INTERNATIONAL Olympiad of Mathematics

Presented by SILVER ZONE
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SILVER ZONE FOUNDATION

Sample Paper

CLASS - 12 Duration : 60 Minutes Total Questions : 40 Maximum Marks :100

MATHEMATICS

iOM Roll Number

Student's Name

INSTRUCTIONS

1. **Write your 12 digit iOM roll number and your name on top of the question paper in the given space.**
2. **Filling up improper roll number may lead to unavailability of 'Result'.**
3. **This question paper consist of 40 questions. Each question carries equal marking of 2.5 marks each.**
4. **Mark your answer (A, B, C, D or E) on the Answer Sheet with HB Pencil or Black/Blue Ball point Pen.**
5. **This question paper contains 4 pages.**
6. **Do not start attempting the test paper until you are asked to do so.**

Note: Return this question paper along with answer sheet

1. If A = $\{(x, y) : x^2 + y^2 = 25\}$ and B = $\{(x, y) : x^2+9y^2 = 144\}$, then $A \cap B$ contains

(A) One point (B) Three points
(C) Two points (D) Four points
(E) None of these

2. On the set N of all natural numbers define the relation R by aRb if and only if the G.C.D. of a and b is 2, then R is

(A) reflexive, but not symmetric
(B) symmetric only
(C) reflexive and transitive
(D) reflexive, symmetric and transitive
(E) None of these

3. The value of $(A \cup B \cup C) \cap (A \cap B^c \cap C^c)^c \cap C^c$, is

(A) $B \cap C^c$ (B) $B^c \cap C^c$
(C) $B \cap C$ (D) $A \cap B \cap C$
(E) None of these

4. The rational number, which equals the number $2.\overline{357}$ with recurring decimal is

(A) $\frac{2355}{1001}$ (B) $\frac{2355}{888}$
(C) $\frac{2355}{999}$ (D) $\frac{2350}{888}$
(E) None of these

5. Sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+$............... is equal to

(A) $2^n - n - 1$ (B) $1 - 2^{-n}$
(C) $n + 2^{-n} - 1$ (D) 2^{n+1}
(E) None of these

6. Suppose a, b, c are in A. P and a^2, b^2, c^2 are in G.P. If a< b< c and a + b + c = $\frac{3}{2}$, then the value of a is

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$
(C) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}-\frac{1}{\sqrt{2}}$
(E) None of these

7. The equation of the circle passing through (1, 1) and the points of intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ is

(A) $4x^2 + 4y^2 - 30x - 10y - 25 = 0$
(B) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$
(C) $4x^2 + 4y^2 - 17x - 10y + 25 = 0$
(D) All of these
(E) None of these

8. **If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$, $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is.**

(A) $2 \text{ or } -\frac{3}{2}$ (B) $-2 \text{ or } -\frac{3}{2}$

(C) $2 \text{ or } \frac{3}{2}$ (D) $-2 \text{ or } \frac{3}{2}$

(E) None of these

9. **If A and B are two events such that P(A) > 0, and P(B) $\neq 1$, then $P\left(\frac{\overline{A}}{\overline{B}}\right)$ is equal to**

(A) $1 - P\left(\frac{A}{B}\right)$ (B) $1 - P\left(\frac{\overline{A}}{B}\right)$

(C) $\frac{1 - P(A \cup B)}{P(\overline{B})}$ (D) $\frac{P(\overline{A})}{P(\overline{B})}$

(E) None of these

10. **Let A, B, C be three mutually independent events. Consider the two statements S_1 and S_2**
S_1 : A and $B \cup C$ are independent
S_2 : A and $B \cap C$ are independent
Then,

(A) Both S_1 and S_2 are true
(B) Only S_1 is true
(C) Only S_2 is true
(D) Neither S_1 nor S_2 is true
(E) None of these

11. **If f(x) =**

$$\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$$

then f(100) is equal to

(A) 0 (B) 1
(C) 100 (D) –100
(E) None of these

12. **If A and B are square matrices of equal degree, then which one is correct among the following?**

(A) A + B = B + A
(B) A + B = A – B
(C) A – B = B – A
(D) AB = BA
(E) None of these

13. **If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then**

(A) $\alpha = a^2 + b^2$, $\beta = ab$
(B) $\alpha = 2ab$, $\beta = a^2 + b^2$
(C) $\alpha = a^2 + b^2$, $\beta = a^2 - b^2$
(D) $\alpha = a^2 + b^2$, $\beta = 2ab$
(E) None of these

14. **Let f : R $\to$ R be any function. Define g : R $\to$ R by g(x) = |f(x)| for all x. Then g is**

(A) onto if f is onto
(B) one-one if f is one-one
(C) continuous if f is continuous
(D) differentiable if f is differentiable.
(E) None of these

15. **Let E = {1, 2, 3, 4} and F = {1, 2}. Then the number of onto functions from E to F is**

(A) 14 (B) 16
(C) 12 (D) 8
(E) None of these

16. **If $y = (\sin x)^{\tan x}$, then $\frac{dy}{dx}$ is equal to**

(A) $(\sin x)^{\tan x} (1 + \sec^2 x \log \sin x)$
(B) $\tan x (\sin x)^{\tan x - 1} \cos x$
(C) $(\sin x)^{\tan x} \sec^2 x \log \sin x$
(D) $\tan x (\sin x)^{\tan x - 1}$
(E) None of these

17. **Let f : $(0, \infty) \to R$ and $F(x) = \int_0^x f(t)dt$.**

If $F(x^2) = x^2(1 + x)$, then f(4) equals

(A) $\frac{5}{4}$ (B) 7
(C) 4 (D) 2
(E) None of these

18. **If $x^2 + y^2 = 1$, then**

(A) $yy'' - 2(y')^2 + 1 = 0$
(B) $yy'' + (y')^2 + 1 = 0$
(C) $yy'' + (y')^2 - 1 = 0$
(D) $yy'' + 2(y')^2 + 1 = 0$
(E) None of these

19. **The value of the integral $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx$ is**

(A) $\sin x - 6 \tan^{-1}(\sin x) + c$
(B) $\sin x - 2 (\sin x)^{-1} + c$
(C) $\sin x - 2 (\sin x)^{-1} - 6\tan^{-1}(\sin x) + c$
(D) $\sin x - 2 (\sin x)^{-1} + 5\tan^{-1}(\sin x) + c$
(E) None of these

20. If $\int_{\sin x}^{1} t^2 f(t)dt = 1 - \sin x$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{3}}$

(C) 3 (D) $\sqrt{3}$

(E) None of these

21. The value of the integral $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}} dx$ is

(A) $\pi/4$ (B) $\pi/2$

(C) π (D) All of these

(E) None of these

22. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

(A) 1 (B) 2

(C) $2\sqrt{2}$ (D) 4

(E) None of these

23. The area bounded by the parabolas $y = (x+1)^2$ and $y = (x-1)^2$ and the line $y = 1/4$ is

(A) 4 sq. units (B) $\frac{1}{6}$ sq. units

(C) $\frac{4}{3}$ sq. units (D) $\frac{1}{3}$ sq. units

(E) None of these

24. If for a real number y, [y] is the greatest integer less than or equal to y, then the value of the integral $\int_{\pi/2}^{3\pi/2} [2\sin x]\, dx$ is

(A) $-\pi$ (B) 0

(C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

(E) None of these

25. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is

(A) $y = 2$ (B) $y = 2x$

(C) $y = 2x - 4$ (D) $y = 2x^2 - 4$

(E) None of these

26. Statement I: The number of ways distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Statement II: The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(A) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.

(B) Statement I is true, Statement II is false.

(C) Statement I is false, Statement II is true.

(D) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.

(E) None of these

27. If y = y(x) and it follows the relation x cos y + y cos x= π, then y" (0)

(A) 1

(B) -1

(C) π

(D) $-\pi$

(E) None of these

28. The scalar $\vec{A}.(\vec{B}+\vec{C})\times(\vec{A}+\vec{B}+\vec{C})$ equals:

(A) 0

(B) $[\vec{A}\ \vec{B}\ \vec{C}]+[\vec{B}\ \vec{C}\ \vec{A}]$

(C) $[\vec{A}\ \vec{B}\ \vec{C}]$

(D) All of these

(E) None of these

29. Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is

(A) the Arithmetic Mean of a and b

(B) the Geometric Mean of a and b

(C) the Harmonic Mean of a and b

(D) equal to zero

(E) None of these

30. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a}-\vec{b}, 2\vec{b}-\vec{c}, 2\vec{c}-\vec{a}\right]$=

(A) 0 (B) 1

(C) $-\sqrt{3}$ (D) $\sqrt{3}$

(E) None of these

31. **The equation of tangent to the curve $y = 2\cos x$ at $x = \frac{\pi}{4}$ is**

(A) $y - \sqrt{2} = 2\sqrt{2}\left(x - \frac{\pi}{4}\right)$

(B) $y + \sqrt{2} = \sqrt{2}\left(x - \frac{\pi}{4}\right)$

(C) $y - \sqrt{2} = -\sqrt{2}\left(x - \frac{\pi}{4}\right)$

(D) $y - \sqrt{2} = \sqrt{2}\left(x - \frac{\pi}{4}\right)$

(E) None of these

32. **The order and degree of the differential equation, $y = x\frac{dy}{dx} + \sqrt{a^2\left(\frac{dy}{dx}\right)^2 + b^2}$ are**

(A) (1, 2) (B) (2, 1)
(C) (1, 1) (D) (2, 2)
(E) None of these

33. **The shortest distance between lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. is**

(A) $\sqrt{30}$ (B) $2\sqrt{30}$

(C) $5\sqrt{30}$ (D) $3\sqrt{30}$

(E) None of these

34. **Local maximum value of the function $\frac{\log x}{x}$ is**

(A) e (B) 1

(C) $\frac{1}{e}$ (D) 2e

(E) None of these

35. **For a moderately skewed distribution, quartile deviation and the standard deviation are related by**

(A) $S.D = \frac{2}{3}$ Q.D (B) $S.D = \frac{3}{2}$ Q.D

(C) $S.D = \frac{3}{4}$ Q.D (D) $S.D = \frac{4}{3}$ Q.D

(E) None of these

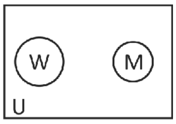
36. **Two lines $\frac{x - x_1}{l_i} = \frac{y - y_1}{m_i} = \frac{z - z_1}{n_i}$ (i = 1, 2) are perpendicular to each other if their direction ratios satisfy**

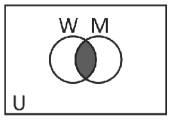
(A) $l_i = m_i = n_i$
(B) $l_1l_2 + m_1m_2 + n_1n_2 = 0$
(C) $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$
(D) All of these
(E) None of these

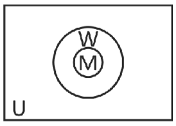
37. **The equation of the plane through (1, 2, 3) and parallel to the plane 2x + 3y – 4z = 0 is**

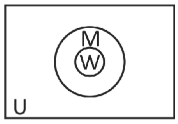
(A) $2x + 3y + 4z + 4 = 4$
(B) $2x + 3y + 4z + 4 = 0$
(C) $2x - 3y + 4z + 4 = 0$
(D) $2x + 3y - 4z + 4 = 0$
(E) None of these

38. **Which of the following Venn diagram corresponds to the statement "All mothers are women"? (M is the set of all mothers, W is the set of all women)**

(A)


(B)


(C)


(D)


(E) None of these

39. **If for two functions g and f, gof of both injective and surjective, then which of the following is true?**

(A) g and f should be injective and surjective
(B) g should be injective and surjective
(C) f should be injective and surjective
(D) None of them may be surjective and injective
(E) None of these

40. **The function $f(x) = \begin{cases} x, \text{if } & 0 \le x \le 1 \\ 1, \text{if } & 1 < x \le 2 \end{cases}$ is**

(A) Continuous at all x, $0 \le x \le 2$ and differentiable at all x, except x = 1 in the interval [0, 2]

(B) Continuous and differentiable at all x in [0, 2]

(C) Not continuous at any point in [0, 2]

(D) Not differentiable at any point [0, 2]

(E) None of these