Instructions

1. The theoretical examination lasts for 5 hours and is worth a total of 300 marks.

2. Dedicated IOAA **Summary Answer Sheets** are provided for writing your answers. Enter the final answers into the appropriate boxes in the corresponding **Summary Answer Sheet**. On each **Summary Answer Sheet**, please fill in
   - Student Code (Country Code and 1 digit)

3. There are **Answer Sheets** for carrying out detailed work/rough work. On each **Answer Sheet**, please fill in
   - Student Code (Country Code and 1 digit)
   - Question no.
   - Page no. and total number of pages.

4. Start each problem on a separate Answer Sheet. Please write only on the printed side of the sheet. Do not use the reverse side. If you have written something on any sheet which you do not want to be evaluated, cross it out.

5. Use as many mathematical expressions as you think may help the evaluator to better understand your solutions. The evaluator may not understand your language. If it is necessary to explain something in words, please use short phrases (if possible in English).

6. You are not allowed to leave your work desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, need more Answer Sheets, etc.), please draw the attention of the proctor using the Help card.

7. The beginning and end of the examination will be indicated by a long sound signal. Additionally, there will be a buzzer sound, fifteen minutes before the end of the examination (before the final sound signal).

8. At the end of the examination you must stop writing immediately. Sort and put your sheets in separate stacks,
   a) Stack 1: Summary Answer Sheets, Answer Sheets of part 1
   b) Stack 2: Summary Answer Sheets, Answer Sheets of part 2
   c) Stack 3: Summary Answer Sheets, Answer Sheets of part 3
   d) Stack 4: question papers and paper sheets you do not want to be graded.

9. Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the examination room.

10. A list of constants and a table of the mark distribution for this exam are given on the next two pages.

Table of constants

Mass ($M_\oplus$)	5.98×10^{24} kg	**Earth**
Radius ($R_\oplus$)	6.38×10^{6} m	
Acceleration of gravity (g)	$9.8\ \mathrm{m \cdot s^{-1}}$	
Obliquity of Ecliptic	23°27’	
Length of Tropical Year	365.2422 mean solar days	
Length of Sidereal Year	365.2564 mean solar days	
Albedo	0.39	
Mass($M_☾$)	7.35×10^{22} kg	**Moon**
Radius ($R_☾$)	1.74×10^{6} m	
Mean distance from Earth	3.84×10^{8} m	
Orbital inclination with the Ecliptic	5.14°	
Albedo	0.14	
Apparent magnitude (mean full moon)	-12.74	
Mass ($M_\odot$)	1.99×10^{30} kg	**Sun**
Radius ($R_\odot$)	6.96×10^{8} m	
Luminosity ($L_\odot$)	3.83×10^{26} W	
Absolute Magnitude ($\mathcal{M}_\odot$)	4.80 mag	
Angular diameter	0.5 degrees	
Rotational velocity in the Galaxy	$220\ \mathrm{km\ s^{-1}}$	
Distance from Galactic centre	8.5 kpc	
Mass	1.89×10^{27} kg	**Jupiter**
Orbital semi-major axis	5.20 au	
Orbital period	11.86 year	
Mass	5.68×10^{26} kg	**Saturn**
Orbital semi-major axis	9.58 au	
Orbital period	29.45 year	
1 au	1.50×10^{11} m	**Physical constants**
1 pc	206 265 au	
Gravitational constant (G)	$6.67 \times 10^{-11}\ \mathrm{N \cdot m^2 \cdot kg^{-2}}$	
Planck constant (h)	$6.62 \times 10^{-34}\ \mathrm{J \cdot s}$	
Boltzmann constant (k_B)	$1.38 \times 10^{-23}\ \mathrm{J \cdot K^{-1}}$	
Stefan-Boltzmann constant (σ)	$5.67 \times 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$	
Hubble constant (H_0)	$67.8\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$	
Speed of light in vacuum (c)	$299\,792\,458\ \mathrm{m \cdot s^{-1}}$	
Proton mass	$938.27\ \mathrm{MeV \cdot c^{-2}}$	
Deuterium mass	$1875.60\ \mathrm{MeV \cdot c^{-2}}$	
Neutron mass	$939.56\ \mathrm{MeV \cdot c^{-2}}$	
Helium-3 mass	$2808.30\ \mathrm{MeV \cdot c^{-2}}$	
Helium-4 mass	$3727.40\ \mathrm{MeV \cdot c^{-2}}$	

Mark distribution of this exam

Problem number	Marks
T1	10
T2	10
T3	10
T4	10
T5	10
T6	15
T7	20
T8	20
T9	20
T10	25
T11	50
T12	40
T13	60
Total	300

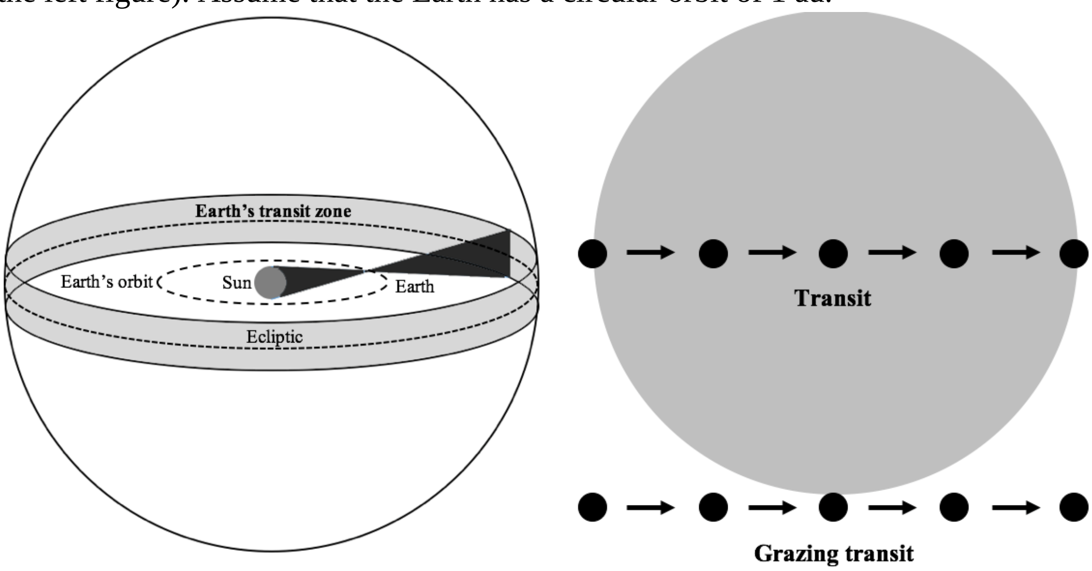
Part 1

(T1) The Large Magellanic Cloud in Phuket **[10 marks]**

The coordinates of the Large Magellanic Cloud (LMC) are R.A. = 5h 24min and Dec = $-70^\circ 00'$. The latitude and longitude of Phuket are $7^\circ 53'$ N and $98^\circ 24'$ E, respectively. What is the date when the LMC culminates at 9pm as seen from Phuket in the same year? You may note that the Greenwich Sidereal Time, GST, at 00h UT 1st January is about 6h 43min, and Phuket is in the UT+7 time zone. [10]

(T2) Earth's Transit Zone **[10 marks]**

Earth's transit zone is an area where extrasolar observers (located far away from the Solar System) can detect the Earth transiting across the Sun. For observers on the Earth, this area is the projection of a band around the Earth's ecliptic onto the celestial plane (light grey area in the left figure). Assume that the Earth has a circular orbit of 1 au.



a) Find the angular width of that part of the Earth's transit zone in degrees, in which the extrasolar observers can detect Earth's total transit (when the whole of the Earth's disk passes in front of the Sun). [5]

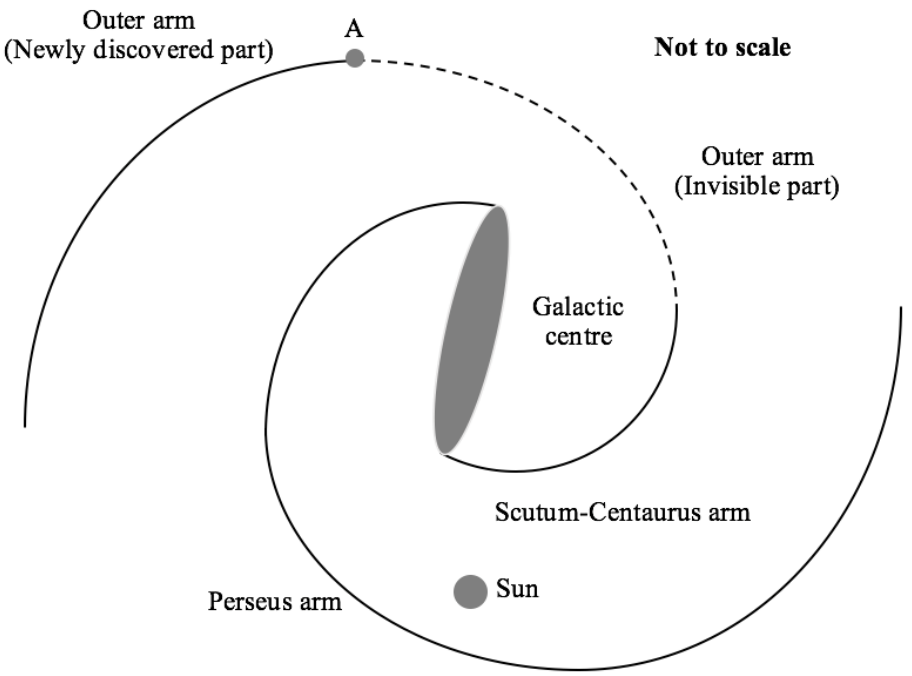
b) Find the angular width of that part of the Earth's transit zone in degrees, where the extrasolar observers can detect at least Earth's grazing transit (when any part of the Earth's disk passes in front of the Sun). [5]

(T3) The Milky Way's Distant Outer Arm **[10 marks]**

In 2011, Dame and Thaddeus found a new part of the outer arm of the Milky Way by studying the CO line using the CfA 1.2m telescope. They found that the CO line was detected at galactic longitude $\ell = 13.25^\circ$ (marked **A** in the figure) where it had a radial velocity of 20.9 km s^{-1} towards the Sun. Assume that the galactic rotation curve is flat beyond 5 kpc from the Galactic centre. The distance between the Sun and the Galactic centre is 8.5 kpc. The velocity of the Sun around the Galactic centre is 220 km s^{-1}.

a) Find the distance from the start of the arm (point **A**) to the Galactic centre. [7]

b) Find the distance from the start of the arm (point **A**) to the Sun. [3]



(T4) 21-cm HI galaxy survey **[10 marks]**

A radio telescope is equipped with a receiver which can observe in a frequency range from 1.32 to 1.52 GHz. Its detection limit is 0.5 mJy per beam for a 1-minute integration time. In a galaxy survey, the luminosity of the HI spectral line of a typical target galaxy is 10^{28} W with a linewidth of 1 MHz. For a large beam, the HI emitting region from a far-away galaxy can be approximated as a point source. The HI spin-flip spectral line has a rest-frame frequency of 1.42 GHz.

What is the highest redshift, z, of a typical HI galaxy that can be detected by a survey carried out with this radio telescope, using 1-minute integration time? You may assume in your calculation that the redshift is small and the non-relativistic approximation can be used. Note that 1 Jy = 10^{-26} W m^{-2} Hz^{-1}. [10]

(T5) A Synchronous Satellite **[10 marks]**

A synchronous satellite is a satellite which orbits the Earth with its period exactly equal to the period of rotation of the Earth. The height of these satellites is 35786 km above the surface of the Earth. A satellite is put in an inclined synchronous orbit with an inclination of $\theta = 6.69°$ to the equatorial plane. Calculate the precise value of the maximum possible altitude of the satellite for an observer at latitude of $\phi = 51.49°$. Ignore the effect of refraction due to the Earth's atmosphere. [10]

Part 2

(T6) Supernova 1987A **[15 marks]**

Supernova SN 1987A was at its brightest with an apparent magnitude of +3 on about 15th May 1987 and then faded, finally becoming invisible to the naked eye by 4th February 1988. It is assumed that brightness B varied with time t as an exponential decline, $B = B_0 e^{-t/\tau}$, where B_0 and τ are constant. The maximum apparent magnitude which can be seen by the naked eye is +6.

a) Determine the value of τ in days. [5]

b) Find the last day that observers could have seen the supernova if they had a 6-inch (15.24-cm) telescope with transmission efficiency $T = 70\%$. Assume that the average diameter of the human pupil is 0.6 cm. [10]

(T7) Life on Other Planets **[20 marks]**

One place to search for life is on planets orbiting main sequence stars. A good starting point is the planets that have an Earth-like temperature range and a small temperature fluctuation. Assume that for a main sequence star, the relation between the luminosity L and the mass M is given by

$$L \propto M^{3.5}.$$

You may assume that the total energy E released over the lifetime of the star is proportional to the mass M of the star. For the Sun, it will have a main sequence lifetime of about 10 billion years. The stellar spectral types are given in the table below. Assume that the spectral subclasses of stars (0-9) are assigned on a scale that is linear in $\log M$.

Spectral Class	O5V	B0V	A0V	F0V	G0V	K0V	M0V
Mass ($M_\odot$)	60	17.5	2.9	1.6	1.05	0.79	0.51

a) If it takes at least 4×10^9 years for an intelligent life form to evolve, what is the spectral type (accurate to the subclass level) of the most massive star in the main sequence around which astronomers should look for intelligent life? [6]

b) Assume that the target planet has the same emissivity ε and albedo a as the Earth. In order to have the same temperature as the Earth, express the distance d, in au, of the planet to its parent main sequence star, of mass M. [6]

c) The existence of a planet around a star can be shown by the variation in the radial velocity of the star about the star-planet system centre of mass. If the smallest Doppler shift in the wavelength detectable by the observer is $(\Delta\lambda/\lambda) = 10^{-10}$, calculate the lowest mass of such a planet in b), in units of Earth masses, that can be detected by this method, around the main sequence star in a). [8]

(T8) The Star of Bethlehem **[20 marks]**

A great conjunction is a conjunction of Jupiter and Saturn for observers on Earth. Assume that Jupiter and Saturn have circular orbits in the ecliptic plane.

The time between successive conjunctions may vary slightly as viewed from the Earth. However, the average time period of the great conjunctions is the same as that of an observer at the centre of the Solar system.

a) Find the average great conjunction period (in years) and average heliocentric angle between two successive great conjunctions (in degrees). [6]

b) The next great conjunction will be on 21st December 2020 with an elongation of 30.3° East of the Sun. Suggest the constellation in which the conjunction on 21st December 2020 will occur. (Give the IAU Latin name or IAU three-letter abbreviation of the constellation, i.e. Ursa Major or UMa) [2]

In 1606, Johannes Kepler determined that in some years the great conjunction can be happen three times in the same year due to the retrograde motions of the planets. He also determined that such an event happened in the year 7 BC, which could have been the event commonly known as "The Star of Bethlehem". For the calculations below you may ignore the precession of the axis of the Earth.

c) Suggest the constellation in which the great conjunctions in 7 BC occurred. (Give the IAU Latin name or IAU three-letter abbreviation of the constellation, i.e. Ursa Major or UMa) [8]

d) During the second conjunction of the series of three conjunctions in 7 BC, suggest the constellation the Sun was in as viewed by the observer on Earth. (Give the IAU Latin name or IAU three-letter abbreviation of the constellation, i.e. Ursa Major or UMa) [4]

(T9) Galactic Outflow **[20 marks]**

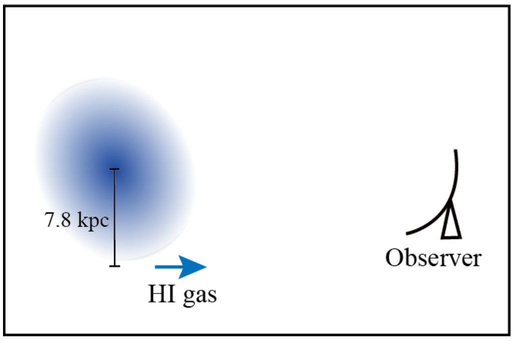
Cannon et al. (2004) conducted an HI observation of a disk starburst galaxy, IRAS 0833+6517, with the Very Large Array (VLA). The galaxy is located at a distance of 80.2 Mpc with an approximate inclination angle of 23 degrees. According to the HI velocity map, IRAS 0833+6517 appears to be undergoing regular rotation with an observed radial velocity of the HI gas of roughly 5850 km s^{-1} at a distance of 7.8 kpc from the centre (the left panel of the figure below).

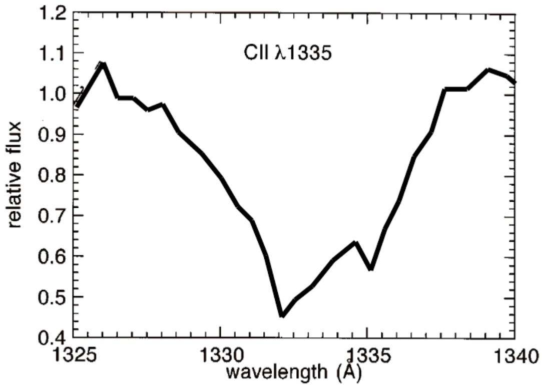
Gas outflow from IRAS 0833+6517 is traced by using the blueshifted interstellar absorption lines observed against the backlight of the stellar continuum (the right panel of the figure). Assuming that this galaxy is gravitationally stable and all the stars are moving in circular orbits,

a) Determine the rotational velocity (v_{rot}) of IRAS 0833+6517 at the observed radius of HI gas. [5]

b) Calculate the escape velocity for a test particle in the gas outflow at the radius of 7.8 kpc. [9]

c) Determine if the outflowing gas can escape from the galaxy at this radius by considering the velocity offset of the C II λ1335 absorption line, which is already corrected for the cosmological recessional velocity. (The central rest-frame wavelength of the CII absorption line is 1335 Å.) (YES / NO) [6]





(T10) GOTO **[25 marks]**

The Gravitational-Wave Optical Transient Observer (GOTO) aims to carry out searches of optical counterparts of any Gravitational Wave (GW) sources within an hour of their detection by the LIGO and VIRGO experiments. The survey needs to cover a big area on the sky in a short time to search all possible regions constrained by the GW experiments before the optical burst signal, if any, fades away. The GOTO telescope array is composed of 4 identical reflective telescopes, each with 40-cm diameter aperture and f-ratio of 2.5, working together to image large regions of the sky. For simplicity, we assume that the telescopes' fields-of-view (FoV) do not overlap with one another.

a) Calculate the projected angular size per mm at the focal plane, i.e. plate scale, of each telescope. [6]

b) If the zero-point magnitude (i.e. the magnitude at which the count rate detected by the detector is 1 count per second) of the telescope system is 18.5 mag, calculate the minimum time needed to reach 21 mag at Signal-to-Noise Ratio (SNR) = 5 for a point source. We first assume that the noise is dominated by both the Read-Out Noise (RON) at 10 counts/pixel and the CCD dark (thermal) noise (DN) rate of 1 count/pix/minute. The CCDs used with the GOTO have a 6-micron pixel size and gain (conversion factor between photo-electron and data count) of 1. The typical seeing at the observatory site is around 1.0 arcsec. [8]

The Signal-to-Noise Ratio is defined as

$$\mathrm{SNR} \equiv \frac{\text{Total Source Count}}{\sqrt{\Sigma_i \mathrm{Noise}_i^2}} = \frac{\text{Total Source Count}}{\sqrt{\sigma_{\mathrm{RON}}^2 + \sigma_{\mathrm{DN}}^2 + \ldots}},$$

$$\sigma_{\mathrm{RON}} = \sqrt{N_{\mathrm{pix}} \cdot \mathrm{RON}^2}, \quad \sigma_{\mathrm{DN}} = \sqrt{N_{\mathrm{pix}} \cdot \mathrm{DN} \cdot t},$$

where t is the exposure time.

c) Normally when the exposure time is long and the source count is high then Poisson noise from the source is also significant. Determine the relation between SNR and exposure time in the case that the noise is dominated by Poisson noise of the source. Recalculate the minimum exposure time required to reach 21 mag with SNR = 5 from part b) if Poisson noise is also taken into consideration. The Poisson noise (standard deviation) of the source is given by $\sigma_{\text{source}} = \sqrt{\text{Source Count}}$. In reality, there is also the sky background which can be important source of Poisson noise. For our purpose here, please ignore any sky background in the calculation. [6]

d) The typical localisation uncertainty of the GW detector is about 100 square-degrees and we would like to cover the entire possible location of any candidate within an hour after the GW is detected. Estimate the minimum side length of the square CCD needed for each telescope in terms of the number of pixels. You may assume that the time taken for the CCD read-out and the pointing change are negligible. [5]

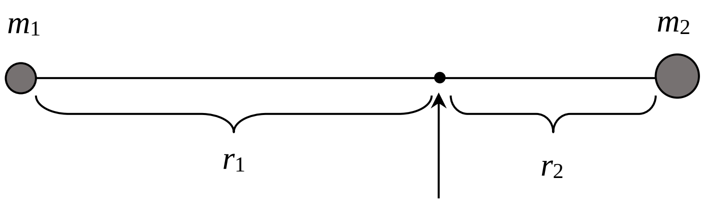
Part 3

(T11) Mass of the Local Group **[50 marks]**

The dynamics of M31 (Andromeda) and the Milky Way (MW) can be used to estimate the total mass of the Local Group (LG). The basic idea is that galaxies currently in a binary system were at approximately the same point in space shortly after the Big Bang. To a reasonable approximation, the mass of the local group is dominated by the masses of the MW and M31. Via Doppler shifts of the spectral lines, it was found that M31 is moving towards the MW with a speed of 118 km s^{-1}. This may be surprising, given that most galaxies are moving away from each other with the general Hubble flow. The fact that M31 is moving towards the MW is presumably because their mutual gravitational attraction has eventually reversed their initial velocities. In principle, if the pair of galaxies is well-represented by isolated point masses, their total mass may be determined by measuring their separation, relative velocity and the time since the universe began. Kahn and Woltjer (1959) used this argument to estimate the mass in the LG.

In this problem we will follow this argument through our calculation as follows.

a) Consider an isolated system with negligible angular momentum of two gravitating point masses m_1 and m_2 (as observed by an inertial observer at the centre of mass).



Write down the expression of the total mechanical energy (E) of this system in mathematical form connecting m_1, m_2, r_1, r_2, v_1, v_2, and the universal gravitational constant G, where v_1 and v_2 are the radial velocities of m_1 and m_2, respectively.

[5]

b) Re-write the equation in a) in terms of r, v, μ, M, and G, where $r \equiv r_1 + r_2$ is the separation distance between m_1 and m_2, v is the changing rate of the separation distance, $\mu \equiv \dfrac{m_1 m_2}{m_1 + m_2}$ is the reduced mass of the system, and $M \equiv m_1 + m_2$ is the total mass of the system.

[10]

c) Show that the equation in b) yields

$$v^2 = (2GM)\left(\frac{1}{r} - \frac{1}{r_0}\right), \text{ where } r_0 \text{ is a new constant.}$$

Find r_0 in terms of μ, M, G and E.

[5]

The solution of the equation in b) is given below in parametric form, under the initial condition $r = 0$ at $t = 0$:

$$r(\theta) = \frac{r_0}{2}(1 - \cos\theta),$$

$$t(\theta) = \left(\frac{r_0^3}{8GM}\right)^{\frac{1}{2}}(\theta - \sin\theta),$$

where θ is in radians.

d) From the above parametric equations, show that an expression for $\frac{vt}{r}$ is

$$\frac{vt}{r} = \frac{(\sin\theta)(\theta - \sin\theta)}{(1-\cos\theta)^2}$$ [10]

e) Now we consider m_1 and m_2 as the MW and M31, respectively, such that the current values of v and r are $v = -118$ km s^{-1} and $r = 710$ kpc, and t may be taken to be the age of the Universe (13700 million years). Find θ using numerical iteration. [10]

f) Use the value of θ from e) to calculate the maximum distance between M31 and the MW, $r_{\max}$, and hence also obtain the value of M in solar masses. [10]

(T12) Shipwreck [40 marks]

You are shipwrecked on an island. Fortunately, you are still wearing a watch that is set to Bangkok time, and you also have a compass, an atlas and a calculator. You are initially unconscious, but wake up to find it has recently become dark. Unfortunately, it is cloudy. An hour or so later you see Orion through a gap in the clouds. You estimate that the star "Rigel" is about 52.5° above the horizon and with your compass you find that it has an astronomical azimuth of 109°. Your watch says it is currently 01:00 on the 21st November 2017. You happen to remember from your astronomy class that Greenwich Sidereal Time (GST) at 00h UT 1st January 2017 is about 6h 43min and that the R.A. and Dec of Rigel are 5h 15min and $-8^\circ\ 11'$, respectively. Bangkok is in the UT+7 time zone.

a) Find the Local Hour Angle (LHA) of Rigel. [10]
b) Find the current Greenwich sidereal time (GST). [10]
c) Find the longitude of the island. [5]
d) Find, to the nearest arcminute, the Latitude of the island. [15]

(T13) Exomoon [60 marks]

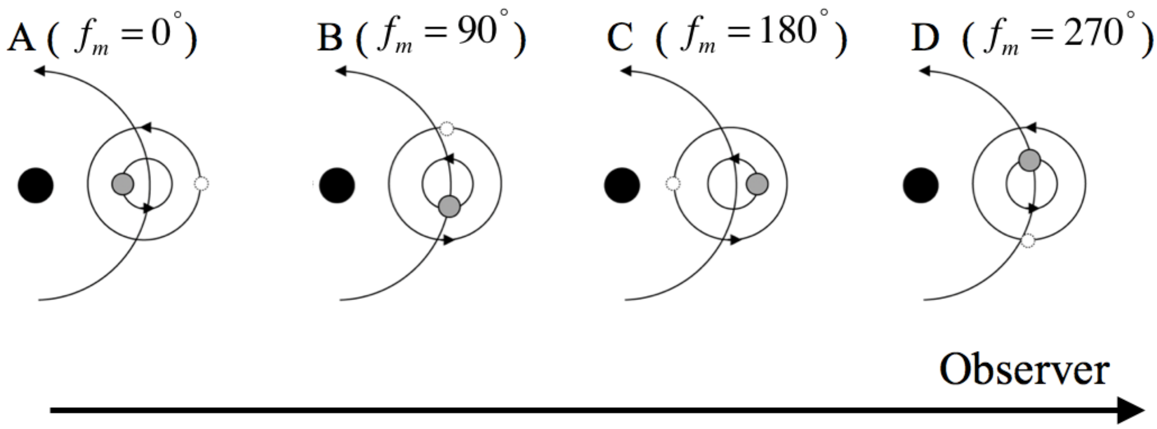
Exomoons are natural satellites of exoplanets. The gravitational influence of such a moon will affect the position of the planet relative to the planet-moon barycentre, resulting in Transit Timing Variations (σ_{TTV}, TTVs) as the observed transit of the planet occurs earlier or later than the predicted time of transit for a planet without a moon.

The motion of the planet around the planet-moon barycentre will also induce Transit Duration Variations (σ_{TDV}, TDVs) as the observed transit duration is shorter or longer than the predicted transit duration for a planet without a moon.

We will consider edge-on circular orbits with the following parameters

- M_P is the planet mass
- M_m is the moon mass
- P_P is the planet-moon barycentre's period around the host star
- P_m is the moon's period around the planet
- a_p is the distance of the planet-moon barycentre to the star
- a_m is the distance of the moon to the planet-moon barycentre
- f_m is the moon phase, $f_m = 0^\circ$ when the moon is in opposition to the star
- τ is the mean transit duration of the planet (as if it has no moon)

We will only consider the orbit of a prograde moon orbiting in the same plane as the planet's orbit. Example phases of the moon, as observed by distant observers, are shown in the figure below.



Phase of the moon.
Black, grey and white circles represent the star, planet and moon, respectively.

a) We define $\sigma_{TTV} \equiv t_m - t$ where t is the predicted transit time without the moon, and t_m is the observed transit time with the moon. Show that

$$\sigma_{TTV} = \left[\frac{a_m M_m P_p}{2\pi a_p M_p}\right]\sin(f_m)$$

A positive value of σ_{TTV} indicates that the transit occurs later than the predicted time of transit for a planet without a moon. [10]

b) Similarly, we define $\sigma_{TDV} \equiv \tau_m - \tau$ where τ is the predicted transit duration without the moon, and τ_m is the observed transit duration with the moon. We can assume that the planet's velocity around the star is much bigger than the moon's velocity around the planet-moon barycentre, and also the moon does not change phase during the transit. Show that

$$\sigma_{TDV} = \tau\left[\frac{P_p M_m a_m}{P_m M_p a_p}\right]\cos(f_m)$$

A positive value of σ_{TDV} indicates that the transit duration is longer than the predicted transit duration without a moon. [13]

An exoplanet is observed transiting a main-sequence solar-type star (1 $M_\odot$, 1 $R_\odot$, spectral class: G2V). The planet has an edge-on circular orbit with a period of 3.50 days. From the observational data, the planet has a mass of 120 $M_\oplus$ and a radius of 12 $R_\oplus$. The observed relation between σ^2_{TTV} and σ^2_{TDV} can be written as

$$\sigma^2_{TDV} = -0.7432\sigma^2_{TTV} + 1.933\times10^{-8}\ \text{days}^2$$

c) Assume that the moon's mass is much smaller than the planet's mass. Find the mean transit duration of the planet (τ) in days. [6]

d) Find the moon's period (P_m) in days [7]

e) Estimate the distance of the moon to the planet-moon barycentre (a_m) in units of Earth radii. Also find the moon mass (M_m) in units of Earth mass. [7]

f) The Hill sphere is a region around a planet within which the planet's gravity dominates. The radius of the Hill sphere can be written as

$$R_h = a_p \sqrt[3]{\frac{M_p}{xM_*}}$$

where M_* is the host star mass.

Find the value of the constant x (Hint: for a massive host star, the radius of the Hill sphere of the system is approximately equal to the distance between the planet and the Lagrange point L_1 or L_2). Hence, find the radius of the Hill sphere of this planetary system in units of Earth radii. [11]

g) The Roche limit is the minimum orbital radius at which a satellite can orbit without being torn apart by tidal forces. Take the Roche limit as

$$R_r = 1.26 R_p \sqrt[3]{\frac{\rho_p}{\rho_m}}$$

where ρ_p and ρ_m are the density of the planet and moon, respectively and R_p is the planet's radius. Assuming that the moon is a rocky moon with the same density as the Earth, find the Roche limit of the system. [3]

h) Does the moon have a stable orbit? (YES / NO) [3]