x + \lambda y + z = 3$, $x + 2y + z = \mu$ and $x + 5y + 3z = 9$, have infinitely many solutions, then (λ, μ) is

(A) $(1, 6)$. (B) $(1, -6)$. (C) $(6, 1)$ (D) $(-1, 6)$.

48. If the line $\frac{x-1}{2} = \frac{y-3}{\alpha} = \frac{z+1}{3}$ lies in the plane $\beta x + 2y + 3z - 4 = 0$, then

(A) $\alpha = \frac{11}{2}, \beta = 1$. (B) $\alpha = -\frac{11}{2}, \beta = -1$.

(C) $\alpha = -\frac{11}{2}, \beta = 1$. (D) $\alpha = 1, \beta = -\frac{11}{2}$.

49. The value of $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\, dx$ is

(A) $\frac{3}{2}$. (B) 1. (C) $\frac{1}{2}$. (D) $\frac{1}{2}$.

50. The function $f(x) = 2\sin x + \cos(2x)$ $(0 \leq x \leq 2\pi)$

(A) increases in $(0, \pi/2)$. (B) decreases in $(0, \pi/2)$.

(C) increases in $(0, \pi/6) \cup (\pi/2, 5\pi/6)$. (D) None.