ZCO 2015, Afternoon Session

Problem 1 Covering

An *interval* is a pair of positive integers $[a, b]$ with $a \leq b$. It is meant to denote the set of integers that lie between the values a and b. For example $[3, 5]$ denotes the set $\{3, 4, 5\}$ while the interval $[3, 3]$ denotes the set $\{3\}$.

We say that an interval $[a, b]$ is covered by an integer i, if i belongs to the set defined by $[a, b]$. For example interval $[3, 5]$ is covered by 3 and so is the interval $[3, 3]$.

Given a set of intervals I, and a set of integers S we say that I is covered by S if for each interval $[a, b]$ in I there is an integer i in S such that $[a, b]$ is covered by i. For example, the set $\{[3, 5], [3, 3]\}$ is covered by the set $\{3\}$. The set of intervals $\{[6, 9], [3, 5], [4, 8]\}$ is covered by the set $\{4, 5, 8\}$. It is also covered by the set $\{4, 7\}$.

We would like to compute, for any set of intervals I, the size of the smallest set S that covers it. You can check that for the set of intervals $\{[6, 9], [3, 5], [4, 8]\}$ the answer is 2 while for the set of intervals $\{[3, 5], [3, 3]\}$ the answer is 1.

Your program should take the set of intervals as input and output the size of the smallest set that covers it as the answer.

Input format

- The first line contains a single integer N, giving the number of intervals in the input.
- This is followed by N lines, each containing two integers separated by a space describing an interval, with the first integer guaranteed to be less than or equal to the second integer.

Output format

Output a single integer giving the size of the smallest set of integers that covers the given set of intervals.

Test data

You may assume that all integers in the input are in the range 1 to 10^8 inclusive.

Subtask 1 (100 Marks) $1 \leq N \leq 5000$.

Sample input 1

```
2
3 5
3 3
```

Sample output 1

```
1
```

Sample input 2

```
3
6 9
3 5
4 8
```

Sample output 2

```
2
```

Limits

- *Memory limit* : 256MB
- *Time limit* : 2s

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Problem 2 Rectangle

We have a rectangular region that is 100000 units along the X-axis and 500 units along the Y-axis.

We assume that the origin $(0, 0)$ is at the bottom-left corner of this region, so that the top-left corner is at $(0, 500)$, the bottom-right at $(100000, 0)$ and the top-right corner at $(100000, 500)$. We are also given the coordinates of a set of N points inside this region. The points have only integer coordinates and do not appear along the X-axis or Y-axis.

We would like to draw a rectangle, with its base on the X-axis, of maximum area within the region such that it does not contain any of the N points in its interior. More specifically, the points may appear on the boundary but cannot be properly inside the rectangle.

For example, if there are 5 points: $(1, 4), (2, 3), (3, 2), (5, 1)$ and $(5, 2)$. Then the rectangle whose bottom-left and top-right corners are given by $(0, 0)$ and $(2, 3)$ is a possibility and its area is 6. Another possibility is the rectangle with bottom-left and top-right corners at $(3, 0)$ and $(5, 500)$ with area 1000. The rectangle with bottom-left at $(2, 3)$ and top-right at $(100000, 500)$ is not valid since its base does not lie on the X-axis. The largest rectangle that meets the requirements in this case is the one with its bottom-left corner at $(5, 0)$ and top-right at $(100000, 500)$ with area 49997500.

Your program should take a description of the N points and output the size of the maximum rectangle satisfying the above property that can be drawn within the 100000×500 region.

Input format

- The first line contains a single integer N, giving the number of points marked in the region.
- This is followed by N lines, each containing two integers separated by a space describing the coordinates of one point.

Output format

Output a single integer giving the area of the largest rectangle that may be drawn with its base on the X-axis and which does not contain any of the given N points in its interior.

Test data

In both subtasks, the X-coordinate of each of the N points is in the range 1 to 99999 inclusive, and the Y-coordinate of each of the N points is in the range 1 to 499 inclusive.

Subtask 1 (40 Marks) $1 \leq N \leq 5000$.

Subtask 2 (60 Marks) $1 \leq N \leq 100000$.

Sample input

```
5
1 4
2 3
```

```
3 2
5 1
5 2
```

Sample output

```
49997500
```

Limits

- *Memory limit* : 256MB
- *Time limit* : 2s