Ist Amity International Olympiad

INDIA - 2012

Mathematical Competition

22nd May, 2012

Please read this first:

1. The time available for the Mathematical Competition is 3 hours. There are four questions.
2. Use only the pen provided.
3. Use of calculator, mobile or any electronic items is not allowed.
4. You are provided with ***Writing sheet*** and additional paper. You can use the additional paper for drafts of your solutions but these papers will not be checked. Your final solutions which will be evaluated should be on the ***Writing sheets.*** Please use as little text as possible. You should mostly use equations, numbers, figures and plots.
5. Use only the front side of ***Writing sheets.*** Write only inside the boxed area.
6. Begin each question on a separate sheet of paper.
7. Fill the boxes at the top of each sheet of paper with your country (**Country)**, your student code (**Student Code**), the question number (**Question Number**), the progressive number of each sheet (**Page Number**), and the total number of **Writing Sheets** used (**Total Number of Pages**). If you use some blank ***Writing sheets*** for notes that you do not wish to be evaluated, put a large X across the entire sheet and do not include it in your numbering.
8. At the end of the exam, place the papers inside the envelope and leave everything on your desk. You are not allowed to take any paper out of the room.

Problem 1 (7 points)

Let $\phi(n)$ denote the number of positive integers less than or equal to n and relatively prime to n. Find the value of k such that $\phi(18522) = \phi(1800) + k$. Results required to be used while solving this problem should be proved.

Problem 2 (7 points)

In a triangle ABC right angled at C, E is the mid-point of AC, BD bisects $\angle$ABC and BE bisects $\angle$ABD. Prove that

$$\frac{5}{2} < (AB/BC) < 3.$$

Problem 3 (7 points)

Let a and b be natural numbers such that $\frac{b}{a} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots - \frac{1}{1570} + \frac{1}{1571}$

Prove that 2357 divides b.

Problem 4 (7 points)

Let m and n be positive integers such that the equation $x^2 - mx + n = 0$ has real roots α and β. Prove that α and β are integers if and only if $[m\alpha] + [m\beta]$ is the square of an integer. (Here $[x]$ denotes the largest integer not exceeding x).