

# INSTITUTE OF ACTUARIES OF INDIA

## EXAMINATIONS

03<sup>rd</sup> May 2016

**Subject CT1 – Financial Mathematics**

**Time allowed: Three Hours (10.30 – 13.30 Hrs)**

**Total Marks: 100**

### *INSTRUCTIONS TO THE CANDIDATES*

- 1. Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. Mark allocations are shown in brackets.*
- 3. Attempt all questions, beginning your answer to each question on a separate sheet.*
- 4. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

**AT THE END OF THE EXAMINATION**

**Please return your answer book and this question paper to the supervisor separately.**

- Q. 1) i)** Explain and differentiate
- a)** Static and dynamic hedging
  - b)** Arbitrage and Hedging (3)
- ii)** The current exchange rates of the INR/EUR, INR/USD, and EUR/USD pairs are 74.321, 67.5790, and 1.07980 respectively. Discuss any arbitrage opportunity that exists for a forex trader, given that a lot for each currency pair is 1,000 units. (3)
- [6]**
- Q. 2)** Write short notes on the following:
- i)** Risks faced by the investor of a swap. (2)
  - ii)** Increase in yield margins on the corporate bonds. (2)
  - iii)** Characteristics of property as an investment compared to the bonds. (2)
- [6]**
- Q. 3)** On 1<sup>st</sup> November 1985, a man was in receipt of three annuities mentioned below:
- Annuity 1: INR 200 p.a. payable annually on 1<sup>st</sup> February each year, the final payment being 1<sup>st</sup> February 2007
- Annuity 2: INR 320 p.a. payable quarterly on 1<sup>st</sup> January, 1<sup>st</sup> April, 1<sup>st</sup> July and 1<sup>st</sup> October each year, the final payment being 1<sup>st</sup> January 2002
- Annuity 3: INR 180 p.a. payable monthly on the first day of each month, the final payment being 1<sup>st</sup> August 2004
- Immediately after receiving the monthly payment due on 1<sup>st</sup> November 1985, the man requested that these three annuities be combined into a single annuity payable half-yearly on 1<sup>st</sup> February and 1<sup>st</sup> August in each subsequent year, the final payment being 1<sup>st</sup> February 2007. The man's request was granted.
- Find the amount of the revised annuity, given that it was calculated at an interest rate of 8% p.a. effective, all months are assumed to have equal length. (6)
- [6]**
- Q. 4) i)** Explain the relationship  $d = iv$  by general reasoning where  $d$  is the effective annual rate of discount and  $i$  is the effective annual rate of interest. (2)
- ii)** Given  $i = 0.08$ , Calculate
- a.**  $d^{(12)}$
  - b.**  $i^{(365)}$
  - c.**  $\delta$
  - d.**  $i^{(1/2)}$  (4)
- iii)** Derive, from the first principles, the formula for  $(Ia^{\cdot})_n$  for  $i > 0$  (2)
- [8]**

**Q. 5)**  $\delta(t)$ , the force of interest per annum at time  $t$  years is defined as:-

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \leq t \leq 5 \\ 0.06 & \text{for } 5 \leq t \leq 10 \\ 0.04 & \text{for } t \geq 10 \end{cases}$$

**i)** Using  $\delta(t)$  Derive the expression for  $v(t)$ , the present value of 1 due at time  $t$  (2)

**ii)** An investor affects a contract under which he will pay 15 premiums annually in advance into an account which will accumulate according to  $\delta(t)$  above. Each premium amounts to INR 600 and the first premium will be paid at time 0.

The investor, in return will receive either

**a)** The accumulated amount of the account one year after the final premium is paid

Or

**b)** The level annuity payable annually for eight years, the first payment being made one year after the final premium is paid.

Find the lump sum payment under option (a) using the expression derived in (i) and the amount of the annual annuity under option (b) (6)

**[8]**

**Q. 6)** An investor bought a 4-year forward contract on 1<sup>st</sup> June 2015 to buy INR 10,000 nominal of a bond. The bond has a tenure of 6 years and pays a coupon of 4.5% p.a. payable half yearly on 31<sup>st</sup> March and 30<sup>th</sup> September. Additionally the bond is expected to pay 5% lump sum every year on 1<sup>st</sup> July and final redemption at 75% of the par value on 31<sup>st</sup> March 2021. The bond is also expected to yield 8% p.a effective if purchased on 1<sup>st</sup> June 2015 and held until redemption.

**i)** Calculate the forward price of the contract, if the risk free rate of interest is 7% p.a. (5)

**ii)** Calculate the value of the contract on 1<sup>st</sup> August 2017, when the stock price is at INR 11,000 nominal value. (3)

**[8]**

**Q. 7) i)** Define the term “real rate of interest” (1)

**ii)** Assuming positive inflation, state whether the real rate of interest will be greater than the money rate of interest. (1)

**iii)** State the relationship between real and money rate of interest (1)

**iv)** Ravi has INR 100,000 in his bank account. From this account he makes quarterly payments for a loan in arrears for 5 years at the rate of INR 3,000 p.a., the payment increasing by INR 1,000 p.a. The payments are deferred for one year, i.e., the first quarterly payment is at the end of 15 months from now.

Calculate the amount remaining in the corpus at the end of 6 years from now if the bank account earns interest at the rate of 8% p.a. convertible monthly for the first 3 years and 12% p.a. convertible half - yearly for the next 3 years.

(6)  
[9]

**Q. 8)** During the period 1<sup>st</sup> January 2014 to 1<sup>st</sup> January 2015, the unit prices of two funds a “property fund” and “Equity Fund” are as mentioned below:

	Unit prices (2014)				Unit prices (2015)
	1 <sup>st</sup> January	1 <sup>st</sup> April	1 <sup>st</sup> July	1 <sup>st</sup> October	1 <sup>st</sup> January
<b>Property Fund</b>	12.4	13.1	14.8	15.8	16.4
<b>Equity Fund</b>	12.1	9.2	10.3	13.1	15.5

i) Find the time-weighted rate of return for each of the fund for the year 2014 (1)

ii) An investor bought units in the property fund on 1<sup>st</sup> January, 1<sup>st</sup> April, 1<sup>st</sup> July and 1<sup>st</sup> October 2014, and sold his units on 1<sup>st</sup> January 2015, find the yield on the complete transaction, if

a) He bought the same number of units on each date. (2)

b) He invested the same sum of money on each date. (2)

iii) Repeat the calculation in ii(a) and ii(b) above, except that the investments are done in equity fund. (4)

[9]

**Q. 9)** A loan of INR 160,000 was issued to be repaid by a level annuity certain payable annually in arrears over a term of 10 years and calculated at an interest rate of 8% p.a. The terms of the loan provided that at any time the lender could alter the rate of interest, in which case the amount of the annual repayment would be revised appropriately.

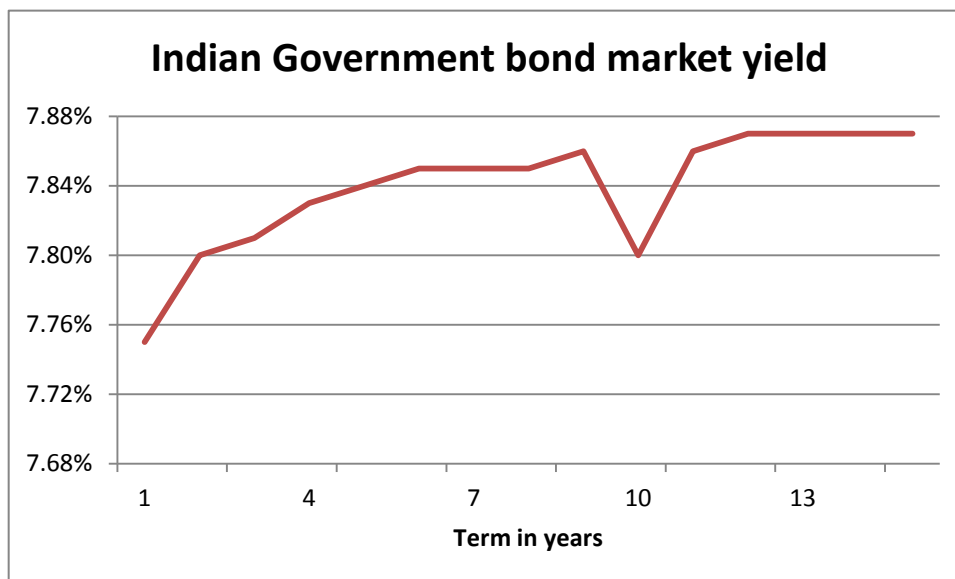
i) Find the initial amount of the annual repayment (2)

ii) Immediately after the fourth repayment, the annual rate of interest was increased to 10% p.a. Find the revised amount of the level annual repayment. (4)

iii) Immediately after the seventh repayment, the annual rate of interest was reduced to 9% p.a. There was no further change to the rate of interest. Find the final amount of the level annual repayment and the effective rate of interest paid by the borrower on the complete transaction. (4)

[10]

**Q. 10)** An Indian Government bond market (spot) yield curve is given below:



i) Describe using three relevant theories

- Interest rate variation
- term of investment
- Shape of the above yield curve.

(6)

ii) A Company issues two bonds with a coupon rate of 6% and 6.5 % per annum respectively payable annually in arrears with redemption of 4 years. These bonds are redeemed at par.

Calculate the duration of the bonds at a gross redemption yield of 8% p.a. effective.

(3)

iii) Comment on ii) above

(1)

[10]

**Q. 11)** A 3 year index linked bond was issued on 1<sup>st</sup> April 2014. The stock had a nominal coupon rate of 5% p.a. payable half yearly in arrears and a nominal redemption price of 100%. The actual coupon and the redemption payments were index linked referenced to Gold price index on the month of the payment.

An investor not subject to tax purchased INR 1 mn nominal of stock on 2<sup>nd</sup> October 2015 and was looking to hold until redemption. Following information about the gold prices and retail price index is known or expected in future:

Gold Price Index	2014	2015	2016	2017
April	25,200	---	23,200	26,900
October	---	24,800	25,300	---
Retail Price Index				
April	131	---	147	159
October	---	142	154	---

\* Figures shown in above table for period after 2<sup>nd</sup> October 2015 are best estimates of future

- i) Calculate the coupon payments and the redemption payments expected by the investor from the bond. (2)
- ii) Calculate the purchase price, the investor should pay on 1<sup>st</sup> October 2015 to achieve a real yield of 4% p.a. effective on the investment. (4)
- iii) Re-calculate purchase price the investor should pay on 1<sup>st</sup> October 2015 to achieve a real yield of 4% p.a. effective on investment when:
- a) Probability of default on maturity amount is 10%
- b) Probability of default on any coupon payment is 5% (2)
- iv) A future based on the said bond is currently priced at INR X. Comment on its price movement for a change in expectation of:
- a) Higher Gold price in future years than the current expectation.
- b) Lower future inflation level i.e. lower increase in rate of retail index than the current expectation. (2)
- [10]**

**Q. 12)** The Return on funds earned by a fund manager has a mean of 10% and a standard deviation 20%, and is independent of return earned in previous years. Each year the value of  $(1 + i_t)$  is lognormally distributed, where  $i_t$  is the return earned in the  $t^{th}$  year.

- i) Determine the lognormal distribution parameters (3)
- ii) Derive the accumulated value of return in the  $n^{th}$  year of the fund manager (3)
- iii) Determine the probability that the value of a single investment of INR 25,000 at time 0 will grow to an accumulation between INR 32,500 and INR 35,000 in 4 years' time. (4)
- [10]**

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