2011
[December]
ECONOMICS
Mathematics for Economists

Full Marks: 75; Time: 3 hours
The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting at least **one** from each Credit
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CREDIT – I

1. (a) Use examples and diagrams to explain different types of functions used in Economics.

   5

   (b) Prove that $(A \cap B) - (A \cap C) = A \cap (B - C)$

   5

   (c) Find the length of the difference between two vectors, $V_1' = (x_1, y_1)$ *and* $V_2' = (x_2, y_2)$, using appropriate diagram.

   5

2. (a) Provide a numerical example of an Idempotent Matrix (M) which is not an Identity Matrix and examine its characteristics.

   7

   (b) What is orthogonality between two vectors? Explain the concept of orthogonality between price vector and budget line.

   5

   (c) Given $Y_1 = 3X_1 + 4X_2$; $Y_2 = 9X_1^2 - 24X_1X_2 + 16X_2^2$

   Use Jacobian determinant to verify if Y_2 is dependent on Y_1.

   3

CREDIT – II

3. Given the C.E.S. production function $Q = A\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-\frac{1}{\rho}}$ where
   $A > 0, 0 < \delta < 1, -1 < \rho \neq 0$. L and K represent labor and capital respectively.
   a. Interpret the parameters $A, \delta, and\ \rho$;
   b. Prove that the given production function is homogeneous of degree one;
   c. Prove that elasticity of substitution is constant for the given production function.

   2+3+10

4. Formulate a simple linear program of maximization. State and prove the duality theorem of linear programming.

   5+10

CREDIT – III

5. (a) Solve the following first order first degree linear differential equation having variable coefficient (u) and variable term (w) where u, y and w are functions of time:

$$\frac{dy}{dt} + uy = w$$

10

(b) Write an explanatory note on Phase Diagram.

5

6. (a) Solve: $\frac{dy}{dt} + ty = 3ty^2$

7

(b) Provide mathematical treatment to examine and explain Domar growth model.

8

CREDIT – IV

7. (a) Solve: $Y_{t+1} + aY_t = c$ where a and c are constants.

8

(b) Given the following demand and supply functions, find inter temporal equilibrium price and determine whether the equilibrium price is stable:

$$Q_{dt} = 10 - 2P_t; \; S_{st} = -5 + 3P_{t-1}$$

7

8. (a) Write an explanatory note on Cobweb model.

10

(b) Solve the following:

$$Q_{dt} = 12 - 2P_t$$
$$Q_{st} = -4 + 2P_{t-1}$$
$$P_{t+1} - P_t = -0.25(Q_{st} - Q_{dt})$$

5