INSTITUTE OF MATHEMATICS AND APPLICATIONS
BHUBANESWAR
ENTRANCE EXAMINATION-2013
B.Sc.(Hons.) in Mathematics & Computing

Full Marks: 100 **Time: 2 Hours**

This question booklet contains 50 questions. For each question, four answers are suggested and given against (A), (B), (C) and (D) of which, only one will be the correct answer. Choose the correct answer in each question. Each correct answer will be awarded 2 marks. More than one answer marked against a question will be deemed as incorrect.

1. Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6\}$. Then the number of sets C such that $A \cap B \subseteq C \subseteq A \cup B$ is

   (A) 12 (B) 10 (C) 8 (D) 7

2. In a certain town 25% families own a cell phone, 15% families own a scooter, and 65% families own neither a cell phone nor a scooter. If 1500 families own both a cell phone and a scooter, then the total number of families in the town is

   (A) $10,000$ (B) $25,000$ (C) $30,000$ (D) $40,000$

3. If A and B are two sets containing 5 and 7 elements, respectively, then the number of relations on the set $A \times B$ is

   (A) 2^{35} (B) 2^{49} (C) 2^{70} (D) $2^{35\times 35}$

4. The domain of the function $f(x) = \log_2 \log_3 \log_4(x)$ is

   (A) $[3, \infty)$ (B) $(4, \infty)$ (C) $(-\infty, 4)$ (D) $(-\infty, 2)$

5. If $[x]$ denotes the greatest integer $\leq x$, then $\left[\frac{2}{3}\right] + \left[\frac{2}{3} + \frac{1}{99}\right] + \left[\frac{2}{3} + \frac{2}{99}\right] + \cdots + \left[\frac{2}{3} + \frac{98}{99}\right]$ is equal to

   (A) 99 (B) 98 (C) 66 (D) 65

6. Which one of the following functions is one-to-one ?

(A) $f(x) = \sin x, x \in [\pi, \pi)$.
(B) $f(x) = \cos x, x \in [\pi, 2\pi)$.
(C) $f(x) = \sin x, x \in \left[-\frac{3\pi}{2}, -\frac{\pi}{4}\right]$.
(D) $f(x) = \cos x, x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.

7. If a function f satisfies $2f(x) + f(1-x) = x^2$ for all real x, then $f(x)$ is equal to

(A) $\frac{x^2+3x-1}{3}$ (B) $\frac{x^2+2x-1}{3}$ (C) $\frac{x^2+2x-1}{6}$ (D) $\frac{x^2-3x+1}{6}$

8. Let ω be a complex root of $x^n = 1$. Then $(5-\omega)(5-\omega^2)\cdots(5-\omega^{n-1})$ is equal to

(A) 1 (B) $\frac{5^n+1}{4}$ (C) 5^{n-1} (D) $\frac{5^{n-1}}{4}$

9. If the imaginary part of $\frac{2z+1}{iz+1}$ is -4, then the locus of the point representing z in the complex plane is a

(A) straight line passing through the origin.
(B) circle.
(C) an ellipse.
(D) parabola.

10. The number of four-letter words that can be formed(the words may not be meaningful) using the letters of the word $MEDITERRANEAN$ such that the first letter is E and the last letter is R, is

(A) $\frac{11!}{2!\,2!\,2!}$ (B) $\frac{11!}{3!\,2!\,2!}$ (C) $\frac{11!}{3!\,3!\,2!}$ (D) 59

11. The number of ways in which 5 ladies and 7 gentlemen can be seated in a round table so that no two ladies sit together, is

(A) 720 (B) $7(720)^2$ (C) $7(360)^2$ (D) $\frac{7}{2}(720)^2$

12. If two numbers p and q are chosen randomly from the set $\{1, 2, 3, 4\}$ with replacement, then the probability that $p^2 \geq 4q$ is

(A) $\frac{1}{4}$ (B) $\frac{3}{16}$ (C) $\frac{7}{16}$ (D) $\frac{9}{16}$

13. A die has four blank faces and two faces marked 3. The chance of getting a total of 12 in 5 throws is

(A) $5_{C_4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)$
(B) $5_{C_4}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^4$
(C) $5_{C_4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)$
(D) $5_{C_4}\left(\frac{1}{6}\right)^5$

14. Suppose that two persons *Ram* and *Shyam* solve the equation $x^2+ax+b=0$. While solving, *Ram* commits a mistake in constant term and finds the roots as 6 and 3 and *Shyam* commits a mistake in the coefficient of x and finds the roots as -7 and -2. Then the equation is

(A) $x^2+9x+14=0$ (B) $x^2-9x+14=0$ (C) $x^2+9x-14=0$
(D) $x^2-9x-14=0$

15. A student read the common difference of an A.P. as -3 instead of 3, and obtained the sum of first 10 terms as -30. Then the actual sum of the first 10 terms is

(A) 240 (B) 120 (C) 180 (D) 300

16. Which one of the following statement is false ?

(A) $\sim[p \vee (\sim q)] \equiv (\sim p) \wedge q$. (B) $\sim[p \wedge (\sim p)]$ is a tautology.

(C) $(p \wedge q) \wedge (\sim p)$ is a contradiction. (D) $\sim(p \vee q) \equiv (\sim p) \vee (\sim q)$.

17. The coefficient of x in the expansion of $(1+x)(1+2x)(1+3x)\cdots(1+100x)$ is

(A) 1100 (B) 4950 (C) 5050 (D) 5151

18. If, in the binomial expansion of $(x-y)^n, n \geq 5$, the sum of 5th and the 6th terms is zero, then the value of $\frac{x}{y}$ is

(A) $\frac{n-5}{6}$ (B) $\frac{6}{n-4}$ (C) $\frac{5}{n-4}$ (D) $\frac{n-4}{5}$

19. If α and β are roots of the equation $ax^2+bx+c=0$, then $\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(A) 1 (B) $\frac{1}{2}(\alpha-\beta)^2$ (C) $\frac{a^2}{2}(\alpha-\beta)^2$ (D) $\alpha-\beta$

20. $\lim_{x\to 0}\left(\frac{4^x+9^x}{2}\right)^{\frac{1}{x}}$ is equal to

(A) 2 (B) 4 (C) 6 (D) 13

21. If the function $f(x) = \begin{cases} x, & x \leq 1 \\ cx+d, & 1 < x < 4 \\ -2x, & x \geq 4 \end{cases}$ is continuous everywhere, then the values of c and d are, respectively

(A) $-3, -5$ (B) $-3, 5$ (C) $-3, -4$ (D) $-3, 4$

22. Let $f(x) = \begin{cases} (x-1)\sin\left(\dfrac{1}{x-1}\right), & x \neq 1 \\ 0, & x = 1 \end{cases}$. Then which one of the following is true ?

(A) f is neither differentiable at $x = 0$ nor at $x = 1$.

(B) f is differentiable at $x = 0$ and at $x = 1$.

(C) f is differentiable at $x = 0$ but not at $x = 1$.

(D) f is differentiable at $x = 1$ but not at $x = 0$.

23. If $y = \left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)\cdots\left(1+\frac{n}{x}\right)$, and $x \neq 0$, then $\frac{dy}{dx}$ at $x = -1$ is

(A) $n!$ (B) $(n-1)!$ (C) $(-1)^n (n-1)!$ (D) $(-1)^n n!$

24. If $\alpha = \int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$ and $\beta = \int_0^1 \frac{\cos x}{\sqrt{x}}\,dx$, then which one of the following is true ?

(A) $\alpha > \frac{2}{3}$ and $\beta > 2$ (B) $\alpha < \frac{2}{3}$ and $\beta < 2$ (C) $\alpha < \frac{2}{3}$ and $\beta > 2$

(D) $\alpha > \frac{2}{3}$ and $\beta < 2$

25. The value of $\int_0^1 \sqrt{x}e^{\sqrt{x}}\,dx$ is equal to

(A) $\frac{e-2}{2}$ (B) $2(e-2)$ (C) $2e-1$ (D) $2(e-1)$

26. $\int e^{3\log x}(x^4+1)^{-1}\,dx$ is equal to

(A) $\frac{e^{3\log x}}{x^4+1}+C$ (B) $\frac{x^4}{x^4+1}+C$ (C) $\frac{1}{4}\log(x^4+1)+C$ (D) $\frac{1}{3}\log(x^4+1)+C$

27. $\int \frac{\sec x}{\sqrt{\cos 2x}}\,dx$ is equal to

(A) $2\sin^{-1}(\tan x)+C$ (B) $\sin^{-1}(\tan x)+C$ (C) $\frac{1}{2}\sin^{-1}(\tan x)+C$

(D) $\frac{1}{2}\tan^{-1}(\tan 2x)+C$

28. A spherical iron ball of radius 10 cm, coated with a layer of ice of uniform thickness, melts at a rate of 100π cm^3/min. The rate at which the thickness of ice decreases when the thickness of ice is 5 cm. is

(A) $\frac{1}{9}$cm/min. (B) $\frac{1}{9\pi}$cm/min. (C) $\frac{1}{36}$cm/min. (D) $\frac{1}{54}$cm/min.

29. The minimum value of the function $f(x) = \dfrac{1}{\sin x + \cos x}$ in the interval $\left[0, \frac{\pi}{2}\right]$ is

(A) $\dfrac{1}{\sqrt{2}}$ (B) $-\dfrac{1}{\sqrt{2}}$ (C) $\dfrac{2}{1+\sqrt{3}}$ (D) $-\dfrac{2}{1+\sqrt{3}}$

30. The order and degree of the differential equation $\left(\dfrac{d^3y}{dx^3}\right)^{\frac{1}{3}} = 2\dfrac{d^2y}{dx^2} + \sqrt[3]{\cos^2 x}$ are, respectively

(A) 3 and 1 (B) 3 and 3 (C) 1 and 3 (D) 3 and 2

31. An integrating factor of the differential equation $xdy - ydx + x^2e^x dx = 0$ is

(A) x (B) $\dfrac{1}{x}$ (C) $\dfrac{1}{1+x^2}$ (D) $\sqrt{1+x^2}$

32. If the equation $\dfrac{x^2}{9-c} + \dfrac{y^2}{5-c} = 1$ represents an ellipse, then the foci are

(A) $(\pm 3, 0)$ (B) $(\pm 2, 3)$ (C) $(\pm 4, 0)$ (D) $(\pm 2, 0)$

33. The area of the plane region bounded by the curve $x+2y^2 = 0$ and $x+3y^2 = 1$ is equal to

(A) $\dfrac{5}{3}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{2}{3}$ (D) $\dfrac{4}{3}$

34. If the straight lines $\dfrac{x-1}{k} = \dfrac{y-2}{2} = \dfrac{z-3}{3}$ and $\dfrac{x-2}{3} = \dfrac{y-3}{k} = \dfrac{z-1}{2}$ intersect at a point, then the integer k is equal to

(A) -5 (B) 5 (C) 2 (D) -2

35. If $\cos^{-1}\left(\dfrac{5}{13}\right) - \sin^{-1}\left(\dfrac{12}{13}\right) = \cos^{-1} x$, then x is equal to

(A) 1 (B) -1 (C) 0 (D) $\dfrac{\sqrt{3}}{2}$

36. $\begin{vmatrix} a+x & b & c \\ a & b+x & c \\ a & b & c+x \end{vmatrix}$ is equal to

(A) $abc\left(1+\frac{x}{a}+\frac{y}{b}+\frac{z}{c}\right)$

(B) $abc\left(1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)$

(C) $xyz\left(1+\frac{x}{a}+\frac{y}{b}+\frac{z}{c}\right)$

(D) $xyz(1+a+b+c)$

37. If the matrix M_k is given by $M_k = \begin{bmatrix} k & k-1 \\ k-1 & k \end{bmatrix}, k = 1, 2, \cdots,$ then the value of $\det(M_1) + \det(M_2) + \cdots + \det(M_{2008})$ is

(A) 2007 (B) 2008 (C) $(2008)^2$ (D) $(2009)^2$

38. The value of λ for which the system of equations: $3x + \lambda y - 2z = 0$, $x + \lambda y + 3z = 0, 2x + 3y - 4z = 0$ has a non-trivial solution over the set of rational numbers, is

(A) $\frac{2}{33}$ (B) $\frac{1}{33}$ (C) $-\frac{23}{2}$ (D) $\frac{33}{2}$

39. If $A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$, then A^4 is equal to

(A) $729A$ (B) $243A$ (C) $81A$ (D) $27A$

40. If $(3, -2)$ is the centre of a circle and $4x + 3y + 19 = 0$ is a tangent to the circle, then the equation of the circle is

(A) $x^2 + y^2 - 6x + 4y + 9 = 0$

(B) $x^2 + y^2 - 6x + 4y - 12 = 0$

(C) $x^2 + y^2 - 6x + 4y + 12 = 0$

(D) $x^2 + y^2 - 6x + 4y + 25 = 0$

41. The values of λ for which the plane $x + y + z = \sqrt{3}\lambda$ touches the sphere $x^2 + y^+ z^2 - 2x - 2y - 2z - 6 = 0$ are

(A) $\pm\sqrt{3}$ (B) $\sqrt{3} \pm 1$ (C) $3 \pm \sqrt{3}$ (D) $\sqrt{3} \pm 3$

42. In a $\Delta\, ABC$, $a = 13$ cm, $b = 12$ cm, and $c = 5$ cm. Then the distance of A from BC is

(A) $\frac{25}{13}$ cm (B) $\frac{60}{13}$ cm (C) $\frac{65}{12}$ cm (D) $\frac{144}{13}$ cm

43. If e_1 is the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$, and e_2 is the eccentricity of the hyperabola $\frac{x^2}{9} - \frac{y^2}{7} = 1$, then $e_1 + e_2$ is equal to

(A) $\frac{16}{7}$ (B) $\frac{16}{9}$ (C) $\frac{25}{12}$ (D) $\frac{25}{4}$

44. $A + B = 45°$, then $(\cot A - 1)(\cot B - 1)$ is equal to

(A) -2 (B) -1 (C) 1 (D) 2

45. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then $\cos 2\alpha + \cos 2\beta$ is equal to

(A) $-2\sin(\alpha+\beta)$ (B) $-2\cos(\alpha+\beta)$ (C) $-2\cos(\alpha-\beta)$ (D) $2\sin(\alpha-\beta)$

46. The distance of the point $(1, 2)$ from the line $x + y + 5 = 0$ measured along the line parallel to $3x - y = 7$ is equal to

(A) $4\sqrt{10}$ (B) $10\sqrt{2}$ (C) $\sqrt{40}$ (D) $2\sqrt{20}$

47. If $\vec{a} = p\hat{i} - \hat{j} + 8\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + q\hat{k}$ and $(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b}) = 0$, then $p - q$ is equal to

(A) $\frac{63}{2}$ (B) $\frac{63}{4}$ (C) $\frac{17}{32}$ (D) $-\frac{63}{2}$

48. Let $g(x) = \begin{cases} e^{2x}, & x \le 1 \\ \log(x-1), & x > 1 \end{cases}$. The equation of normal to $y = g(x)$ at the point $(3, \log 2)$ is

(A) $y - 2x = 6 + \log 2$ (B) $y + 2x = 6 + \log 2$ (C) $y + 2x = -6 + \log 2$

(D) $y - 2x = 6 - \log 2$

49. The equation of the line through the point $(1, 2)$ whose distance from the point $(3, 1)$ has the greatest value, is

(A) $y - 3x + 1 = 0$ (B) $4y - x - 7 = 0$ (C) $x + 2y - 5 = 0$ (D) $y - 2x = 0$

50. Equation of the directrix of the conic $x^2 + 4y + 4 = 0$ is

(A) $y = 0$ (B) $x = 0$ (C) $y = -1$ (D) $x = 1$