1. The number of mappings which are not one-one on a set $A = \{a, b, c, d\}$ is
(A) 24 (B) 256 (C) 232 (D) 16

2. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is
(A) $R - \{0\}$
(B) The open interval $(-\infty, 0)$
(C) The open interval $(0, \infty)$
(D) The closed interval $(-1, 1)$

3. If $N = 100\,!$, then $\frac{1}{\log_2 N} + \frac{1}{\log_3 N} + \dots + \frac{1}{\log_{100} N}$ is
(A) 100 (B) 2 (C) 0 (D) 1

4. Which one of the following subset in R^2 is not convex ?
(A) $\{(x, y) : x^2 + y^2 \leq 25\} \cup \{(x, y) : x^2 + y^2 = 1\}$
(B) $\{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 2\} \cup \{(x, y) : |x| \leq 2, |y| \leq 2\}$
(C) $\{(x, y) : |x| \leq 1, |y| \leq 1\} \cup \{(x, y) : 2 \leq x \leq 5, 3 \leq y \leq 5\}$
(D) $\{(x, y) : 0 \leq x \leq 2 \text{ and } y \leq x\}$

5. Let $f : R \to R$ and $g : R \to R$ be continuous functions, where R is the set of all real numbers. Then the value of the integral $\int_{-\pi/2}^{\pi/2} [f(x) + f(-x)][g(x) - g(-x)]dx$ is
(A) π (B) -1 (C) 1 (D) 0

6. If 1, $\alpha_1, \alpha_2, \dots, \alpha_{24}$ are the 25^{th} roots of unity, then $(1 - \alpha_1)(1 - \alpha_2) \dots (1 - \alpha_{24})$ is
(A) 24 (B) 25 (C) 1 (D) -1

7. The curve represented by $\text{Im}\left(\frac{1}{z}\right) = c$, where $c \neq 0$ and z is a complex variable, is
(A) a straight line
(B) a circle
(C) a rectangular hyperbola
(D) a parabola

8. If $l, m, n \in R$, $l \neq 0$, and the quadratic equation $lx^2 + mx + n = 0$ has no real roots, then
(A) $l + m + n = 0$
(B) $(l + m + n)\, n < 0$
(C) $lm + ln + mn = 0$
(D) $(l + m + n)\, n > 0$

9. The system $x + y + 2z = a_1$, $-2x - z = a_2$, $x + 3y + 5z = a_3$ has no solution if

(A) $a_3 = a_2$ and $a_1 \neq 0$
(B) $a_3 = a_2 = a_1 = 0$
(C) $a_3 = 3a_1$ and $a_2 = 0$
(D) $a_2 = -3a_1$ and $a_3 = 0$

10. If $A = \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & -1 \end{bmatrix}$ then $A^{100} - A^{50} + A^{25} - A + I$ is

(A) 0
(B) A
(C) –A
(D) I

11. The straight line $x + y = a$ touches the parabola $x^2 - x + y = 0$ if

(A) $a = 1$
(B) $a = -1$
(C) $a = 0$
(D) a takes any value

12. What points P(x, y) satisfy the inequality $x^2 + y^2 - 2x - 4y - 4 < 0$?

(A) P lies inside the ellipse with focus (1, 2) and eccentricity 2
(B) P lies outside the ellipse with focus (1, 2) and eccentricity 2
(C) P lies inside the circle of radius 3 with centre (1, 2)
(D) P lies outside the circle of radius 3 with centre (1, 2)

13. The maximum number of points of intersection of a circle and a parabola is

(A) 1
(B) 2
(C) 3
(D) 4

14. The angle between the lines whose direction cosines are (1, –1, 0) (–1, –1, –1) is

(A) 0
(B) $\pi/4$
(C) $\pi/3$
(D) $\pi/2$

15. The minimum number of points needed to determine a sphere is

(A) 4
(B) 3
(C) 2
(D) 1

16. $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dy \, dx$ is also equal to

(A) $\int_0^\infty \int_y^\infty \frac{e^{-y}}{y} \, dydx$
(B) $\int_0^\infty \int_0^y \frac{e^{-y}}{y} \, dydx$
(C) $\int_0^\infty \int_y^\infty \frac{e^{-x}}{x} \, dxdy$
(D) $\int_0^\infty \int_0^y \frac{e^{-x}}{x} \, dydx$

17. If a > 0, then the integral $\int_a^{\infty} \sin x \, dx$

(A) converges
(B) diverges
(C) neither converges nor diverges
(D) is equal to ± 1

18. $\frac{dy}{dx}$ of $y = \int_0^{x^2} \cos t \, dt$ is

(A) $2x \cos x^2$
(B) $2x \sin x^2$
(C) $2x \sin 2x$
(D) $2x \cos 2x$

19. The equation of the tangent to the curve $x = t \cos t$, $y = t \sin t$ at the origin is

(A) $y = 0$
(B) $x = 0$
(C) $x = y$
(D) $x = -y$

20. Let $f : R \to R$ be a differentiable function and $f(1) = 4$. Then the value of $\lim_{x \to 1} \int_4^{f(x)} \frac{2t}{x-1} dt$ is

(A) $f'(1)$
(B) $2f'(1)$
(C) $4f'(1)$
(D) $8f'(1)$

21. Let $\{f_n\}$ be a sequence of continuous functions on [0, 1] converging pointwise to a function f on [0, 1]. For f to be continuous on [0, 1], the uniform convergence of $\{f_n\}$ to f on [0, 1] is

(A) sufficient, but not necessary
(B) necessary, but not sufficient
(C) necessary and sufficient
(D) neither necessary nor sufficient

22. For the series $\sum_{n=1}^{\infty} \frac{e^{inx}}{n}$, x in $[0, 2\pi]$, which of the following statements hold ?

(A) The series converges uniformly on the closed interval $[0, 2\pi]$
(B) The series converges uniformly on the open interval $(0, 2\pi)$
(C) The series converges uniformly on compact subsets of $[0, 2\pi]$
(D) The series converges only at a finite number of points in $[0, 2\pi]$

23. Let $\{f_n\}$, $\{g_n\}$ be two sequences of complex valued functions on a set S, each converging uniformly on S. Then which of the following statements is not necessarily true ?

(A) $\{f_n + g_n\}$ is uniformly bounded on S
(B) $\{f_n g_n\}$ is uniformly bounded on S
(C) $\{f_n + g_n\}$ is uniformly convergent on S
(D) $\{f_n g_n\}$ is uniformly convergent on S

24. From the following sequences of functions, pick the sequence which is uniformly convergent on [0, 1].

(A) $\{x^n\}$ (B) $\{(x - 1)\, x^n\}$ (C) $\{(x + 1)x^n\}$ (D) $\{(1 + x^2)\, x^n\}$

25. If the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n z^n$ is 2, then the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n z^{n^2}$ is

(A) 2 (B) $\sqrt{2}$ (C) 4 (D) 1

26. Let e^z denote the exponential function. For $z = x + iy$ in $\mathbb{C}$, $|e^{iz}|$ has the value

(A) 1 (B) $e^{|z|}$ (C) e^x (D) e^{-y}

27. Pick the region in which there does not exist an analytic branch of the logarithm.

(A) $\{z : |z - 1| < 1\}$ (B) $\{z : 0 < |z| < 1\}$

(C) $\mathbb{C} \sim \{z : z \leq 0\}$ (D) $\mathbb{C} \sim \{ z : z \geq 0\}$

28. Suppose a function f defined on a disk D has a power series expansion on D. Then which of the following statements is false ?

(A) f is analytic on D

(B) f is infinitely many times differentiable on D

(C) f does not have a primitive in D

(D) exp {f(z)} is analytic on D

29. The function $\frac{z^6 - 1}{(z - 1)^2}$ $(z \in \mathbb{C},\ z \neq 1)$ has at $z = 1$

(A) a simple pole (B) a removable singularity

(C) a pole of order 2 (D) an essential singularity

30. Which of the following subsets of $I = [0, 1]$ has a positive Lebesgue measure ?

(A) $\{x \in I : x$ has a decimal expansion $x = a_1, a_2 \ldots$ with $a_n = 0$ for $n > 1000\}$

(B) $\{x \in I : x$ has a ternary expansion $x = \frac{a_1}{3} + \frac{a_2}{3^2} + \ldots$ with $a_n = 0$ or $1\}$

(C) $\{x \in I : x$ has a binary expansion$\}$

(D) all rational points in [0, 1]

31. Let $\alpha(x) = \frac{1}{2}$ on $\left[0, \frac{1}{2}\right]$

$= -\frac{1}{2}$ on $\left(\frac{1}{2}, 1\right]$

Then $\int_0^1 x^2 \, d\,\alpha(x)$ has the value

(A) 0 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) $-\frac{1}{4}$

32. Let f be defined on [0, n] where n is a positive integer by

$f(x) = k$ if $k - 1 < x \leq k$, $k = 1, 2, \ldots n$ and $f(0) = 0$.

Let $\alpha(x) = [x]$ be the greatest integer function. Then $\int_0^n f(x)\, d\alpha\,(x)$ has the value

(A) $n(n-1)$ (B) n^2 (C) $n(n+1)$ (D) $\frac{n}{2}(n+1)$

33. Let $\{f_n\}$ be a sequence of non-negative measurable functions on a measurable set E of IR. Suppose $f_n(x) \to f(x)$ almost everywhere on E. If

$\alpha = \int_E f(x)dx$ and $\beta = \underline{\lim} \int_E f_n(x)\, dx$, then

(A) $\alpha < \beta$ (B) $\alpha \leq \beta$ (C) $\beta \leq \alpha$ (D) $\beta < \alpha$

34. If γ is the positively oriented unit circle, then $\int_\gamma \frac{e^z}{z} dz$ has the value

(A) 0 (B) 1 (C) $2\pi i$ (D) $-2\pi i$

35. If $\gamma(t) = 1 + 2e^{it}$, $0 \leq t \leq 2\pi$, then $\frac{1}{2\pi i} \int_\gamma \frac{z^2+3}{z-2} dz$ has the value

(A) 0 (B) 1 (C) 7 (D) 5

36. If $|a| < 1$, the Mobius transformation $\frac{z-a}{1-\bar{a}z}$ maps the disk $D = \{z:|z|<1\}$ onto

(A) D
(B) a proper subset of D
(C) 2 D
(D) The upper half plane

37. Let f be analytic in the disk $\{z:|z|<1\}$ with f (0) = 0 and $|f(z)| \leq 1$ for all z in the disk. Then which of the following statements does not hold ?

(A) $\left|f\left(\frac{1}{4}\right)\right| \leq \frac{1}{4}$ (B) $|f'(0)| \leq 1$ (C) $\left|f\left(\frac{1}{2}\right)\right| > \frac{1}{2}$ (D) $\left|f\left(-\frac{1}{2}\right)\right| \leq \frac{1}{2}$

38. Let f be an entire function with $f(z) \to 1$ as $|z| \to \infty$. Then f (0) has the value

(A) 1 (B) –1 (C) 0 (D) 2

39. Let f = u + iv be analytic on the unit disk D with f(0) = 1. If v(x, y) = 2xy for x + iy in D, then u(x, y) is equal to

(A) $x^2 - y^2$ (B) $x^2 + y^2$ (C) $x^2 - y^2 + 1$ (D) $x^2 - y^2 - 1$

40. A Mobius transformation different from identity has

(A) atmost one fixed point
(B) atleast two fixed points
(C) atmost two fixed points
(D) no fixed point

41. Which of the following is an irreducible polynomial over Z_2 ?

(A) $x^3 + 1$ (B) $x^4 + x^2 + x + 1$ (C) $x^4 + x^2 + 1$ (D) $x^4 + x + 1$

42. Let f(x) and g(x) be polynomials of degree 5 over a field F. Which of the following is a possible degree of f(x) + g(x) ?

(A) 10 (B) 8 (C) 6 (D) 4

43. Which of the following is a zero divisor in the ring Z_{10} ?

(A) 3 (B) 5 (C) 7 (D) 9

44. Let D be a Euclidean domain with Euclidean valuation ε. Let a, b $\in$ D with $\varepsilon(a) = \varepsilon(b)$. Which of the following is necessarily true ?

(A) a = b
(B) ab = 1
(C) a = bc for some $c \in D$
(D) none of the above

45. Which of the following is an integral domain ?

(A) Z_4 (B) Z_5 (C) Z_6 (D) Z_{10}

46. Let $\varphi : Z \to \mathbb{Q}$ be a homomorphism of rings where Z is the ring of integers and $\mathbb{Q}$ is the ring of rationals. Suppose that $\varphi(z) \neq 0$. Then which of the following is true about Ker φ ?

(A) Ker $\varphi = \langle p \rangle$ where $\langle p \rangle$ is the ideal generated by a prime p

(B) Ker $\varphi = \langle m \rangle$ where m is a non-prime

(C) Ker $\varphi = (0)$

(D) Ker $\varphi = Z$

47. The characteristic of the field of complex numbers is
(A) 0 (B) 1
(C) 2 (D) 3

48. Let a be algebraic and b be transcendental over a field F. Then which of the following is not true ?
(A) ab is transcendental (B) $a + b$ is transcendental
(C) $a + b$ is algebraic (D) $a^2 + b^2$ is transcendental

49. The degree of the splitting field of $x^3 - 2$ over $\mathbb{Q}$ is
(A) 2 (B) 3 (C) 5 (D) 6

50. Which of the following pairs of fields are isomorphic. (Here $\mathbb{C}$ is the field of complex numbers R is the field of reals and $\mathbb{Q}$ is the field of rationals. Also x is an indeterminate)
(A) $\mathbb{C}$ and R (B) $\mathbb{Q}$ and $\mathbb{Q}\left(\sqrt{2}\right)$
(C) $\mathbb{Q}(x)$ and $\mathbb{Q}(x^2)$ (D) $\mathbb{Q}(x)$ and $R(x)$

51. Which of the following sets are linearly independent in $\mathbb{R}^3$?
(A) {(1, 2, 1), (1, 3, 1), (1, 4, 1)}
(B) {(2, 4, 2), (2, 5, 2), (2, 6, 2)}
(C) {(3, 4, 3), (3, 5, 5), (3, 6, 7)}
(D) {(3, 1, 3), (4, 1, 4), (1, 1, 2)}

52. Let V be the vector space of all polynomials of degree ≤ 5 over the reals $\mathbb{R}$.Then dimension of V is
(A) 5 (B) 6 (C) 10 (D) 12

53. Let V be the space of all polynomials of degree ≤ 3 over $\mathbb{R}$. Which of the following is a subspace of V ?
(A) $\{f(x) \in V : f(0) = 1\}$ (B) $\{f(x) \in V : f(1) = 1\}$
(C) $\{f(x) \in V : f(1) = 0\}$ (D) $\{f(x) \in V : f(1) \neq 0\}$

54. Which of the following is an eigen value of the matrix $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$?
(A) 0 (B) 2
(C) 3 (D) 4

55. Which of the following pairs of matrices are conjugates of each other ?

(A) $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$ and $\begin{bmatrix}2 & 1\\0 & 2\end{bmatrix}$ (B) $\begin{bmatrix}2 & 1\\0 & 2\end{bmatrix}$ and $\begin{bmatrix}4 & 1\\0 & 1\end{bmatrix}$

(C) $\begin{bmatrix}3 & 1\\0 & 3\end{bmatrix}$ and $\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}$ (D) $\begin{bmatrix}1 & 1\\4 & 1\end{bmatrix}$ and $\begin{bmatrix}3 & 0\\0 & -1\end{bmatrix}$

56. Which of the following is an even permutation ?

(A) (1 2 3 4) (B) (1 2 3) (2 4) (C) (1 2 3) (1 3 4) (D) (1 2) (1 3 4)

57. The number of homomorphisms from the cyclic group Z_5 to the cyclic group Z_6 is

(A) 1 (B) 2 (C) 3 (D) 4

58. The number of subgroups of order 5 in a group of order 20 is

(A) 1 (B) 2 (C) 5 (D) 6

59. Let S_5 be the symmetric group and A_5 be the alternating group on 5 symbols. Let $\varphi : A_5 \to S_5$ be a non-trivial homomorphism. Then which of the following is true ?

(A) φ is one-to-one (B) φ is onto

(C) Im φ contains odd permutations (D) Im φ is a subgroup of index 5 in S_5

60. The order of the element $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$ in the multiplicative group of non singular 3×3 matrices is

(A) 2 (B) 3 (C) 4 (D) infinite

61. Let $\mathbb{R}^3$ be the metric space with Euclidean metric. Which of the following is not a point on the unit circle in $\mathbb{R}^3$?

(A) (1,0, 0) (B) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

(C) (1, 0, 1) (D) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

62. Let C [0, 1] be the metric space of all continuous real valued functions on [0, 1]; with supremum metric. Let $z \in C$ [0, 1] be defined by z (t) = 0 for all $t \in$ [0, 1]. Which of the following belongs to the open ball of radius 1 centered at z ?

(A) $f(t) = t^2$ (B) $f(t) = 1 + t$ (C) $f(t) = \frac{t^2+1}{3}$ (D) $f(t) = \frac{t+1}{2}$

63. Let $f_n(t) = \begin{cases} \frac{1}{n} : t \le \frac{1}{n} \\ 0 : t > \frac{1}{n} \end{cases}$ be a sequence in C [0, 1]. Which of the following is true ?

(A) f_n converges to f (t) = 0
(B) f_n converges to f (t) =1
(C) f_n converges to f (t) = $\frac{1}{2}$
(D) f_n is not convergent

64. Which of the following is not a complete metric space ?
(A) $\mathbb{R}^2$ with Euclidean metric
(B) $\mathbb{R}^2$ with discrete metric
(C) C [0, 1] with supremum metric
(D) P [0, 1] of all polynomials with supremum metric

65. Let R be the set of reals, Q the set of rationals and S be the set of all irrationals. Let τ be a topology on R given by $\tau = \{R, Q, S, \phi\}$. Let A = {1} then $\overline{A} =$
(A) A (B) R (C) S (D) Q

66. Let X = {1, 2, 3, 4, 5} and $\tau = \{X, \phi, \{1, 2, 3\}, \{2, 3\}\}$. Then the interior of A = {2, 3, 4, 5} in (X, τ) is
(A) A (B) {2, 3} (C) {1, 2, 3} (D) ϕ

67. Which of the following pairs of topological spaces are homeomorphic ? All spaces have topology induced by Euclidean metric.
(A) (0, 1) and $\mathbb{R}$
(B) (0, 1) and [0, 1]
(C) [0, 1] and $\mathbb{R}$
(D) [0, 1] and [0, ∞]

68. Let X, Y be topological spaces and $f : X \to Y$ be a continuous map. Which of the following is not necessarily true ?
(A) $f^{-1}(A)$ is closed in X whenever A is closed in Y
(B) f(B) is closed in Y whenever B is closed in X
(C) $\{f(x_n)\}$ is convergent whenever (x_n) is convergent
(D) $f(\overline{A}) \subseteq \overline{f(A)}$ for all subsets A of X

69. Let X be a connected space with infinitely many points and Y be the two points discrete space {0, 1}. Let $f : X \to Y$ be continuous with f(x) = 1 for some $x \in X$. Then which of the following is true ?

(A) f(y) = 1 for all $y \in X$

(B) $f(y) \neq 1$ whenever $y \neq x$

(C) f is one-to-one

(D) f is onto

70. Let X be the two points discrete space X = {0, 1}. Let Y be a connected space with |Y|>2. Which of the following is true about X×Y ?

(A) X×Y is connected

(B) X×Y is disconnected with exactly two components

(C) X×Y is disconnected with exactly three components

(D) There is a disconnection of X×Y separating any two points z_1 and z_2

71. Let C be the field of complex numbers and A be the linear operator on the complex vector space C^2 defined by $A(x_1, x_2) = (x_2, -x_1)$. Let I be the identity operator. Then the null space of A – iI is the span of

(A) {(1, –i)}

(B) {(1, –1)}

(C) {(1, i)}

(D) {(1, 1)}

72. Let X be a normed linear space. Then a subspace Y of X is bounded iff

(A) Y = {0}

(B) Y is finite dimensional

(C) Y is infinite dimensional

(D) $Y \neq \overline{Y}$

73. Let X be the normed linear space C_{00} with norm $\| \|_\infty$. Then $\overline{X}$ is

(A) C

(B) C_0

(C) C_{00}

(D) l^∞

74. The Hilbert space in which the Legendre polynomials are orthogonal is

(A) $L^2[-\pi, \pi]$

(B) $L^2\left[-\pi/2, \pi/2\right]$

(C) $L^2[-1, 1]$

(D) $L^2[0, \infty]$

75. Let H be the complex Hilbert space of square summable sequences of complex numbers and let e_j = (0, 0, ... 0, 1, 0, ...), where 1 occurs in the j^{th} coordinate. If x = (1, 2, ..., 100, 0, 0, ...), then $\sum_{j=1}^{\infty} |\langle x, e_j \rangle|^2$ is

(A) 100

(B) 100^2

(C) 1+2+3+ ... + 100

(D) $1^2 + 2^2 + ... + 100^2$

76. Let $X = C[-1, 1]$ with L^2- innerproduct and $S = \{f \in X : f(-t) = f(t) \forall\ t \in [-1, 1]\}$. Then $S^{\perp}$ is

(A) $\{0\}$

(B) X

(C) $\{f \in X : f(t) = c\ \forall\ t \in [-1, 1]$, where c is a constant

(D) $\{f \in X : f(-t) = -f(t)\ \forall\ t \in [-1, 1]\}$

77. Let X be an innerproduct space and for $x, y \in X$, $f(x) = f(y)$ for every $f \in X'$. Then

(A) $x = y = 0$ (B) $x = y$ (C) $x \perp y$ (D) $x = -y$

78. Let H be a Hilbert space. If $x, y \in H$ are such that $||x|| = 6$, $||x + y|| = 16$ and $||x-y|| = 4$, then $||y||$ is

(A) 2 (B) 8 (C) 10 (D) 12

79. Let H be the complex Hilbert space C^3, where C is the field of complex numbers.

If a linear operator A on H is represented by the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & -i \end{bmatrix}$ with respect to the standard basis, then A* (the adjoint of A) is represented by the matrix.

(A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & i \end{bmatrix}$ (B) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & -i \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & -i \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & i \end{bmatrix}$

80. Let R^2 and R be the normed linear spaces with the Euclidean norm, where R is the field of real numbers. If $T : R^2 \to R$ is defined by $T(x_1, x_2) = x_1$ then

(A) T is bounded but not open

(B) T is open but not bounded

(C) T is bounded and open

(D) T is neither bounded nor open