INTERNATIONAL Olympiad of Mathematics

Presented by SILVER ZONE ISO 9001: 2000 Certified

Organized by Mathematics Olympiad Foundation New Delhi, India

SILVER ZONE FOUNDATION

Sample Paper

CLASS - 11 Duration : 60 Minutes Total Questions : 40 Maximum Marks :100

MATHEMATICS

iOM Roll Number | | | | | | | | | | | | |

Student's Name

INSTRUCTIONS

1. Write your 12 digit iOM roll number and your name on top of the question paper in the given space.
2. Filling up improper roll number may lead to unavailability of 'Result'.
3. This question paper consist of 40 questions. Each question carries equal marking of 2.5 marks each.
4. Mark your answer (A, B, C, D or E) on the Answer Sheet with HB Pencil or Black/Blue Ball point Pen.
5. This question paper contains 4 pages.
6. Do not start attempting the test paper until you are asked to do so.

Note: Return this question paper along with answer sheet

1. If $A = \sin^2 x + \cos^4 x$, then for all real x

(A) $\frac{13}{16} \le A \le 1$ (B) $1 \le A \le 2$

(C) $\frac{3}{4} \le A \le \frac{13}{16}$ (D) $\frac{3}{4} \le A \le 1$

(E) None of these

2. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

(A) -132 (B) -144

(C) 132 (D) 144

(E) None of these

3. $\lim_{x \to 2} \left(\frac{\sqrt{1 - \{\cos 2(x-2)\}}}{x - 2} \right)$

(A) equals $\sqrt{2}$ (B) equals $-\sqrt{2}$

(C) equals $\frac{1}{\sqrt{2}}$ (D) does not exist

(E) None of these

4. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is

(A) $P(C|D) \geq P(C)$ (B) $P(C|D) < P(C)$

(C) $P(C|D) = \frac{P(D)}{P(C)}$ (D) $P(C|D) = P(C)$

(E) None of these

5. If $\omega (\neq 1)$ is a cube root of unity and $(1 + \omega)^7 = A + B\omega$, then (A,B) equals to

(A) (1, 1) (B) (1, 0)

(C) (–1, 1) (D) (0, 1)

(E) None of these

6. Let A, B, C be pairwise independent events with $P(C) > 0$ and $P(A \cap B \cap C) = 0$. Then, $P(A^C \cap B^C | C)$ is equal to

(A) $P(A^C) - P(B)$

(B) $P(A) - P(B^C)$

(C) $P(A^C) + P(B^C)$

(D) $P(A^C) - P(B^C)$

(E) None of these

7. The equation of the hyperbola whose foci are (–2, 0) and (2, 0) and eccentricity is 2 is given by

(A) $-3x^2 + y^2 = 3$ (B) $x^2 - 3y^2 = 3$

(C) $3x^2 - y^2 = 3$ (D) $-x^2 + 3y^2 = 3$

(E) None of these

8. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then, the set of all possible values of a is the interval

(A) (–1, 1) (B) $(0, \infty)$

(C) $(1, \infty)$ (D) $(-1, \infty)$

(E) None of these

9. The distance of the point (1, –5, 9) from the plane x – y + z = 5 measured along a straight line x = y = z is

(A) $3\sqrt{5}$ (B) $10\sqrt{3}$

(C) $5\sqrt{3}$ (D) $3\sqrt{10}$

(E) None of these

10. The number of complex numbers z such that |z – 1| = |z + 1| = |z – i| equals

(A) 0 (B) 1

(C) 2 (D) ∞

(E) None of these

11. A person is to count 4500 currency notes. Let a_n denotes the number of notes he counts in the nth minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in AP with common difference –2, then the time taken by him to count all notes, is

(A) 24 min (B) 34 min

(C) 125 min (D) 135 min

(E) None of these

12. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is

(A) y = 0 (B) y = 1

(C) y = 2 (D) y = 3

(E) None of these

13. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and let $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then tan 2α is equal to

(A) $\frac{25}{16}$ (B) $\frac{56}{33}$

(C) $\frac{19}{12}$ (D) $\frac{20}{7}$

(E) None of these

14. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is

(A) there is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

(B) there is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(C) there is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

(D) there is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

(E) None of these

15. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then

(A) A = C

(B) B = C

(C) $A \cap B = \phi$

(D) A = B

(E) None of these

16. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|z|$ is equal to

(A) $\sqrt{3}+1$ (B) $\sqrt{5}+1$

(C) 2 (D) $2+\sqrt{2}$

(E) None of these

17. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is

(A) 0 (B) 2

(C) 7 (D) 8

(E) None of these

18. A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is $\frac{1}{2}$. Then, the length of the semi-major axis is

(A) $\frac{8}{3}$ (B) $\frac{2}{3}$

(C) $\frac{4}{3}$ (D) $\frac{5}{3}$

(E) None of these

19. The conjugate of a complex number is $\frac{1}{i-1}$. Then, that complex number is

(A) $-\frac{1}{i-1}$ (B) $\frac{1}{i+1}$

(C) $-\frac{1}{i+1}$ (D) $\frac{1}{i-1}$

(E) None of these

20. How many numbers can be formed from the digits 1, 2, 3, 4 when the repetition is not allowed?

(A) $^4P_4$

(B) $^4P_3$

(C) $^4P_1 + {}^4P_2 + {}^4P_3$

(D) $^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4$

(E) None of these

21. Match the column in the table given below:

S.No.	Sets		Roster Form
1.	{x:x^2-3=0 and x is a rational number}	**a.**	{4}
2.	{x: x is an even prime number}	**b.**	{-5, 5}
3.	{x: 3<x<5, x is a natural number}	**c.**	{*}
4.	{x: x^2 =25, and x is an odd integer}	**d.**	{2}

(A) 1 – c, 2 – d, 3 – a, 4 – b
(B) 1 – d, 2 – c, 3 – a, 4 – b
(C) 1 – c, 2 – a, 3 – d, 4 – b
(D) 1 – b, 2 – d, 3 – a, 4 – c
(E) None of these

22. Angle between the lines 2x – y –15 = 0 and 3x + y + 4 = 0 is
(A) 60° (B) 180°
(C) 90° (D) 45°
(E) None of these

23. If the line x + 2by + 7 = 0 is a diameter of the circle $x^2 + y^2 – 6x + 2y = 0$, then b =
(A) –1 (B) 3
(C) 5 (D) – 5
(E) None of these

24. If a, b, c are in G.P, then
(A) a^2, b^2, c^2 are in G.P
(B) $a^2(b + c)$, $c^2(a+b)$, $b^2(a + c)$ are in G.P
(C) $\frac{a}{a+b}, \frac{b}{c+a}, \frac{c}{a+b}$ are in G.P
(D) All of these
(E) None of these

25. The distance between the lines 3x – 2y = 1 and 6x + 9 = 4y is
(A) $\frac{1}{\sqrt{52}}$ (B) $\frac{11}{\sqrt{52}}$
(C) $\frac{4}{\sqrt{13}}$ (D) $\frac{6}{\sqrt{13}}$
(E) None of these

26. If the equation $ax^2 + 2hxy + by^2 = 0$ represents two lines $y = m_1x$ and $y = m_2x$, then
(A) $m_1 + m_2 = \frac{2h}{b}$ and $m_1m_2 = -\frac{a}{b}$
(B) $m_1 + m_2 = \frac{2h}{b}$ and $m_1m_2 = \frac{a}{b}$
(C) $m_1 + m_2 = \frac{2h}{b}$ and $m_1m_2 = –ab$
(D) $m_1 + m_2 = \frac{-2h}{b}$ and $m_1m_2 = \frac{a}{b}$
(E) None of these

27. How many words can be formed using the letter A thrice, the letter B twice and the letter C thrice?
(A) 500 (B) 560
(C) 580 (D) 520
(E) None of these

28. The triangle formed by the points (0, 7, 10), (–1, 6, 6), (–4, 9, 6) is
(A) Equilateral
(B) Isosceles
(C) Right angled
(D) Right angled isosceles
(E) None of these

29. Argument and modulus of $\frac{1+i}{1-i}$ are respectively
(A) $\frac{-\pi}{2}$ and 1 (B) $\frac{\pi}{2}$ and $\sqrt{2}$
(C) $\frac{\pi}{2}$ and 1 (D) 0 and $\sqrt{2}$
(E) None of these

30. X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements; how many elements does $X \cap Y$ have?
(A) 23 (B) 5
(C) 15 (D) 18
(E) None of these

31. If $\frac{3x+4}{x^2-3x+2} = \frac{A}{x-2} - \frac{B}{x-1}$, then (A, B) =

(A) (7, 10)

(B) (10, 7)

(C) (10, –7)

(D) (–10, 7)

(E) None of these

32. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \text{----- } + C_nx^n$, then the value of $C_0+2C_1+3C_2+ \text{ ---- } + (n+1)C_n$ will be

(A) $(n + 2)2^{n-1}$

(B) $(n + 1)2^n$

(C) $(n + 1)2^{n-1}$

(D) $(n + 2)2^n$

(E) None of these

33. The mid point of the line joining the points (–10, 8) and (–6, 12) divides the line joining the points (4, –2) and (–2, 4) in the ratio of:

(A) 1 : 2 internally

(B) 1 : 2 externally

(C) 2 : 1 internally

(D) 2 : 1 externally

(E) None of these

34. If $f(a) = 2, f'(a) = 1, g(a) = -1; g'(a) = 2$, then

$\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$ is equal to

(A) 3

(B) 5

(C) 0

(D) – 3

(E) None of these

35. Two towns A and B are 60 km apart. A school is to be built to serve 150 students in town A and 50 students in town B. If the total distance to be travelled by all 200 students is to be as small as possible, then the school should be built at

(A) town B

(B) 45 km from town A

(C) town A

(D) 45 km from town B

(E) None of these

36. If ${}^{n-1}C_r = (k^2 - 3){}^nC_{r+1}$, then $k \in$

(A) $(-\infty, -2]$

(B) $[2, \infty)$

(C) $[-\sqrt{3}, \sqrt{3}]$

(D) $(\sqrt{3}, 2]$

(E) None of these

37. One angle of an isosceles Δ is 120° and radius of its incircle = $\sqrt{3}$. Then the area of the triangle in sq. units is

(A) $7 + 12\sqrt{3}$

(B) $12 - 7\sqrt{3}$

(C) $12 + 7\sqrt{3}$

(D) 4π

(E) None of these

Direction for questions no. 38 to 40. Read the given passage carefully and answer the questions that follow.

A, B, C, D, E, F are members of a family. They are engineer, stenographer, doctor, draughtsman, lawyer and judge (not in order). A, the engineer is married to the lady stenographer. The judge is married to the lawyer. F, the draughtsman is the son of B and brother of E. C, the lawyer is the daughter-in-law of D. E is the unmarried doctor. D is the grandmother of F. There are two married couples in the family.

38. What is the profession of B?

(A) Judge

(B) Lawyer

(C) Draughtsman

(D) Cannot be determined

(E) None of these

39. Which of the following is/are a couple/ couples?

(A) AD only

(B) BC only

(C) Both AD and BC

(D) Both AC and BD

(E) None of these

40. What is the profession of D?

(A) Judge

(B) Stenographer

(C) Doctor

(D) Cannot be determined

(E) None of these