

Model Question Paper-II with effect from 2016-17

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15MAT41

Fourth Semester B.E.(CBCS) Examination Engineering Mathematics-IV

(Common to all Branches)

Time: 3 Hrs

Max.Marks: 80

**Note: Answer any FIVE full questions, choosing at least ONE question from each module.
Use of statistical tables allowed.**

Module-I

1. (a) Solve $\frac{dy}{dx} = x^2 y^2 + 1$, $y(0) = 1$ using Taylor's series method considering up to fourth degree terms and, find the $y(0.1)$ **(05 Marks)**
- (b) Use Runge - Kutta method of fourth order to solve $10 \frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$, to find $y(0.2)$.
(Take $h = 0.2$). **(05 Marks)**
- (c) Given that $\frac{dy}{dx} = x(1 + y^2)$ and $y(1) = 2$, $y(2.1) = 1.2330$, $y(2.2) = 1.5480$, & $y(2.3) = 1.9790$
find $y(1.4)$, using Adam-Bashforth predictor-corrector method. **(06 Marks)**

OR

2. (a) Solve the differential equation $\frac{dy}{dx} = -xy^2$ under the initial condition $y(0) = 2$ by using modified Euler's method at the point $x = 0.1$. Perform three iterations at each step, taking $h = 0.05$. **(05 Marks)**
- (b) Use fourth order Runge - Kutta method, to find $y(0.2)$, given $\frac{dy}{dx} = 3x + y$, $y(0) = 1$. **(05 Marks)**
- (c) Apply Milne's predictor-corrector formulae to compute $y(1.2)$ given **(06 Marks)**
- $\frac{dy}{dx} = 3x - 4y^2$ with
- | | | | | |
|-----|-----|--------|--------|--------|
| x | 0 | 0.3 | 0.6 | 0.9 |
| y | 1.0 | 1.3020 | 1.3795 | 1.4762 |

Module-II

3. (a) By Runge - Kutta method, solve $\frac{d^2 y}{dx^2} = x \left(\frac{dy}{dx} \right)^2 - y^2$ for $x = 0.2$, correct to four decimal places, using initial conditions $y(0) = 1$, $y'(0) = 0$. **(05 Marks)**
- (b) If α and β are two distinct roots of $J_n(x) = 0$, then prove that $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$ if $\alpha \neq \beta$. **(05 Marks)**
- (c) Express $f(x) = x^3 - 5x^2 + 14x + 5$ in terms of Legendre polynomials. **(06 Marks)**

OR

4. (a) Apply Milne’s predictor-corrector method to compute $y(0.4)$ given the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 2e^x$ and the following table of initial values:

| | | | | |
|------|---|------|------|------|
| x | 0 | 0.1 | 0.2 | 0.3 |
| y | 2 | 2.01 | 2.04 | 2.09 |
| y' | 0 | 0.20 | 0.40 | 0.60 |

(05 Marks)

(b) With usual notation, show that $J_{-1/2}(x) = \sqrt{(2/\pi x)} \cos x$.

(05 Marks)

(c) With usual notation, derive the Rodrigues’s formula viz., $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$.

(06 Marks)

Module-III

5. (a) Derive Cauchy-Riemann equation in cartesian form.

(05 Marks)

(b) evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is the circle $|z|=3$, using Cauchy’s residue theorem.

(05 Marks)

(c) Discuss the transformation $w = z^2$.

(06 Marks)

OR

6. (a) Find the analytic function whose real part is $r^2 \cos 2\theta$.

(05 Marks)

(b) State and prove Cauchy’s theorem.

(05 Marks)

(c) Find the bilinear transformation which maps the points $z = \infty, i, 0$ into the points $w = -1, -i, 1$.

(06 Marks)

Module-IV

7. (a) Derive mean and variance of the Poisson distribution.

(05 Marks)

(b) A random variable X has the following probability function for various values of x :

| | | | | | | | | |
|------------|---|-----|------|------|------|-------|--------|------------|
| $X(= x_i)$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $P(x)$ | 0 | k | $2k$ | $2k$ | $3k$ | k^2 | $2k^2$ | $7k^2 + k$ |

Find (i) the value of k (ii) $P(x < 6)$ (iii) $P(x \geq 6)$

(05 Marks)

(c) Let X be the random variable with the following distribution and Y is defined by X^2 :

| | | | | |
|------------|-----|-----|-----|-----|
| $X(= x_i)$ | -2 | -1 | 1 | 2 |
| $f(x_i)$ | 1/4 | 1/4 | 1/4 | 1/4 |

Determine (i) the distribution of g of Y (ii) joint distribution of X and Y (iii) $E(X)$, $E(Y)$, $E(XY)$.

(06 Marks)

OR

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8. (a) When a coin is tossed 4 times find, using binomial distribution, the probability of getting (i) exactly one head (ii) at most 3 heads (iii) at least 3 heads. (05 Marks)
- (b) In a normal distribution, 31% of the items are under 45 and 8% of the items are over 64%. Find the mean and standard deviation of the distribution. (05 Marks)
- (c) A fair coin is tossed thrice. The random variables X and Y are defined as follows :
 $X=0$ or 1 according as head or tail occurs on the first; $Y=$ Number of heads.
Determine (i) the distribution of X and Y (ii) joint distribution of X and Y . (06 Marks)

Module-V

9. (a) Define the terms:(i)Null hypothesis (ii)Confidence intervals (iii)Type-I and Type-II errors (05marks)
- (b) Ten individuals are chosen at random from a population and their heights in inches are found to be 63,63,66,67,68,69,70,70,71,71. Test the hypothesis that the mean height of the universe is 66 inches. ($t_{0.05} = 2.262$ for 9 d.f.). (05 marks)
- (c) Show that probability matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$ is regular stochastic matrix and find the associated unique fixed probability vector. (06 marks)

OR

9. (a) A manufacture claimed that at least 95%of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 of them were faulty. Test his claim at a significance level of 1% and 5%. (05 marks)
- (b) Explain (i) transient state (ii) absorbing state (iii) recurrent state of a Markov chain. (05marks)
- (c) Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball , find the probabilities that after three throws (i) A has the ball (ii) B has the ball and (iii) C has the ball. (06 marks)
