Entrance Examination for M. A. Economics, 2014

Option A (Series 01)

Time. 3 hours **Maximum marks.** 100

General Instructions. Please read the following instructions carefully:

- Check that you have a bubble-sheet accompanying this booklet. Do **not** break the seal on this booklet until instructed to do so by the invigilator.
- Immediately on receipt of this booklet, fill in your Signature, Name, Roll number and Booklet number (see the top corners of this Booklet) in the space provided below.
- This examination will be checked by a machine. **Therefore, it is very important that you follow the instructions on the bubble-sheet.**
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- You are **not allowed to leave the examination hall** during the first 30 minutes and the last 15 minutes of the examination time.
- When you finish the examination, hand in this **booklet and the bubble-sheet** to the invigilator.

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Before you start

- Check that this booklet has pages 1 through 26. Also check that the top of each page is marked with *EEE 2014 A 01*. Report any inconsistency to the invigilator.
- You may use the blank pages at the end of this booklet, marked **Rough work**, to do your calculations and drawings. No other paper will be provided for this purpose. Your "Rough work" will be neither read nor checked.

You may begin now. Best Wishes!

Part I

- This part of the examination consists of 20 multiple-choice questions. Each question is followed by four possible answers, at least one of which is correct. If more than one choice is correct, choose **only the 'best one'**. The 'best answer' is the one that implies (or includes) the other correct answer(s). Indicate your chosen best answer on the **bubble-sheet** by shading the appropriate bubble.
- For each question, you will get: 1 mark if you choose only the best answer; 0 mark if you choose none of the answers. **However, if you choose something other than the best answer or multiple answers, you will get $-1/3$ mark for that question.**

Question 1. Let $a_n = \left(1 + \frac{1}{n}\right)^{n+1}$, $n = 1, 2, ...$ Then the sequence $(a_n)_{n=1}^{\infty}$

(a) is an increasing sequence.
(b) first increases, then decreases.
(c) is a decreasing sequence.
(d) first decreases, then increases.

Answer (c).

Question 2. Let M, A, B, C be respectively the four matrices below:

$$\begin{pmatrix} 4 & 7 \\ 7 & 9 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 4 & 5 \end{pmatrix}.$$

Then $M = xA + yB + zC$,
(a) but x, y, z are not unique.
(b) $z = -1$
(c) $z = -1$ and $z = -2$ both can hold.
(d) x, y, z are unique but $z = 2$.

Answer (b).

Question 3. Let f be a continuous function from $[a, b]$ to $[a, b]$, and is differentiable on (a, b). We will say that point $y \in [a, b]$ is a *fixed point* of f if $y = f(y)$. If the derivative $f'(x) \neq 1$ for any $x \in (a, b)$, then f has
(a) has multiple, and an odd number of, fixed points.
(b) no fixed points in $[a, b]$.
(c) has multiple, but an even number of, fixed points.
(d) exactly one fixed point in $[a, b]$.

Answer (d).

Question 4. Which of the following statements is true for all real numbers a, b with $a < b$?
(a) $\sin b - \sin a \leq b - a$.
(b) $\sin b - \sin a \geq b - a$.
(c) $|\sin b - \sin a| \geq b - a$.
(d) $|\sin b - \sin a| \leq |b - a|$.

Answer (d).

Question 5. Let $O(0,0)$, $P(3,4)$ and $Q(6,0)$ be the vertices of a triangle OPQ. If a point S in the interior of the OPQ is such that triangles OPS, PQS and OQS have equal area, then the coordinates of S are:
(a) $(4/3, 3)$.
(b) $(3, 2/3)$.
(c) $(3, 4/3)$.
(d) $(4/3, 2/3)$.

Answer (c).

The following data is the starting point for Questions 1-4. Consider an exchange economy with two agents, 1 and 2, and two goods, X and Y. Agent 1's endowment is $(0, 10)$ and Agent 2's endowment is $(11, 0)$. Agent 1 strictly prefers bundle (a, b) to bundle (c, d) if, either $a > c$, or $a = c$ and $b > d$. Agent 2 strictly prefers bundle (a, b) to bundle (c, d) if $\min\{a, b\} > \min\{c, d\}$. For both agents, we say that bundle (a, b) is indifferent to bundle (c, d) if, neither (a, b), nor (c, d), is strictly preferred to the other.

Question 6. This exchange economy has
(a) one competitive equilibrium allocation
(b) two competitive equilibrium allocations
(c) an infinite number of competitive equilibrium allocations
(d) no competitive equilibrium allocations

Answer: (a)

Question 7. Which of the following changes makes $(p_X, p_Y) = (1, 0)$ a competitive equilibrium price vector?
(a) agent 2's endowment changes to $(9, 0)$
(b) agent 2's endowment changes to $(10, 0)$
(c) agent 1's endowment changes to $(0, 12)$
(d) none of the above

Answer: (d)

Question 8. Suppose only agent 2's preferences are changed. The changed preferences of agent 2's become identical to those of agent 1. Then,
(a) there is no equilibrium price ratio
(b) both of the following are true
(c) $p_X/p_Y = 0$ is an equilibrium price ratio
(d) $p_Y/p_X = 0$ is an equilibrium price ratio

Answer: (a)

Question 9. Suppose only agent 2's preferences are changed. The changed preference is such that agent 2 strictly prefers bundle (a, b) to bundle (c, d) if, either $b > d$, or $b = d$ and $a > c$. Then,

(a) there is no equilibrium price ratio
(b) both of the following are true
(c) $p_X/p_Y = 0$ is an equilibrium price ratio
(d) $p_X/p_Y > 0$ is an equilibrium price ratio

Answer: (d)

Question 10. Suppose only agent 1's preferences are changed. After change, agent 1's preferences become identical to those of agent 2. Then,

(a) there is no equilibrium price ratio
(b) both of the following are true
(c) $p_X/p_Y = 0$ is an equilibrium price ratio
(d) $p_Y/p_X = 0$ is an equilibrium price ratio

Answer: (c)

Question 11. Suppose that we classify all households into one of two states, rich and poor. The probability of a particular generation being in either of these states depends only on the state in which their parents were. If a parent is poor today, their child is likely to be poor which probability 0.7. If a parent is rich today, their child is likely to be poor with probability 0.6. What is the probability that the great grandson of a poor man will be poor?

(a) 0.72
(b) 0.67
(c) 0.62
(d) 0.78

Answer (b)

Question 12. Consider the experiment of tossing two fair coins. Let the event **A** be a head on the first coin, the event **C** be a head on the second coin, the event **D** be that both coins match and the event **G** be two heads. Which of the following is **false**?

(a) **C** and **D** are statistically independent

(b) **A** and **G** are statistically independent
(c) **A** and **D** are statistically independent
(d) **A** and **C** are statistically independent

Answer: (b)

Question 13. Let Y denote the number of heads obtained when 3 fair coins are tossed. Then, the expectation of $Z = 4 + 5Y^2$ is
(a) 17
(b) 18
(c) 19
(d) None of the above.

Answer: (c)

Question 14. Let Y denote the number of heads obtained when 3 fair coins are tossed. Then, the variance of $Z = 4 + 5Y^2$ is
(a) 185.5
(b) 178.5
(c) 187.5
(d) None of the above.

Answer: (c)

Question 15. Let events E, F and G be pairwise independent with $Pr(G) > 0$ and $Pr(E \cap F \cap G) = 0$. Let X^C denote the complement of event X. Then, $Pr(E^C \cap F^C|G) =?$
(a) $Pr(E^C) + Pr(F^C)$
(b) $Pr(E^C) - Pr(F^C)$
(c) $Pr(E^C) - Pr(F)$
(d) None of the above.

Answer: (c)

Question 16. 5 men and 5 women are seated randomly in a single row of chairs. The expected number of women sitting next to at least 1 man equals
(a) 11/3
(b) 13/3

(c) 35/9
(d) 37/9

Answer (c).

Question 17. Let M be a 3×3 matrix such that $M^2 = M$. Which of the following is necessarily true?

(a) M is invertible.
(b) $\det(M) = 0$.
(c) $\det(M^5) = \det(M)$.
(d) None of the above.

Answer: (c)

Question 18. Suppose a straight line in $\mathcal{R}^3$ passes through the point $(-1, 3, 3)$ in the direction of the vector $(1, 2, 3)$. The line will hit the xy-plane at point

(a) $(2, -1, 0)$
(b) $(1, 3, 0)$
(c) $(3, 1, 0)$
(d) None of the above

Answer: (a)

Question 19. X is a random variable. Which of the following statements is always true

(a) The expectation of X exists.
(b) The distribution function of X is strictly increasing.
(c) X has a median
(d) None of the above
Answer (c)

Question 20. Consider two disjoint events A and B in a sample space S. Which of the following is correct?

(a) A and B are always independent
(b) A and B cannot be independent
(c) A and B are independent if both of them have positive probability
(d) None of the above

Answer: (d)

End of Part I.

Proceed to Part II of the examination on the next page.

Part II

- This part of the examination consists of 40 multiple-choice questions. Each question is followed by four possible answers, at least one of which is correct. If more than one choice is correct, choose **only the 'best one'**. The 'best answer' is the one that implies (or includes) the other correct answer(s). Indicate your chosen best answer on the **bubble-sheet** by shading the appropriate bubble.
- For each question, you will get: 2 marks if you choose only the best answer; 0 mark if you choose none of the answers. **However, if you choose something other than the best answer or multiple answers, then you will get $-2/3$ mark for that question**.
- The following notational conventions apply wherever the following symbols are used. $\Re$ denotes the set of real numbers. $\Re^n$ denotes the n-dimensional vector space.

Question 21. $\lim_{n\to\infty} \sqrt[n]{n} =$

(*a*) 0
(*b*) 0.5
(*c*) 1
(*d*) 2

Answer: (*c*).

Question 22. $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right) =$

(*a*) -1
(*b*) 0
(*c*) 1
(*d*) The limit does not exist.

Answer (*b*).

Question 23. Suppose $A_1, A_2, \ldots,$ is a countably infinite family of subsets of a vector space. Suppose all of these sets are linearly independent, and that $A_1 \subseteq A_2 \subseteq \ldots$. Then $\cup_{i=1}^{\infty} A_i$ is

(a) a linearly independent set of vectors.
(b) a linearly dependent set of vectors.
(c) linearly independent provided the vectors are orthogonal.
(d) not necessarily either dependent or independent.

Answer (a).

Question 24. If u and v are distinct vectors and k and t are distinct scalars, then the vectors $u + k(u - v)$ and $u + t(u - v)$
(a) are linearly independent.
(b) may be identical.
(c) are linearly dependent.
(d) are distinct.

Answer (d).

Question 25. Let $d\left((x_1, x_2), (y_1, y_2)\right) = \max\left\{|x_1 - y_1|, |x_2 - y_2|\right\}$ be the distance between two points (x_1, x_2) and (y_1, y_2) on the plane. Then the circle centered at the origin with radius length equal to 1 is
(a) a square with side length $= 1$.
(b) a square with side length $= \sqrt{2}$.
(c) a square with side length $= 2$.
(d) a circle with radius $= 1$.

Answer (c).

Question 26. The set of all pairs of positive integers $a, b, a < b$ such that $a^b = b^a$
(a) is an empty set.
(b) consists of a single pair.
(c) consists of multiple, but finite number of, pairs.
(d) is countably infinite.

Answer (b).

Question 27. Suppose c is a given positive real number. The equation $\ln x = cx^2$ must have a solution if
(a) $c < 1/(2e)$.
(b) $c < 1/e$.

(c) $c > 1/(2e)$.
(d) $c > 1/e$.

Answer (a).

Question 28. Sania's boat is at point A on the sea. The closest point on land, point B, is 2 km. away. Point C on land is 6 k.m. from point B, such that triangle (ABC) is right-angled at point B. Sanya wishes to reach point C, by rowing to some point P on the line $\bar{BC}$, and jog the remaining distance to C. If she rows 2 km. per hour and jogs 5 km. per hour, at what distance from point B should she choose her landing point P, in order to minimize her time to reach point C?
(a) $21/\sqrt{4}$
(b) $4/\sqrt{21}$
(c) $4/\sqrt{12}$
(d) $21/\sqrt{21}$

Answer (b).

How much time will she need to reach point C?

Question 29. Suppose $A_j, j = 1, 2, \ldots$ are non-empty sets of real numbers. Define the sets $C_n = \cap_{k=l}^{\infty} \cup_{j=k}^{\infty} A_j,\ n = 1, 2, \ldots$. Which of the choices below must then hold for a given n? (where the symbol $\subset$ stands for 'strict subset').
(a) $C_n \subset C_{n+1}$
(b) $C_{n+1} \subset C_n$
(c) $C_n = C_{n+1}$
(d) None of the above need hold.

Answer: (c)

Question 30. Suppose x and y are given integers. Consider the following statements:
A. If $2x + 3y$ is divisible by 17, then $9x + 5y$ is divisible by 17.
B. If $9x + 5y$ is divisible by 17, then $2x + 3y$ is divisible by 17.
Which of the following is true?
(a) A is true and B is false.
(b) B is true and A is false.

(c) Both A and B are true.
(d) Neither A nor B is true.

Answer (c).

The following data is the starting point for Questions 1-2. Consider an exchange economy with two goods. Suppose agents i and j have the same preferences. Moreover, suppose their preferences have the following property: if (a, b) and (c, d) are distinct bundles that are indifferent to each other, then the bundle $((a+c)/2, (b+d)/2)$ is strictly preferred to (a, b) and (c, d).

Question 31. In a Pareto efficient allocation, i and j
(a) will get the same bundle
(b) may get different bundles
(c) will get the same bundle, provided their endowments are identical
(d) will get the same bundle, provided their endowments are identical and the preferences are monotonically increasing

Answer: (b)

Question 32. In a competitive equilibrium allocation, i and j
(a) will get the same bundle
(b) may get different bundles
(c) will get the same bundle, provided their endowments are identical
(d) will get the same bundle, provided their endowments are identical and the preferences are monotonically increasing

Answer: (c)

The following data is the starting point for Questions 3-5. Two firms produce the same commodity. Let x_1 and x_2 be the quantity choices of firms 1 and 2 respectively. The total quantity is $X = x_1 + x_2$. The inverse demand function is $P = a - bX$, where P is the market price, and a and b are the intercept and slope parameters respectively. Firms 1 and 2 have constant average costs equal to c_1 and c_2 respectively. Suppose $b > 0$, $0 < c_1 < c_2 < a$ and $a + c_1 > 2c_2$.

Question 33. In a Cournot equilibrium,

(a) firm 1 has the larger market share and the larger profit
(b) firm 2 has the larger market share and the larger profit
(c) firm 1 has the larger market share and the smaller profit
(d) firm 2 has the larger market share and the smaller profit

Answer: (a)

Question 34. If a increases, then
(a) the market share of firm 1 increases and price increases
(b) the market share of firm 1 decreases and price increases
(c) the market share of firm 1 increases and price decreases
(d) the market share of firm 1 decreases and price decreases

Answer: (b)

Question 35. If b decreases, then
(a) the price and market share of firm 1 increase
(b) the price and market share of firm 1 decrease
(c) the market shares are unchanged but price increases
(d) neither price, nor market shares, change

Answer: (d)

Question 36. Suppose that an economy has endowment of K units of capital and L units of labour. Two final goods X_1 and X_2 can be produced by the following technologies,

$$X_1 = \sqrt{kl}, \quad X_2 = \sqrt{l}$$

where k is quantity of capital and l is quantity of labour. Find the production possibility frontier.
(a) $X_1^2 + KX_2^2 = KL$
(b) $X_1^2 + X_2^2 = KL$
(c) $X_1 + \sqrt{K}X_2 = \sqrt{KL}$
(d) $X_1 + X_2^2 = KL$

Answer: (a)

Question 37. A two-person two commodity economy has social endowment of $x = 1$ unit of food and $y = 1$ unit of wine. Agents preferences are increasing in own consumption but decreasing in wine consumption of the other person. Preferences of agents A and B are as follows,

$$u_A(x_A, y_A, y_B) = x_A[1+\max(y_A - y_B, 0)], u_B(x_B, y_B, y_A) = x_B[1+\max(y_B - y_A, 0)]$$

where A consumes x_A and y_A units of x and y respectively, similarly B's consumption is x_B and y_B.
Which of the following is a Pareto optimum allocation.

(a) $x_A = \frac{1}{4}, x_B = \frac{3}{4}$, $y_A = y_B = \frac{1}{2}$
(b) $x_A = x_B = \frac{1}{2}$, $y_A = \frac{1}{4}, y_B = \frac{3}{4}$
(c) $x_A = \frac{1}{4}, x_B = \frac{3}{4}$, $y_A = 1, y_B = 0$
(d) $x_A = \frac{1}{4}, x_B = \frac{3}{4}$, $y_A = 0, y_B = 1$

Answer: (d)

Question 38. Consider a two-person two-goods pure exchange economy. The initial endowment vectors are $\mathbf{e}^1 = (1, 0)$ and $\mathbf{e}^2 = (0, 1)$. The two individuals have identical preferences represented by the utility functions:

$$u^1(x, y) = u^2(x, y) = \begin{cases} 1, & \text{when } x + y < 1 \\ x + y, & \text{when } x + y \geq 1, \end{cases}$$

where x is the quantity of the first good and y is the quantity of the second good. For this economy, the set of Pareto optimum allocations

(a) consists of The entire Edgeworth box
(b) is just the qual division of the goods
(c) is a null set
(d) is $\{(0, 0), (1, 1)\}$

Answer: (d)

Question 39. A monopolist seller produces a good with constant marginal cost $c \geq 0$. The monopolist sells the entire output to a consumer whose utility from consuming x units of the product is given by $\theta\sqrt{x} - t$, where t is the payment made by the consumer to the monopolist. Suppose, consumer's outside option is 0, i.e., if she does not buy the good from the monopolist, she gets 0 utility. Then, the monopolist's profit is

(a) $\theta/4c$

(b) $\theta^2/4c$
(c) $c\theta^2$
(d) $c\theta/2$

Answer: (b)

Question 40. Consider an economy consisting of $n \geq 2$ individuals with preference relations defined over the set of alternatives X. Let $S = \{a, b, c, d, e\}$ and $T = \{a, b, c, d\}$ be two subsets of X. Now consider the following statements:

- A. If a is Pareto optimal (PO) with respect to set S, then a is PO with respect to set T.
- B. If a is PO with respect to set T, then a is PO with respect to set S.
- C. If a is PO with respect to set S and b is *not* PO with respect to set T, then a is Pareto superior to b.
- D. If a is the only PO alternative in set S and b is *not* with respect to set S, then a is Pareto superior to b.

How many of the above statements are correct?

(a) 1
(b) 2
(c) 3
(d) All are correct.

Answer: (b)

Question 41. 5 men and 5 women are seated randomly in a single circle of chairs. The expected number of women sitting next to at least 1 man equals
(a) 23/6
(b) 25/6
(c) 4
(d) 17/4

Answer (b).

Question 42. Ms. A selects a number X randomly from the uniform distribution on $[0, 1]$. Then Mr. B repeatedly, and independently, draws numbers $Y_1, Y_2, ...$ from the uniform distribution on $[0, 1]$, until he gets a number larger than $X/2$, then stops. The expected number of draws that Mr. B makes equals

(a) $2 \ln 2$
(b) $\ln 2$
(c) $2/e$
(d) $6/e$

Answer (a).

Question 43. The expected sum of the numbers Mr. B draws, given $X = x$, equals

(a) $\ln 2$
(b) $1/(1 - \frac{1}{2})$
(c) $1/(2 - x)$
(d) $3/(1 - \frac{1}{2})$

Answer (c).

Question 44. There are two fair coins (i.e. Heads and Tails are equally likely for tosses of both). Coin 1 is tossed 3 times. Let X be the number of Heads that occur. After this, Coin 2 is tossed X times. Let Y be the number of Heads that we get with Coin 2. The probability $\text{Prob}(X \geq 2 | Y = 1)$ equals

(a) $1/2$
(b) $4/7$
(c) $2/3$
(d) $11/18$

Answer (d)

Question 45. Two independent random variables X and Y have the same probability density function:

$$f(x) = \begin{cases} c(1+x) & x \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

Then the variance of their sum, $\text{Var}(X + Y)$ equals

(a) 2/9
(b) 13/81
(c) 4/45
(d) 5/18

Answer (b).

Question 46. Suppose two restaurants are going to be located at a street that is ten kilometers long. The location of each restaurant will be chosen randomly. What is the probability that they will be located less than five kilometers apart?

(a) 1/4
(b) 1/2
(c) 3/4
(d) 1/3

Answer (a).

Question 47. Consider the linear regression model: $y_i = \beta_1 D1_1 + \beta_2 D2_i + \varepsilon_i$, where $D1_i = 1$ if $1 < i < N$ and $D1_i = 0$ if $N+1 < i < n$ for some $i < N < n$; and $D2 = 1 - D1$. Can this model be estimated using least squares?

(a) No, because $D1$ and $D2$ are perfectly collinear

(b) Yes, and it is equivalent to running two separate regressions of y on $D1$ and y on $D2$, respectively.

(c) No, because there is no variability in $D1$ and $D2$

(d) Yes, provided an intercept term is included.

Answer: (b)

Question 48. Consider the least squares regression of y on a single variable x. Which of the following statements is true about such a regression?

(a) The coefficient of determination R^2 is always equal to the squared correlation coefficient between y on x

(b) The coefficient of determination R^2 is equal to the squared correlation coefficient between y on x only if there is no intercept in the equation

(c) The coefficient of determination R^2 is equal to the squared correlation coefficient between y on x only if there is an intercept in the equation (*d*) There is no relationship between the coefficient of determination R^2 and the squared correlation coefficient between y on x

Answer: (c)

Question 49. An analyst runs two least squares regressions: first, of y on a single variable x, and second, of x on y. In both cases, she decides to include an intercept term. Which of the following is true of what she finds?

(*a*) The slope coefficient of the first regression will be the inverse of the slope coefficient of the second regression; this will also be true of the associated t-ratios

(*b*) The slope coefficients will be different, the associated t- ratios will also be different, but the R^2 from the two regressions will be the same

(*c*) The slope coefficients will be different, but the associated t-ratios and the R^2 from the two regressions will be the same

(*d*) The slope coefficients will be the inverse of each other, the associated t-ratios will also be the inverse of each other, but the R^2 from the two regressions will be the same.

Answer: (c)

Question 50. Consider the two regression models

(i) $y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u$

(ii) $y = \gamma_0 + \gamma_1 Z_1 + \gamma_2 Z_2 + v$,

where variables Z_1 and Z_2 are distinct from X_1 and X_2. Assume $u \sim N(0, \sigma_u^2)$ and $v \sim N(0, \sigma_v^2)$ and the models are estimated using ordinary least squares. If the true model is (i) then which of the following is true?

(*a*) $E[\hat{\beta}_1] = E[\hat{\gamma}_1] = \beta_1$ and $E[\hat{\sigma}_v^2] = \sigma_u^2$.

(*b*) $E[\hat{\sigma}_v^2] \geq \sigma_u^2$.

(*c*) $E[\hat{\sigma}_v^2] \leq \sigma_u^2$.

(*d*) None of the above as the two models cannot be compared

Answer: (b)

The next Ten questions are based on the following information: Please read them carefully before you proceed to answer.

Consider an economy consisting of N identical firms producing a single final commodity to be used for consumption as well as investment purposes. Each firm is endowed with a Cobb-Douglas production technology, such that

$$Y_t^i = \left(K_t^i\right)^{\alpha} \left(L_t^i\right)^{1-\alpha}; 0 < \alpha < 1,$$

where K_t^i and L_t^i denote the amounts of capital and labour employed by the i-th firm at time period t. The final commodity is the numeraire; wage rate for labour (w_t) and the rental rate for capital (r_t) are measured in terms of the final commodity. The firms are perfectly competitive and employ labour and capital so as to maximize their profits - taking the factor prices as given. The aggregate output produced is thus given by:

$$Y_t = \sum_{i=1}^{N} \left(K_t^i\right)^{\alpha} \left(L_t^i\right)^{1-\alpha} = \left(K_t\right)^{\alpha} \left(L_t\right)^{1-\alpha},$$

where $K_t = \sum_{i=1}^{n} K_t^i$ and $L_t = \sum_{i=1}^{n} L_t^i$ are the total capital and labour employed in the aggregate economy in period t.

Labour and capital on the other hand are provided by the households. There are H identical households, each endowed with k_t^h units of capital and 1 unit of labour at the beginning of period t. Capital stock of the households gets augmented over time due to the savings and investments made by the households. In particular, each household saves and invests exactly half of its total income y_t^h - (which includes its labour as well as capital income) in every period and consumes the rest, such that $\frac{dk_t^h}{dt} = \frac{1}{2} y_t^h$ (There is no depreciation of capital).

The entire capital endowment at the beginning of every period is supplied inelastically to the market at the given rental rate (r_t). Labour supply however is endogenous and responds to the market wage rate. Out of the total endowment of 1 unit of labour, the households optimally supplies l_t^h units so as to maximise his utility:

$$U_t^h = w_t l_t^h - \left(l_t^h\right)^{\delta}; \delta > 1,$$

where the first term captures the (indirect) utility derived from labour earnings while the second term captures the dis-utility of labour.

Question 51. The labour demand schedule for the aggregate economy is given by the following function:

(a) $L_t = \left[\frac{1}{w_t}\right]^{1/\alpha} K_t$

(b) $L_t = N \left[\frac{1-\alpha}{w_t}\right]^{1/\alpha} K_t$

(c) $L_t = \left[\frac{1-\alpha}{w_t}\right]^{1/\alpha} K_t$

(d) None of the above.

Answer: (c)

Question 52. The aggregate labour supply schedule by the households is given by the following function:

(a) $L_t^S = \begin{cases} H \left[\frac{w_t}{\delta}\right]^{1/(\delta-1)} & \text{for } w_t < \bar{w} \equiv (\delta)^{1/(\delta-1)} \\ H & \text{for } w_t \geq \bar{w} \end{cases}$

(b) $L_t^S = \begin{cases} H \left[\frac{w_t}{\delta}\right]^{1/(\delta-1)} & \text{for } w_t < \hat{w} \equiv \delta \\ H & \text{for } w_t \geq \hat{w} \end{cases}$

(c) $L_t^S = \begin{cases} \left[\frac{H w_t}{\delta}\right]^{1/(\delta-1)} & \text{for } w_t < \hat{w} \equiv \delta \\ 1 & \text{for } w_t \geq \hat{w} \end{cases}$

(d) None of the above.

Answer: (b)

Question 53. The market clearing wage rate in the short run (period t) is given by:

(a) $w_t^* = \begin{cases} \left[\frac{K_t \ (1-\alpha)^{1/\alpha}}{H}\right]^{\frac{\alpha(\delta-1)}{\alpha+\delta-1}} & \text{for } K_t < \frac{H}{\delta} \equiv \hat{K} \\ \left[\frac{K_t \ (1-\alpha)^{1/\alpha}}{H}\right]^{\alpha} & \text{for } K_t \geq \hat{K} \end{cases}$

(b) $w_t^* = \begin{cases} \left[\frac{K_t \ (1-\alpha)^{1/\alpha}}{H}\right]^{\frac{\alpha(\delta-1)}{\alpha+\delta-1}} & \text{for } K_t < H \left(\frac{\delta}{1-\alpha}\right)^{1/\alpha} \equiv \bar{K} \\ \left[\frac{K_t \ (1-\alpha)^{1/\alpha}}{H}\right]^{\alpha} & \text{for } K_t \geq \bar{K} \end{cases}$

$$(c)\ w_t^* = \begin{cases} \left[\frac{K_t\ (1-\alpha)^{1/\alpha}\ (\delta)^{1/(\delta-1)}}{H}\right]^{\frac{\alpha(\delta-1)}{\alpha+\delta-1}} & \text{for } K_t < H\left(\frac{\delta}{1-\alpha}\right)^{1/\alpha} \equiv \bar{K} \\ \left[\frac{K_t\ (1-\alpha)^{1/\alpha}}{H}\right]^{\alpha} & \text{for } K_t \geq \bar{K} \end{cases}$$

(d) None of the above.

Answer: (c)

Question 54. Equilibrium output in the short run (period t) is given by:

$$(a)\ Y_t^* = \begin{cases} (K_t)^{\frac{\alpha\delta}{\alpha+\delta-1}}\ \ (H)^{\frac{(1-\alpha)(\delta-1)}{\alpha+\delta-1}} \left[\frac{(1-\alpha)^{1/\alpha}\ (\delta)^{1/(\delta-1)}}{\delta}\right]^{\frac{\alpha(1-\alpha)}{\alpha+\delta-1}} \\ \text{for } K_t < H\left(\frac{\delta}{1-\alpha}\right)^{1/\alpha} \equiv \bar{K};\ \text{ and} \\ (K_t)^{\alpha}\ \ (H)^{1-\alpha} \quad \text{for } K_t \geq \bar{K} \end{cases}$$

$$(b)\ Y_t^* = \begin{cases} (K_t)^{\alpha}\ \ (H)^{1-\alpha} \left[\frac{(1-\alpha)^{1/\alpha}\ (\delta)^{1/(\delta-1)}}{\delta}\right]^{\frac{\alpha(1-\alpha)}{\alpha+\delta-1}} & \text{for } K_t < H\left(\frac{\delta}{1-\alpha}\right)^{1/\alpha} \equiv \bar{K} \\ (K_t)^{\alpha}\ \ (H)^{1-\alpha} & \text{for } K_t \geq \bar{K} \end{cases}$$

$$(c)\ Y_t^* = \begin{cases} (K_t)^{\frac{\alpha\delta}{\alpha+\delta-1}}\ \ (H)^{\frac{(1-\alpha)(\delta-1)}{\alpha+\delta-1}} \left[\frac{(1-\alpha)^{1/\alpha}\ (\delta)^{1/(\delta-1)}}{\delta}\right]^{\frac{\alpha(1-\alpha)}{\alpha+\delta-1}} & \text{for } K_t < \frac{H}{\delta} \equiv \hat{K} \\ (K_t)^{\alpha}\ \ (H)^{1-\alpha} & \text{for } K_t \geq \hat{K} \end{cases}$$

(d) None of the above.

Answer: (a)

Question 55. The aggregate output in this economy

(a) initially increases until $K_t < \hat{K}$, and then reaches a constant value within finite time when $K_t \geq \hat{K}$.

(b) initially increases (until $K_t < \bar{K}$) and then reaches a constant value within finite time when $K_t \geq \bar{K}$.

(c) keep increasing at a decreasing rate and approaches a constant value only in the long run (when $t \to \infty$).

(d) increases at an increasing rate until $K_t < \hat{K}$; increases at a decreasing rate when $K_t \geq \hat{K}$ and approaches a constant value only in the very long run (when $t \to \infty$).

Answer: (c)

Let us now change the labour supply behaviour of households in the above question. Assume that producers' side of the story remain exactly the same as above. The household side story now changes as follows.

Labour supply is now determined by the following rule. Out of the total endowment of 1 unit of labour, the households optimally supplies l_t^h units so as to maximise his utility, U_t^h: $U_t^h = w_t l_t^h - D; (D > 0)$ for any $l_t^h > 0$; and $U_t^h = 0$ for any $l_t^h = 0$. For the case $l_t^h > 0$, the first term captures the (indirect) utility derived from labour earnings while the constat term D captures the dis-utility of labour - which is now independent of how much labour is supplied.

Rest of the assumptions about household behaviour (their endowment, savings and consumption behaviours; capital augmentation equation) remain unchanged.

Question 56. The new aggregate labour supply schedule by the households is given by the following function:

(a) $L_t^S = \begin{cases} 0 \text{ for } w_t < \underline{w} \equiv \frac{D}{H} \\ H \quad \text{ for } w_t \geq \underline{w} \end{cases}$

(b) $L_t^S = H$ for all values of w_t.

(c) $L_t^S = \begin{cases} 0 \text{ for } w_t < \tilde{w} \equiv D \\ H \quad \text{ for } w_t \geq \tilde{w} \end{cases}$

(d) None of the above.

Answer: (b)

Question 57. The new market clearing wage rate in the short run (period t) is given by:

(a) $w_t^* = \begin{cases} D \text{ for } K_t < H \left[\frac{D}{1-\alpha}\right]^\alpha \equiv \tilde{K} \\ \left[\frac{K_t \ (1-\alpha)^{1/\alpha}}{H}\right]^\alpha \text{ for } K_t \geq \tilde{K} \end{cases}$

(b) $w_t^* = D$ for all values of K_t

(c) $w_t^* = \begin{cases} \left[\frac{K_t \ (1-\alpha)^{1/\alpha}}{H}\right]^{\frac{\alpha(\delta-1)}{\alpha+\delta-1}} \text{ for } K_t < \frac{H}{D} \equiv \hat{K} \\ \left[\frac{K_t \ (1-\alpha)^{1/\alpha}}{H}\right]^\alpha \text{ for for } K_t \geq \hat{K} \end{cases}$

(d) None of the above.

Answer: (a)

Question 58. An increase in the number of firms (N)

(a) leaves the wage rate unchanged in the short run (until $K_t < \tilde{K}$) and increases it thereafter

(b) increases the wage rate in the short run (until $K_t < \hat{K}$) and leaves it unchanged thereafter

(c) leaves the wage rate unchanged irrespective of K_t

(d) None of the above.

Answer: (a)

Question 59. The new equilibrium output in the short run (period t) is given by:

$$(a)\ Y_t^* = \begin{cases} K_t \left[\frac{(1-\alpha)^{1/\alpha}}{D}\right]^{1-\alpha} & \text{for } K_t < H \left[\frac{D}{1-\alpha}\right]^{\alpha} \equiv \tilde{K} \\ (K_t)^{\alpha} \ (H)^{1-\alpha} & \text{for } K_t \geq \tilde{K} \end{cases}$$

(b) $Y_t^* = (K_t)^{\alpha} \ (H)^{1-\alpha}$ for all values of K_t.

$$(c)\ Y_t^* = \begin{cases} K_t \left[\frac{(1-\alpha)^{1/\alpha}}{D}\right]^{1-\alpha} & \text{for for } K_t < \frac{H}{D} \equiv \hat{K} \\ (K_t)^{\alpha} \ (H)^{1-\alpha} & \text{for } K_t \geq \hat{K} \end{cases}$$

(d) None of the above.

Answer: (a)

Question 60. The aggregate output in this economy

(a) initially increases until $K_t < \tilde{K}$, and then reaches a constant value within finite time when $K_t \geq \tilde{K}$.

(b) initially increases (until $K_t < \hat{K}$) and then reaches a constant value within finite time when $K_t \geq \hat{K}$.

(c) increases at a constant rate until $K_t < \tilde{K}$; increases at a decreasing rate when $K_t \geq \tilde{K}$ and approaches a constant value only in the very long run (when $t \to \infty$).

(d) None of the above.

Answer: (c)

End of Part II

Rough Work

Rough Work

Rough Work

Rough Work

Rough Work

Rough Work