Ist Amity International Olympiad

INDIA - 2012

Theoretical Competition

22nd May, 2012

Please read this first:

1. The time available for the theoretical competition is 3 hours. There are three questions.

2. Use only the pen provided.

3. Use of calculator, mobile or any electronic items is not allowed.

4. You are provided with ***Writing sheet*** and additional paper. You can use the additional paper for drafts of your solutions but these papers will not be checked. Your final solutions which will be evaluated should be on the ***Writing sheets.*** Please use as little text as possible. You should mostly use equations, numbers, figures and plots.

5. Use only the front side of ***Writing sheets.*** Write only inside the boxed area.

6. Begin each question on a separate sheet of paper.

7. Fill the boxes at the top of each sheet of paper with your country (**Country)**, your student code (**Student Code**), the question number (**Question Number**), the progressive number of each sheet (**Page Number**), and the total number of **Writing Sheets** used (**Total Number of Pages**). If you use some blank ***Writing sheets*** for notes that you do not wish to be evaluated, put a large X across the entire sheet and do not include it in your numbering.

8. At the end of the exam, place the papers inside the envelope and leave everything on your desk. You are not allowed to take any paper out of the room.

Figure 1 shows schematically the principle of operation of a *magnetohydrodynamic*, or *MHD generator*. The function of an MHD generator is to transform the kinetic energy of a hot gas directly into electric energy.

A very hot gas is injected on the left at a high velocity *v*. The gas is made conducting by injecting a salt such as K_2CO_3 that ionizes readily at high temperature, forming positive ions and free electrons. Conductivities of the order of 100 siemens per meter are thus achieved, at temperatures of about 3000 kelvins. (The conductivity of copper is 5.8×10^7 siemens per meter). Take $v^2 << c^2$.

These ions and the electrons are deflected in the magnetic field. With the B shown, the positive ions are deflected downward and the electrons upward. The resulting current flows through a load resistance R, giving a voltage difference V, and hence an electric field *E* between the plates. One such MHD generator is planned to have a power output in the 500 megawatt range. It would use a B of several teslas over a region 20 meters long and 3 meters in diameter.

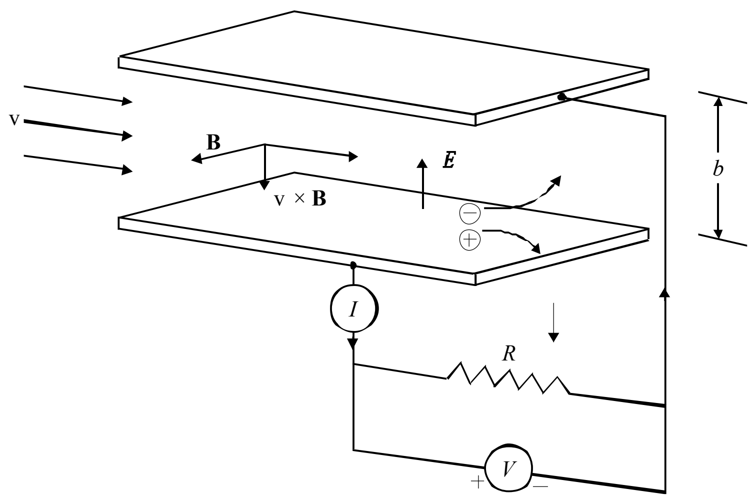


Fig. 1

Problem 1(a) (5 points)

Let us suppose that **E, B**, and the velocity **v** are uniform inside the chamber. These are rather crude assumptions. In particular, **v** is not uniform because (a) the ions and electrons have a vertical component of velocity, and (b) the vertical velocities give $qv \times$ B forces that point backward, for both types of particle; these braking forces slow down the horizontal drift of the charged particles. It is through this deceleration that part of the initial kinetic energy of the gas is transformed into electric energy.

The electrodes each have a surface area *A* and are separated by a distance *b*. Show that

$$I = \frac{V'}{R + R_i},$$

where R_i is the internal resistance of the generator, and V′ is the value of V when R is infinite. Thévenin’s theorem therefore applies here. **[5]**

Note that, because $v^2 << c^2$, the current density is the same in the reference frame of the laboratory as it is in the reference frame of the moving gas.

Problem 1(b) (5 points)

Find an expression for the efficiency

$$\varepsilon = \frac{\text{power dissipated in } R}{\text{total power dissipated}}, \qquad \textbf{[2]}$$

What is the efficiency at $I = 0$? **[0.5]**

What is the value of the current when the efficiency is zero? **[0.5]**

Sketch a curve of ε as a function of I, and output voltage as a function of I **[2]**

Problem 2(a) (6 points)

Underneath the topmost, homogenous covering rock layer which covers the ground and has a horizontal surface, there is another inclined rock plate of different density and composition. The seismic waves generated by the explosion on the surface of the ground are detected at three different places with the help of geophones. The first geophone is at the place of explosion and it detects the reflected seismic wave 0.2 sec after the explosion. The second geophone is at 50 m east, the third is at 50 m west from the place of explosion. The second geophone detected the reflected waves with a time delay of 0.26 seconds, while the third seismic detector measured a delay of 0.34 sec. (consider inclination in east west direction).

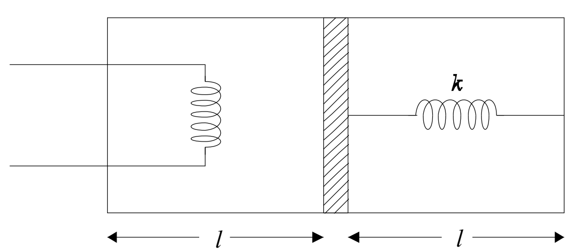
i) What could be the propagation speed of the seismic waves in the top most, covering rock layer? **[04]**

ii) How far is the inclined rock plate from the place of explosion? (shortest distance of rock from point of explosion). **[01]**

iii) What is the angle of inclination of the rock plate in east – west direction? **[01]**

Problem 2(b) (4 points)

A non-conducting cylinder of length $2l$ and filled with monoatomic gas is divided into two equal parts by a piston which is perfectly conducting, massless and it can move freely. Natural length of spring is $\frac{3l}{4}$.Initially piston is in equilibrium.



Left part of cylinder is given heat at constant rate Q J/sec. Find temperature of gas in right part as function of time. Take area of cross-section of piston A, initial pressure and temperature in left part in P_0 and T_0 respectively. (Displacement of piston is small compare to l during the process). Take spring constant k, **[04]**

Problem 3 (4 points) Stability of Circular Orbits

The orbit is stable circular if the total energy equals the minimum value of the effective potential energy, $E = V_{min}$. More generally, however, a circular orbit is allowed for *any* attractive potential, because the attractive force can *always* be made to just balance the centrifugal force by the proper choice of radial velocity. Although circular orbits are therefore always possible in a central, attractive force field, such orbits are not necessarily stable. A circular orbit at $r = \rho$ exists if $\left.\frac{\partial V}{\partial r}\right|_{r=\rho} = 0$ for all t; this is possible if $(\partial V / \partial r)|_{r=\rho} = 0$. But only the effect potential energy has a *true minimum* for stability result. All other equilibrium circular orbits are unstable.

Given an attractive central force with the form

$$F(r) = -\frac{k}{r^n}$$

a) Find the value of n for stability of orbit. **[4]**

b) If satellite is displaced slightly along radius find angular frequency of oscillations. Also discuss the condition of stability using the concept of oscillations. **[6]**