A

16631 **120 MINUTES**

1. A function f(x) is defined as $f(x) = |x| + |x-1| + |x-2|$. Consider the statements:
(a) f is continuous at x = 0, (b) f is continuous at x = 1, (c) f is continuous at x = 2.
Now state which of the following is correct?

A) Only (a) and (b) are true B) Only (a) and (c) are true
C) Only (b) and (c) are true D) (a) (b) and (c) are all true

2. If f and g are functions defined on the set N of natural numbers as f(n) = n+5 and g(n) = 2n for $n \in N$, then which of the following is correct?
A) $fog(n) = gof(n)$
B) $fog(n) > gof(n)$
C) $fog(n) < gof(n)$
D) Both fog and gof are not well defined functions

3. Which of the following can be considered as a binary expansion of the rational number 1/3 using the digits 0 and 1?
A) 0.01010… B) 0.01101…
C) 0.10101… D) 0.01001…

4. Given X_1 and X_2 as two non-empty sets. Suppose A_1 and A_2 are subsets of X_1 and B_1 and B_2 are subsets of X_2. Consider the statements:
(a) $(A_1 \times B_1) \cap (A_2 \times B_2) = (A_1 \cap A_2) \times (B_1 \cap B_2)$
(b) $(A_1 \times B_1) \cup (A_2 \times B_2) = (A_1 \cup A_2) \times (B_1 \cup B_2)$
(c) $(A_1 \times B_1) - (A_2 \times B_2) = (A_1 - A_2) \times (B_1 - B_2)$
Now state which of the following is correct?

A) Only (a) is true B) Only (b) is true
C) Only (c) is true D) Only (b) and (c) are true

5. Let X be a non-empty set. Consider the statements:
(a) If d_1 and d_2 are metrics on X, then $d_1 + d_2$ is also a metric on X
(b) If d_1 and d_2 are metrics on X, then $d_1 - d_2$ is also a metric on X
(c) If d is a metric on X, then for $x, y \in X$ the metric d_1 defined by $d_1(x,y) = d(x,y)/[1 + d(x,y)]$ is also a metric on X
Which of the following is correct?

A) Only (a) is true B) Only (b) is true
C) Only (c) is true D) Only (a) and (c) are true

6. Which of the following is not correct on the real line?
A) The set of all integral points on R has no limit points at all
B) The set of all integral points on R is not closed
C) The empty set $\emptyset$ is open while R is closed
D) Every Cauchy sequence in R converges to a limit in R

7. Suppose $S_1 = \{(1,2,4),(-1,3,2),(-2,0,1)\}$ and $S_2 = \{(1,2,4),(-1,3,2),(4,3,10)\}$ are two subsets of the vector space R^3. Let V_1 and V_2 be the sub-spaces generated by the vectors of S_1 and S_2 respectively. Now which of the following is correct?

A) $V_1 = V_2$ B) $V_1 \supset V_2$
C) $V_1 \subset V_2$ D) Neither $V_1 \subset V_2$ nor $V_1 \supset V_2$

8. In R^3 consider the following subsets:
(a) The plane of vectors (b_1, b_2, b_3) such that $b_1 = 1$
(b) The space of the union of a plane $\{(b_1, b_2, b_3) \in R^3 : b_1 = 0\}$ and a plane $\{(b_1, b_2, b_3) \in R^3 : b_2 = 0\}$
(c) The set of all linear combinations of two vectors $V_1 = (1,1,0)$ and $V_2 = (2,0,1)$
(d) $\{0, 0, 0\}$
Now state which of the above sets is/are not sub-space(s) of R^3

A) Only (a) B) Only (b)
C) (c) and (d) Only D) All (a), (b), (c) and (d)

9. Given (1, 1, 0, 1) and (1, 1, 1, 1) are two vectors in R^4. Then the angle between these two vectors is equal to

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{6}$

10. Consider the following sets: S_1 is the set of all 2 x 2 non-singular matrices of real numbers and S_2 is the set of all 2 x 2 singular matrices of real numbers. Then which of the following is correct?

A) Both S_1 and S_2 are vector spaces
B) Neither S_1 nor S_2 are vector spaces
C) S_1 is a vector space whereas S_2 is not
D) S_2 is a vector space whereas S_1 is not

11. Given a matrix $A = \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}$. If I is a 2 x 2 unit matrix $c_1 = -\frac{1}{6}$, $c_2 = \frac{5}{6}$, then the following two lists are given:

List – 1	List – 2
(i) $c_1 A^2 + c_2 A$	(a) A^3
(ii) $c_1 A + c_2 I$	(b) I
(iii) $c_1 A^3 + c_2 A^2$	(c) A
	(d) A^{-1}

Now state which of the following is the correct match between the items of list-1 and list-2?

A) (i) – (b), (ii) – (d), (iii) – (c)
B) (i) – (a), (ii) – (c), (iii) – (b)
C) (i) – (d), (ii) – (b), (iii) – (a)
D) (i) – (c), (ii) – (a), (iii) – (d)

12. Suppose A = $\begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}$. Then the characteristic roots of A are given by

A) 1, 2 and 9 B) 2, 3 and 7
C) 1, 1 and 10 D) 2, 2 and 8

13. Suppose A and B are two non-empty sets such that $A \cap B \neq \emptyset$. Define the sequence $\{E_n\}$ of sets by $E_n = \begin{cases} A \cup B, \ if\ n\ is\ odd \\ A \cap B, if\ n\ is\ even \end{cases}$. Then lim inf and lim sup of {En} are given in their order as

A) $A, A \cup B$ B) $B, A \cup B$
C) $A \cap B,\ A \cap B$ D) $A \cap B, A \cup B$

14. Consider the following statements:

(i) If E $= \left\{\frac{1}{3}, \frac{1}{3^2}, ...\right\}$ then the Lebesgue measure of E is $\frac{1}{2}$

(ii) If $\left(0, 1 - \frac{1}{n}\right)$, n = 1, 2, … is a sequence of intervals then the limit of the Lebesgue measure of $\left(0, 1 - \frac{1}{n}\right)$ as $n \to \infty$ is equal to 1.

Now state which of the following is correct?

A) Only (i) is true B) Only (ii) is true
C) Both (i) and (ii) are true D) Neither (i) nor (ii) is true

15. If the Stieltjes measures function G(x) is defined by

$$G(x) = \begin{cases} 0, x < 0 \\ x^2, 0 \le x \le 1 \\ 1, x > 1 \end{cases}$$

Then for the set E = $\left(-1, \frac{1}{2}\right] \subset$ R, the Lebesgue – Stieltjes measure of E is equal to

A) $\frac{1}{2}$ B) 1
C) $\frac{1}{4}$ D) 0

16. Suppose A and B are events with probabilities defined by P(A) = p, P(B) = 1 – 2p and P(AUB) = 0.68 where 0< p < 1. Then for what value of p, the events A and B are independent?

A) 0.4 B) 0.3
C) 0.2 D) 0.5

17. Let X = {1, 3, 5, …} represent the set of odd positive integers. Let $\mathcal{F}$ be the class of subsets of X. Suppose set functions are defined on $\mathcal{F}$ through those values on singleton sets of X as given below. Then which set function can be considered as one which induces a probability measure on $\mathcal{F}$?

A) $P\{i\} = \frac{1}{2^{i+1}}, i \in X$

B) $P\{i\} = \frac{1}{3^{i+1}}, i \in X$

C) $P\{i\} = \frac{1}{3^{i}}, i \in X$

D) $P\{i\} = \frac{15}{4^{i+1}}, i \in X$

18. In X = {1, 3, 5,…} we make consecutive disjoint pairs of elements and take the sum of integers in each pair and write B = {4, 12, 20, 28, ….} to represent the set of those sum of integers in each pair. Then with respect to the identified probability measure on $\mathcal{F}$ in question No. 17, what is the probability induced for the singleton sets of elements of B?

A) $P(\{4+8(i-1)\}) = \frac{15 \text{ x } 17}{(4)^{4i}}, i = 1, 2, \dots$

B) $P(\{4+8(i-1)\}) = \frac{15}{(4)^{4i+1}}, i = 1, 2, \dots$

C) $P(\{4+8(i-1)\}) = \frac{2}{3^{i+1}}, i = 1, 2, \dots$

D) $P(\{4+8(i-1)\}) = \frac{1}{2^{i+1}}, i = 1, 2, \dots$

19. A and B are two events such that P(A) = p_1, P(B) = p_2 and P(A∩B) = p_3. Now consider the following two lists of items:

List – 1		List – 2	
(i)	$P(A^c \cup B^c)$	(a)	$p_1 - p_3$
(ii)	$P(A \cap B^c)$	(b)	$(1 - p_1 - p_2 + p_3) / (1 - p_2)$
(iii)	$P(A^c \cap B^c)$	(c)	$(1 - p_3) / (1 - p_2)$
(iv)	$P(A^c / B^c)$	(d)	$1 - p_3$
		(e)	$1 - p_1 - p_2 + p_3$

Which of the following is then a correct matching?

A) (i) – (d), (ii) – (a), (iii) – (e), (iv) – (b)
B) (i) – (d), (ii) – (c), (iii) – (e), (iv) – (a)
C) (i) – (c), (ii) – (e), (iii) – (d), (iv) – (b)
D) (i) – (e), (ii) – (d), (iii) – (a), (iv) – (b)

20. Four identical slips are taken and the symbols A_1, A_2, A_3 are all written on the first slip whereas on each of the other three slips only one of A_1, A_2, A_3 is written on them according to their order without repetition of a letter in more than one slip. Suppose the four slips are shuffled, one drawn randomly and let E_i denote the event that the letter A_i is seen on the drawn slip. Consider now the statements:

(a) $P(E_1) = P(E_2) = P(E_3) = \frac{1}{2}$

(b) E_1, E_2 and E_3 are pair wise independent

(c) $P(E_1 E_2 E_3) = P(E_1) . P(E_2) . P(E_3)$

Then which of the following is correct?

A) Only (a) is true
B) Only (a) and (b) are true
C) Only (a) and (c) are true
D) (a),(b) and (c) are all true

21. There are two identical urns. First urn contains 3 black and 2 white balls whereas the second urn contains 2 black and 3 white balls. One of the urn is selected and a ball drawn at random. It was observed as a white ball. Then what is the probability that it was drawn from the second urn?

A) 2/5
B) 1/5
C) 3/5
D) 3/10

22. (X_1, X_2) is a two dimensional random variable such that X_1 is discrete with probability mass function $p(x_1) = \left(\frac{1}{2}\right)^{x_1}, x_1 = 1, 2, ...$ and X_2 is a continuous random variable with conditional pdf of X_2 given $X_1 = x_1$ has the expression $x_1(1-x_2)^{x_1-1}$over the interval (0,1). Then the unconditional distribution of the random variable X_2 is given by the pdf

A) $f_{x_2}(x_2) = 2x_2,\ 0 < x_2 < 1$

B) $f_{x_2}(x_2) = 1,\ 0 < x_2 < 1$

C) $f_{x_2}(x_2) = \frac{(log\ 2)^{-1}}{1+x_2}, 0 < x_2 < 1$

D) $f_{x_2}(x_2) = \frac{2}{(1+x_2)^2}, 0 < x_2 < 1$

23. Given a random sample of five observations from a distribution with cdf F(x). If so, identify the statistic for which the distribution function is $5[F(x)]^4 - 4\ [F(x)]^5$, from among the following?

A) First order statistic
B) Median of the sample
C) Largest order statistic
D) Fourth order statistic

24. If the pdf of two random variables are $f_1(x)$ and $f_2(x)$ with $F_1(x)$ and $F_2(x)$ as the corresponding cdf's, then state which of the following is not a pdf?

A) $\frac{f_1(x)+f_2(x)}{2}$

B) $\frac{f_1(y)}{2F_1(x)}+\frac{f_2(y)}{2F_2(x)}$, for $-\infty < y \leq x$

C) $\frac{f_1(y)}{3(1-F_1(x))}+\frac{2f_2(y)}{3(1-F_2(x))}$, $y \geq x$

D) $\frac{f_1(y)}{3F_1(x)}+\frac{2f_2(y)}{3F_2(x)}$, $y \geq x$

25. If the moment generating function of a random variable is given by $(e^{-3e^t}-1)/(e^3-1)$, then the distribution of the random variable is

A) Binomial distribution with n = 3 and p = $\frac{1}{2}$
B) Poisson distribution with $\lambda = 3$
C) Geometric distribution with p = $^1/_3$
D) Zero truncated Poisson distribution with $\lambda = 3$

26. Identify the distribution whose characteristic function $\emptyset$ (t) is equal to $\frac{e^{it}(1-e^{nit})}{n(1-e^{it})}$

A) Continuous uniform distribution
B) Discrete uniform distribution
C) Truncated geometric distribution
D) Truncated binomial distribution

27. If $\emptyset$ (t) is a characteristic function, consider the functions
(a) $\emptyset_1(t) = \overline{\emptyset(t)}$ where $\overline{\emptyset(t)}$ is the complex conjugate of $\emptyset$ (t)
(b) $\emptyset_2(t) = e^{\emptyset(t)-1}$
(c) $\emptyset_3(t) = |\emptyset(t)|^2$
Then which of the following is correct?

A) Only $\emptyset_1$ (t) is a characteristic function
B) Only $\emptyset_1$ (t) and $\emptyset_2$ (t) are characteristic functions
C) Only $\emptyset_3$ (t) is a characteristic function
D) $\emptyset_1$ (t) , $\emptyset_2$ (t) and $\emptyset_3$ (t) are all characteristic functions

28. Let X be a random variable with mean μ and standard deviation σ and consider the statements:
(a) $P(\mu - 2\sigma < X < \mu + 2\sigma) \geq \frac{3}{4}$
(b) $P(\mu - 3\sigma < X < \mu + 3\sigma) \geq \frac{8}{9}$
Now state which of the following is correct?

A) Only (a) is true
B) Only (b) is true
C) Both (a) and (b) are true
D) Neither (a) nor (b) is true

29. Let $\{X_n)$ and (Y_n) be sequences of random variables and let a and b be constants not equal to zero. In the following two lists, items in list – 2 contain results attained by items of list – 1 which are to be identified

List – 1		List - 2	
(i)	$X_n \xrightarrow{L} X,\ Y_n \xrightarrow{P} 0$	(a)	$Y_n \xrightarrow{L} X$
(ii)	$X_n \xrightarrow{L} X,\ Y_n \xrightarrow{P} b$	(b)	$X_n \xrightarrow{as} X$
(ii)	$X_n - Y_n \xrightarrow{P} 0,\ X_n \xrightarrow{L} X$	(c)	$X_n Y_n \xrightarrow{P} 0$
(iv)	$X_n \xrightarrow{P} a,\ Y_n \xrightarrow{P} b$	(d)	$X_n + Y_n \xrightarrow{L} X+b$
		(e)	$X_n / Y_n \xrightarrow{P} a/b$

Now which of the following is the correct matching in terms of results arrived due to the items in list – 1

A) (i) – (e), (ii) – (c), (iii) – (d), (iv) – (b)
B) (i) – (c), (ii) – (d), (iii) – (a), (iv) – (e)
C) (i) – (c), (ii) – (e), (iii) – (d), (iv) – (a)
D) (i) – (d), (ii) – (b), (iii) – (e), (iv) – (a)

30. Let $\{X_i\}$be a sequence of iid Cauchy random variables with location parameter μ and scale parameter σ. Consider now the statements:

(a) $\overline{X}_n$ has an asymptotic normal distribution
(b) M_n the median of the first n random variables has an asymptotic normal distribution

Now state which of the following is correct?

A) Only (a) is true
B) Only (b) is true
C) Neither (a) nor (b) is true
D) Both (a) and (b) are true

31. If P(r; n, p) is the pmf of a binomial distribution with parameters n and p, then for what values of r one may observe $P(r-1; n, p) < P(r; n, p)$?

A) $r < (n+1)p$
B) $r > (n+1)p$
C) $r < \frac{n}{2}$
D) $r > \frac{n}{2}$

32. If X is a random variable with pmf given by

$$P(r,p) = \begin{cases} p(1-p)^r, & r = 0,1,2,\ldots;\ 0 < p < 1 \\ 0, & \text{otherwise} \end{cases}$$

Then the conditional probability $P(X \geq 10 / X \geq 5)$ is equal to

A) $(1-p)^5$
B) $(1-p)^4$
C) $(1-p)^6$
D) p^5

33. The moment generating function of negative binomial distribution with pmf $P(x, r, p) = \binom{-r}{x} p^r (-q)^x$, x = 0, 1, 2, …; 0 < p < 1, q = 1 – p is given by

A) $\left[\frac{p}{1+qe^t}\right]^r$ B) $\left[\frac{1-qe^t}{p}\right]^r$

C) $\left[\frac{p}{1-qe^t}\right]^r$ D) $\left[\frac{1+qe^t}{p}\right]^r$

34. Let X be a random variable with standard exponential distribution Define $T = 1 - e^{-X}$. Then the mean and variance of T are given by

A) 1 and 1 B) 1 and $\frac{1}{2}$

C) $\frac{1}{2}$ and $\frac{1}{12}$ D) $\frac{1}{2}$ and 1

35. The cumulative distribution function of a random variable is given by

$$F_X(x) = 1 - \sum_{j=0}^{r-1} e^{-\lambda x} \frac{(\lambda x)^j}{j\,!}, \text{ x > 0 and r a positive integer}$$

Then the distribution of the random variable is known as

A) Poisson distribution
B) Exponential distribution
C) Gamma distribution
D) Rayleigh distribution

36. Let $\overline{X}$ be the mean of n iid Cauchy random variables with location parameter μ and scale parameter σ. Then the distribution of $\overline{X}$ is

A) Cauchy distribution with location parameter μ and scale parameter $\frac{\sigma}{\sqrt{n}}$
B) Cauchy distribution with location parameter n μ and scale parameter σ
C) Cauchy distribution with location parameter μ and scale parameter σ
D) Cauchy distribution with mean μ and standard deviation $\sigma/\sqrt{n}$

37. If X is distributed as uniform over the interval (–a, a), for a > 0 then the distribution of Y = |X| is

A) Uniform over (0, 2a)
B) Uniform over (0, a)
C) Right triangular over (0, a)
D) Triangular over (0, a)

38. If X_1 and X_2 are independent standard exponential random variables, then the pdf of $Y = \frac{X_1}{X_2}$ is

A) $f(y) = \frac{1}{(1+y)^2}, \ y > 0$

B) $f(y) = \frac{2}{\pi(1+y^2)}, \ y > 0$

C) $f(y) = \frac{1}{\beta\left(\frac{1}{2}, \frac{1}{2}\right)} \frac{y^{\frac{1}{2}-1}}{(1+y)}, y > 0$

D) $f(y) = 1, \ 0 < y < 1$

39. If X_1 , X_2, X_3, X_4, X_5 are iid standard normal variables, then the median of these random variables is

A) Symmetrically distributed about zero
B) Positively skewed
C) Negatively skewed
D) Distributed with bimodals

40. X_1 , X_2, X_3, X_4, X_5 are 5 independent observations drawn from a distribution with pdf $f(x) = \frac{1}{2} e^{-x/2}, x > 0$. Consider the following two lists

List – 1	List – 2
(i) Distribution of $T = \sum_{i=1}^{5} X_i$	(a) Chi-square distribution with 5 df
(ii) Distribution of $U = \frac{4(X_1 + X_2 + X_3)}{6(X_4 + X_5)}$	(b) F–distribution with (3, 2) df
(iii) E(T)	(c) Chi-square distribution with 10 df
(iv) E(U)	(d) 10
	(e) 2
	(f) F–distribution with (6, 4) df

Now state which of the following is the correct match?

A) (i) – (a), (ii) – (b), (iii) – (d), (iv) – (e)
B) (i) – (c), (ii) – (f), (iii) – (d), (iv) – (e)
C) (i) – (a), (ii) – (b), (iii) – (e), (iv) – (d)
D) (i) – (a), (ii) – (f), (iii) – (e), (iv) – (d)

41. If X_1 , X_2, X_3, X_4, X_5, X_6 are six independent standard normal N(0, 1) random variables, then what is the distribution of $T = \frac{1}{3}\left(\frac{X_1}{X_2} + \frac{X_3}{X_4} + \frac{X_5}{X_6}\right)$?

A) The distribution is $N\left(0, \frac{1}{3}\right)$ B) The distribution is N (0, 1)

C) The distribution is C (0, 1) D) The distribution is $C\left(0, \frac{1}{3}\right)$

(C(0, 1) is standard Cauchy)

42. Suppose X_1 , X_2, …, X_{10} are 10 independent observations drawn from $N(0, \sigma^2)$. Then which of the following has a student's t-distribution with 5 df?

A) $\frac{X_1 + X_2 + X_3 + X_4 + X_5}{\sqrt{X_6^2 + X_7^2 + X_8^2 + X_9^2 + X_{10}^2}}$ B) $\frac{\sqrt{5}\,(X_1 + X_2 + X_3 + X_4 + X_5)}{\sqrt{X_6^2 + X_7^2 + X_8^2 + X_9^2 + X_{10}^2}}$

C) $\frac{X_1 + X_2 + X_3 + X_4 + X_5}{\sqrt{5\,(X_6^2 + X_7^2 + X_8^2 + X_9^2 + X_{10}^2}}$ D) $\frac{5\,(X_1 + X_2 + X_3 + X_4 + X_5)}{\sqrt{X_6^2 + X_7^2 + X_8^2 + X_9^2 + X_{10}^2}}$

43. Suppose $F_{xy}(x, y)$ is a bivariate cdf with marginal cdf's $F_x(x)$ and $F_y(y)$. Now consider the statements:
(a) $F_{xy}(x, y) \geq F_x(x) + F_y(y) - 1$ (b)) $F_{xy}(x, y) \leq \sqrt{F_x(x)\, F_y(y)}$
Then which of the following is correct?

A) Only (a) B) Only (b)
C) Both (a) and (b) D) Neither (a) nor (b)

44. A bivariate distribution of a random variable (X, Y) is derived from a base line distribution with cdf F(x) and pdf f(x) and is defined by the bivariate density

$$f_{r,s:n}^{(x,y)} = \frac{n!}{(r-1)!(s-r-1)!(n-s)!} [F(x)]^{r-1} [F(y) - F(x)]^{s-r-1} [1 - F(y)]^{n-s} f(x) f(y)$$

where $-\infty < x < \infty$ and n,r,s are positive integers such that $1 \leq r < s \leq n$. Then which of the following is the marginal pdf of X?

A) $\frac{n!}{(s-1)!(n-s)!} [F(x)]^{s-1} [1-F(x)]^{n-s} f(x)$

B) $\frac{n!}{(r-1)!(n-r)!} [F(x)]^{r-1} [1-F(x)]^{n-r} f(x)$

C) $n\, [F(x)]^{n-1} f(x)$

D) $n[1-F(x)]^{n-1} f(x)$

45. A bivariate distribution is defined by the following pdf:

$$f(x,y) = \frac{x}{a\,\theta^2} e^{-\left(\frac{y}{a}+1\right)\frac{x}{\theta}} \quad , x > 0, y > 0$$

Consider now the following statements on the marginal pdf's $f_1(x)$ and $f_2(y)$:

(a) $f_1(x) = \frac{1}{\theta} e^{-x/\theta}, x > 0$

(b) $f_2(y) = \frac{1}{a}\left[\left(\frac{y}{a}\right)+1\right]^{-2}, \; y > 0$

Now state which of the following is correct?

A) Both (a) and (b) are correct
B) Only (a) is correct
C) Only (b) is correct
D) Neither (a) nor (b) is correct

46. If X_1 and X_2 are two observations drawn from N (μ, 1), then for what values of a and b, $aX_1 + bX_2$ is sufficient for μ?
A) a and b are such that a + b = 1
B) a and b are such that a = b
C) a and b can assume any real values
D) a and b are positive reals

47. Let M_n be the median of a random sample of size n drawn from the distribution with pdf

$f(x, \theta) = \frac{1}{\pi} \frac{1}{1+(x-\theta)^2}$, $-\infty < x < \infty$. Consider the statements - on $\{M_n\}_{n=3}^{\infty}$:

(a) M_n is unbiased and consistent estimator of θ
(b) M_n is consistent and asymptotically and normally distributed estimator of θ
(c) M_n is a sufficient and consistent estimator of θ

Now which of the above statement(s) is (are) correct?

A) (a) and (b)
B) (b) and (c)
C) Only (a)
D) All (a), (b), and (c)

48. Suppose T_1 and T_2 are two unbiased estimators of a parameter θ such that var (T_1) = 2, var (T_2) = 4, correlation between T_1 and T_2 is $\frac{1}{2}$. Then for what values of C_1 and C_2 one can consider $C_1 T_1 + C_2 T_2$ as the best linear unbiased estimator of θ based on T_1 and T_2?

A) $C_1 = \frac{1}{2},\ C_2 = 2$

B) $C_1 = 1, C_2 = 1$

C) $C_1 = \frac{4-\sqrt{2}}{2\left(3-\sqrt{2}\right)},\ C_2 = \frac{2-\sqrt{2}}{2\left(3-\sqrt{2}\right)}$

D) $C_1 = \frac{2-\sqrt{2}}{2\left(3-\sqrt{2}\right)},\ C_2 = \frac{4-\sqrt{2}}{2\left(3-\sqrt{2}\right)}$

49. Suppose $X_{1:n}, X_{2:n}, \ldots, X_{n:n}$ are the order statistics of a random sample of size n drawn from a uniform distribution over (θ, θ+2). Consider the statements given below on an estimator?

$$T = \frac{X_{1:n} + X_{n:n}}{2} - 1$$

(a) T is a method of moment estimator of θ

(b) T is an unbiased estimator of θ

(c) T is the MLE of θ

(d) T is UMVUE of θ

Now state which of the above statements are true.

A) (a) and (d) B) (b) and (d)

C) All of (a), (b), (c) and (d) D) (b) and (c)

50. In a genetic trial a total of N progenies are classified into four classes AB, Ab, aB, ab with frequencies n_1, n_2, n_3 and n_4 respectively. Suppose the expected proportions with which an individual is classidied in the classes AB, Ab, aB, ab are $\frac{p}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{p}{2}$ respectively, then the MLE of p is given by

A) $\frac{n_1 + n_2}{N}$ B) $\frac{n_3 + n_4}{N}$

C) $\frac{n_1 + n_4}{N}$ D) $\frac{n_2 + n_3}{N}$

51. Out of n observations drawn from a Bernoulli distribution with p as the probability of success, it was observed that n_1 of the observations have value 0 and $n - n_1$ of the observations have the value 1. Then the minimum chi-square method estimate of θ is

A) $\frac{n - n_1}{n}$ B) $\frac{n_1}{n}$ C) $\frac{n_1\left(n - n_1\right)}{n^2}$ D) $\frac{n_1^2}{n^2}$

52. X_1 , X_2 are 2 independent observations drawn from U(0, θ). Based on these observations one has to carry out a test of $H_0 : \theta = 1$ against $H_1 : \theta = 2$ by the test function

$$\Phi(x_1, x_2) = \begin{cases} 1, \text{if } x_1 + x_2 \geq 0.75 \\ 0, \quad \text{otherwise} \end{cases}$$

Consider the statements:

(a) Size of the test $\frac{23}{32}$ (b) Power of the test $\frac{107}{128}$

Now state which of the following is correct?

A) Only (a) is true
B) Only (b) is true
C) Both (a) and (b) are true
D) Neither (a) nor (b) is true

53. State which of the following are true with the most Powerful Test ϕ_α of the Neyman-Pearson Lemma?

(a) Power of the test ϕ_α < size of the test ϕ_α

(b) If there is a sufficient statistic then ϕ_α can be defined in terms of T

(c) If $\alpha_1 < \alpha_2$ then $\phi_{\alpha 1}(x) < \phi_{\alpha 2}(x)$ for almost all x

A) Only (a) and (b)
B) Only (a) and (c)
C) Only (b) and (c)
D) All (a), (b) and (c)

54. For a data collected from a population, a two parameter logistic distribution was fitted and thereby the following table of observed and expected frequencies obtained

Class Numbers	1	2	3	4	5	6
Observed frequency	2	6	8	10	7	2
Expected frequency	2	5	7	14	5	2

Now the possible values of the degrees of freedom and χ^2-test statistic for testing the goodness- of- fit are given below

Serial No-i	1	2	3
Degrees of freedom di	1	3	5
χ^2- test statistic Ti	$\frac{74}{245}$	$\frac{10}{49}$	$\frac{69}{49}$

Then which of the following is the correct choice of degrees of freedom di and the statistic Ti?

A) (d_3, T_1)
B) (d_3, T_3)
C) (d_2, T_1)
D) (d_1, T_2)

55. A sample of 10 observations are given below:

1.2, 2.8, 1.7, 1.4, 1.8, 2.3, 1.9, 3.0, 2.4, 1.5

To test if the given observations are drawn randomly, each observation is compared with the sample median to affix + or – sign on them according as it exceeds or not exceeds the median. Then consider the following two lists of items

	List – 1		List – 2
(i)	No. of runs of + and – symbols ‘d’	(a)	6
(ii)	E (d)	(b)	$\frac{20}{9}$
(iii)	Var (d)	(c)	5
		(d)	$\frac{9}{20}$

Which of the following is the correct match?

A) (i) – (c), (ii) – (a), (iii) – (b)
B) (i) – (a), (ii) – (c), (iii) – (d)
C) (i) – (a), (ii) – (c), (iii) – (b)
D) (i) – (c), (ii) – (d), (iii) – (b)

56. Given the following 10 observations drawn from a distribution:

0.21, 0.58, 0.36, 0.43, 0.14, 0.72, 0.69, 0.27, 0.85, 0.48. Then in order to test the null hypothesis H_0 : The given sample is drawn from the uniform distribution over (0, 1), the value of Kolmogoror – Smirnov statistic is equal to

A) 0.52
B) 0.15
C) 0.53
D) 0.10

57. Suppose $\overline{X}$ is the mean of a large sample of size n drawn from a distribution with pdf

$$f(x,\theta) = \begin{cases} \theta, e^{-\theta x}, & x > 0, 0 > 0 \\ 0, & \text{otherwise} \end{cases}$$

Then a 95% confidence interval for θ is

A) $\left(\bar{X} - \frac{1.96}{\sqrt{n}},\ \bar{X} + \frac{1.96}{\sqrt{n}}\right)$

B) $\left(\frac{1}{\bar{X}} - \frac{\sqrt{n}}{1.96},\ \frac{1}{\bar{X}} + \frac{\sqrt{n}}{1.96}\right)$

C) $\left(\frac{1}{\bar{X}\left[1+\frac{1.96}{\sqrt{n}}\right]},\ \frac{1}{\bar{X}\left[1-\frac{1.96}{\sqrt{n}}\right]}\right)$

D) $\left(\frac{\bar{X}}{\left[1+\frac{1.96}{\sqrt{n}}\right]},\ \frac{\bar{X}}{\left[1-\frac{1.96}{\sqrt{n}}\right]}\right)$

58. Suppose $X_1, X_2 \ldots, X_n$ are the observations of a random sample of size n from a Bernoulli distribution with θ as the probability of success. Then the posterior Bayes estimator of θ (1 – θ) with respect to a uniform prior distribution is

A) $\frac{(\sum Xi)\ (n - \sum Xi)}{(n+2)(n+1)}$

B) $\frac{\{\sum Xi+1\}\ \{(n+1) - \sum Xi\}}{(n+3)(n+2)}$

C) $\bar{X}$

D) $\frac{(\sum Xi+1)\ (n - \sum Xi - 1)}{(n+2)(n+3)}$

59. Given distribution with pdf given by

$$f(x) = \begin{cases} 2(1-x), & 0 < x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Suppose 0.7500, 0.1900, 0.3600, 0.9375 and 0.6975 are five random numbers selected from the interval (0,1]. Then which of the following shall be considered as a correspondingly generated random sample size of 5 from the above distribution?

A) 0.50, 0.10, 0.20, 0.75, 0.45
B) 0.25, 0.01, 0.40, 0.5625, 0.2025
C) 0.25, 0.81, 0.64, 0.0625, 0.3025
D) 0.75, 0.19, 0.36, 0.9375, 0.6975

60. In stratified sampling with L stratums with h-th stratum size N_h, stratum weight $W_h = N_h/N$, $N = \sum_{h=1}^{L} N_h$, stratum variance S_h^2 for h = 1, 2, …, L which of the following characteristics of the stratified sample mean $\bar{y}_{st}$ with h-th stratum sample size n_h and sampling fraction $f_h = n_h / N_h$ are true?

(a) $V(\bar{y}_{st}) = \sum_{h=1}^{L} W_h^2 \frac{S_h^2}{n_h} (1 - f_h)$

(b) Unbiased estimate of $V(\bar{y}_{st})$ is $\frac{1}{N^2} \sum_{h=1}^{L} N_h (N_h - n_h) \frac{s_h^2}{n_h}$ where s_h^2 is the variance of units selected from the h-th stratum to obtain $\bar{y}_{st}$

(c) With proportional allocation $n_h = \frac{n N_h}{N}$, unbiased estimate of $V(\bar{y}_{st})$ is $\frac{1-f}{n} \sum_{h=1}^{L} W_h\ s_h^2$ where $f = \frac{n}{N}$

A) Only (a) and (b)
B) Only (a) and (c)
C) Only (b) and (c)
D) All (a), (b), and (c)

61. With usual notation in ratio method of estimation which of the following is/are true?

(a) $V(\hat{Y}_R) = (1-f)\frac{Y^2}{n}\left(\frac{S_y^2}{\overline{Y}^2} + \frac{S_x^2}{\overline{X}^2} - \frac{2S_{yx}}{\overline{X}\,\overline{Y}}\right)$

(b) If the sample size n is large $\hat{Y}_R$ has smaller variance than that of $\hat{Y} = N\bar{y}$ if $\rho > \frac{Coefficient\ of\ variation\ of\ xi}{2(Coefficient\ of\ variation\ of\ yi)}$

A) Only (a) is true B) Only (b) is rue
C) Neither (a) nor (b) is true D) Both (a) and (b) are true

62. In linear regression estimates with usual notations, consider the following statements?

(a) With simple random sampling, for the regression parameter assigned with a given value b_0, the linear regression estimate $\overline{y}_{LR} = \overline{y} + b(\overline{X} - \overline{x})$ is unbiased estimator of $\overline{Y}$

(b) The variance of Y_{LR} is equal to $\frac{1-f}{n}(S_y^2 - 2b_0\, S_{yx} + b_0^2\, S_x^2)$

(c) $V(\overline{y}_{LR})$ is minimum when $b_0 = \frac{S_x^2}{S_{yx}}$

Now state which of the above statements are correct?

A) (a) only B) (b) only
C) (a) and (b) only D) All (a), (b), and (c)

63 $(Y, A\theta, \sigma^2 I)$ is a linear model in which Y is n x1 vector, θ is k x1 vector, A is a matrix of nxk constants, σ^2 is a scalar and I is the unit matrix of order n. Below is given an assertion (A) and reason (R).
Assertion: A – Every parametric function bθ is estimable
Reason:R – Rank of the matrix is k
Now state which of the following is correct?

A) Both A and R are true and R is the correct explanation of A
B) Both A and R are true, but R is not the correct explanation of A
C) A is true but R is false
D) Both A and R are false

64. In a linear model $(Y, A\theta, \sigma^2 I)$, given that bθ is estimable. Consider the statements:
(a) Rank of (A, b)= Rank of (A)
(b) Rank of $(A$A,b) = Rank of (A)
(c) bθ is invariant for all solutions of θ of the equation AAθ = AY
Now which of the above statement(s) is(are) correct?

A) Only (a) B) Only (a) and (b)
C) Only (a) and (c) D) All (a), (b), and (c)

65. Suppose the observational equations of an experiment is given by $Y_{ij} = \mu + \alpha_i + bX_{ij} + e_{ij}$, $j = 1, 2, \ldots, n_i$; $i = 1, 2, .., k$ and $n = \sum_{i=1}^{k} n_i$, wherein μ, α_i, b are constants, Y_{ij} and X_{ij} are the values of the variable of primary interest and that of a concomitant variable and e_{ij}'s are errors, then under $H_o : \alpha_1 = \alpha_2 = \ldots = \alpha_k = \alpha$, the estimate of b is given by

A) $\dfrac{\sum_{ij} X_{ij} Y_{ij} - \sum_{i} \frac{X_{i.}Y_{i.}}{n_i}}{\sum_{ij} X_{ij}^2 - \sum_{i} \frac{X_{i.}^2}{n_i}}$

B) $\dfrac{\sum_{ij} X_{ij} Y_{ij} - \frac{X_{..}Y_{..}}{n}}{\sum X_{ij}^2 - \frac{X_{..}^2}{n}}$

C) $\dfrac{\sum_{i} \frac{X_{i.}Y_{i.}}{n_i} - \frac{X_{..}Y_{..}}{n}}{\sum_{i} \frac{X_{i.}^2}{n_i} - \frac{X_{..}^2}{n}}$

D) $\dfrac{\sum_{ij} X_{ij} Y_{ij} + \sum_{i} \frac{X_{i.}Y_{i.}}{n_i} - 2\frac{X_{..}Y_{..}}{n}}{\sum_{ij} X_{ij}^2 + \sum_{i} \frac{X_{i.}^2}{n_i} - 2\frac{X_{..}^2}{n}}$

66. Consider the following 4 x 4 Latin squares

L_1 :

C	B	D	A
B	C	A	D
D	A	C	B
A	D	B	C

L_2 :

ϑ	β	δ	α
δ	α	ϑ	β
α	δ	β	ϑ
β	ϑ	α	δ

L_3 :

b	c	a	d
a	d	b	c
d	b	c	a
c	a	d	b

Consider the following assertion and reason

Assertion A: L_1, L_2 and L_3 are mutually orthogonal Latin squares

Reason R: L_1 & L_2 are orthogonal, L_1 & L_3 are orthogonal and L_2 & L_3 are orthogonal

Now state which of the following is correct?

A) Both A and R are true and R is the correct explanation of A
B) Both A and R are true, but R is not the correct explanation of A
C) A is true but R is false
D) Both A and R are false

67. In a RBD with 3 blocks and 5 treatments one observation is missing as per the details given below

Block No	Treatments				
	1	2	3	4	5
1	4	6	6	5	8
2	5	7	-	6	9
3	6	8	7	7	10

Then state which of the following statements is / are correct?

(a) Estimate of the missing cell is 8.75
(b) SE of the mean of third treatment of $\sigma/\sqrt{2}$
(c) SE of the mean of fifth treatment of $\sigma/\sqrt{3}$

A) (a) Only
B) (a) and (b) Only
C) (a) and (c) Only
D) All (a), (b), and (c)

68. The following is a design with 9 treatments and 12 blocks

Block No	Treatments		
1	1	2	3
2	4	7	8
3	5	6	9
4	1	4	5
5	2	6	7
6	3	8	9
7	1	7	9
8	2	5	8
9	3	4	6
10	1	6	8
11	2	4	9
12	3	5	7

Consider the following statements:

(a) The error d*f* of the design is 16
(b) The efficiency of this design relative to a comparable RBD is larger than one
(c) SE of difference between two treatment means is $\sigma\sqrt{2/3}$

Which of the following is then true?

A) (a) and (b) Only
B) (a) and (c) only
C) (b) and (c) Only
D) All (a), (b), and (c)

69. In a 2^4 experiment with factors A, B, C and D some interactions are confounded into the blocks and the principal block of the design is {(1), abc, ad, bcd}. Now which of the following interactions given in

(a) BC (b) AD (c) ABD (d) ACD (e) ABCD

are altogether confounded?

A) (a), (b) and (e) B) (b), (c) and (d)

C) (c), (d) and (e) D) (a), (c), and (d)

70. If the same set of interactions are confounded in 4 repetitions of the experiment and each with principal block as given in question no.69, then the error d*f* in the ANOVA is

A) 45 B) 36 C) 24 D) 42

71. A multivariate random vector X is divided into X $=\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ with the corresponding division of the mean vector μ into $\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$ and that of the dispersion matrix Σ into $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$. Then if X ~ N(μ, Σ) then the conditional distribution of X_1 given $X_2 = x_2$ is

A) $N(\mu_1, \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}$

B) $N(\mu_1, \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21})$

C) $N(\mu_1 + \Sigma_{12} \Sigma_{22}^{-1} (x_2 - \mu_2), \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21})$

D) $N(\mu_1 + \Sigma_{12} \Sigma_{22}^{-1} (x_2 - \mu_2), \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21})$

72. If X follows a multivariate normal distribution N (μ, Σ), we have the following two lists of items in which $\bar{X}$ is the sample mean and A is the matrix of corrected sum of products

List – 1		List – 2	
(i)	the MLE of μ	(a)	A/N
(ii)	the MLE of Σ	(b)	$\frac{N}{N-1}$ A
(iii)	Unbiased estimate of Σ	(c)	A / (N-1)
		(d)	$\bar{X}$

Which of the following is then the correct match?

A) (i) – (d), (ii) – (b), (iii) – (a)

B) (i) – (d), (ii) – (a), (iii) – (b)

C) (i) – (d), (ii) – (a), (iii) – (c)

D) (i) – (d), (ii) – (c), (iii) – (a)

73. If R is the multiple correlation coefficient of a dependent variable with k independent variables computed based on a sample of size n, then which of the following statistic has an F-distribution?

A) $\dfrac{R^2/(k+1)}{(1-R^2)/(n-k)}$ B) $\dfrac{R^2/(k+1)}{\left(\sqrt{1-R^2}\right)/(n-k-1)}$

C) $\dfrac{R^2}{\left(\sqrt{1-R^2}\right)/(n-2)}$ D) $\dfrac{R^2/k}{(1-R^2)/n-k-1)}$

74. Suppose $\bar{X}_i$ and A_i are the mean and corrected sum of products matrix of a sample of size n_i drawn from a multivariate normal distribution N_p (μ_i, Σ), for i = 1, 2, then which of the following is taken as a F statistic for testing $H_0 : \mu_1 = \mu_2$?

A) $\frac{n_1 n_2}{n_1+n_2}\left(\bar{X}_1-\bar{X}_2\right)^1 (A_1+A_2)^{-1}\left(\bar{X}_1-\bar{X}_2\right)$

B) $\frac{n_1+n_2-p-1}{p}\frac{n_1 n_2}{n_1+n_2}\left(\bar{X}_1-\bar{X}_2\right)^1\left(\frac{A_1+A_2}{p_1+A_2-2}\right)^{-1}\left(\bar{X}_1-\bar{X}_2\right)$

C) $\frac{n_1+n_2-2}{p}\frac{n_1 n_2}{n1+n2}\left(\bar{X}_1-\bar{X}_2\right)^{-1}(A_1+A_2)^{-1}\left(\bar{X}_1-\bar{X}_2\right)$

D) $\frac{n_1+n_2-p}{p}.\frac{n_1 n_2}{n_1+n_2}\left(\overline{X_1}-\overline{X_2}\right)^1\left(\frac{A_1+A_2}{n_1+n_2-2}\right)^{-1}\left(\overline{X_1}-\overline{X_2}\right)$

75. Suppose S is a set of states contained in the set X of states of a Markov chain and $\bar{S}$ = X – S. Now consider the different statements given in the following two lists

List – 1	List – 2
(i) S is a closed set	(a) S and $\bar{S}$ are both closed
(ii) Absorbing state	(b) If all states of the Markov chain communicate each other
(iii) Reducible Markov chain	(c) If a closed set contains only one state
	(d) S is such that $P_{ij} = 0$, for all i ε S and j ε $\bar{S}$

Which of the following is then the correct match?

A) (i) – (d), (ii) – (c), (iii) – (a) B) (i) – (c), (ii) – (d), (iii) – (b)
C) (i) – (b), (ii) – (c), (iii) – (d) D) (i) – (d), (ii) – (c), (iii) – (b)

76. If P = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ is the transition probability matrix of a Markov chain, then which of the following is correct?

A) The chain is aperiodic
B) All states of the chain are of period two
C) All states of the chain are of period three
D) Periods of the state 1, 2, 3 are 3, 2, 1 respectively

77. In a Poisson input queue consider the statements regarding expected number of customers in the system – L, expected waiting time in the system – W, expected queue size – Lq and expected waiting time in the queue – Wq all under steady state conditions and under the representation of λ^{-1} as the mean inter-arrival time

(a) $L = \lambda W$ (b) $Lq = \lambda W$ (c) $Lq = \lambda W_q$ (d) $L = \lambda W_q$

Then state which of the following is correct?

A) (a) and (c) Only B) (b) and (d) only

C) (a) and (b) Only D) (c) and (d) only

78. If $X_1(t)$ and $X_2(t)$ are two independent Poisson processes with parameters a_1 and a_2 respectively. Then $P\{X_1(t) = k \mid X_1(t) + X_2(t) = n\}$ is equal to

A) $\frac{e^{-a_2 t}(a_2 t)^{n-k}}{(n-k)!}$

B) $nC_k\, p^k (1-p)^{n-k}$, $p = \frac{a_1}{a_1+a_2}$

C) $nC_k\, p_1^k (1-p_1)^{n-k}$, $p_1 = \frac{a_2}{a_1+a_2}$

D) $\frac{e^{-p_i t}(p_i t^k)}{k!}$, $p_1 = \frac{a_2}{a_1+a_2}$

79. Circular test is one of the adequacy tests for index numbers. Now state which of the following index numbers satisfy this test?

(a) Laspeyre's index number (b) Paasche's index number

(c) Fisher's index number

A) (a) and (b) Only B) (a) and (c) only

C) All (a), (b) and (c) D) None of (a), (b) and (c)

80. Given that $x_1, x_2, \ldots$ and $y_1, y_2, \ldots$, are two correlated time series data. Now consider the following correlations

(a) Correlation between the observations y_t and y_{t+k}, t = 1, 2, …, N

(b) Correlation between the observations x_t and y_{t+k}, t = 1, 2, …, N

(c) Correlation between the observations x_t and x_{t+k}, t = 1, 2, …, N

Now state the type of correlations which the above correlations represent?

A) (a) and (c) are lag correlation of lag k whereas (b) is the autocorrelation of order k

B) (a) and (c) are serial correlations of order k while (b) is the lag correlation of lag k

C) (a) and (c) are Pearson's correlations whereas (b) is the serial correlation of order k

(D) (a) and (c) are lag correlation of lag k whereas (b) is the Pearson's correlation

……………………